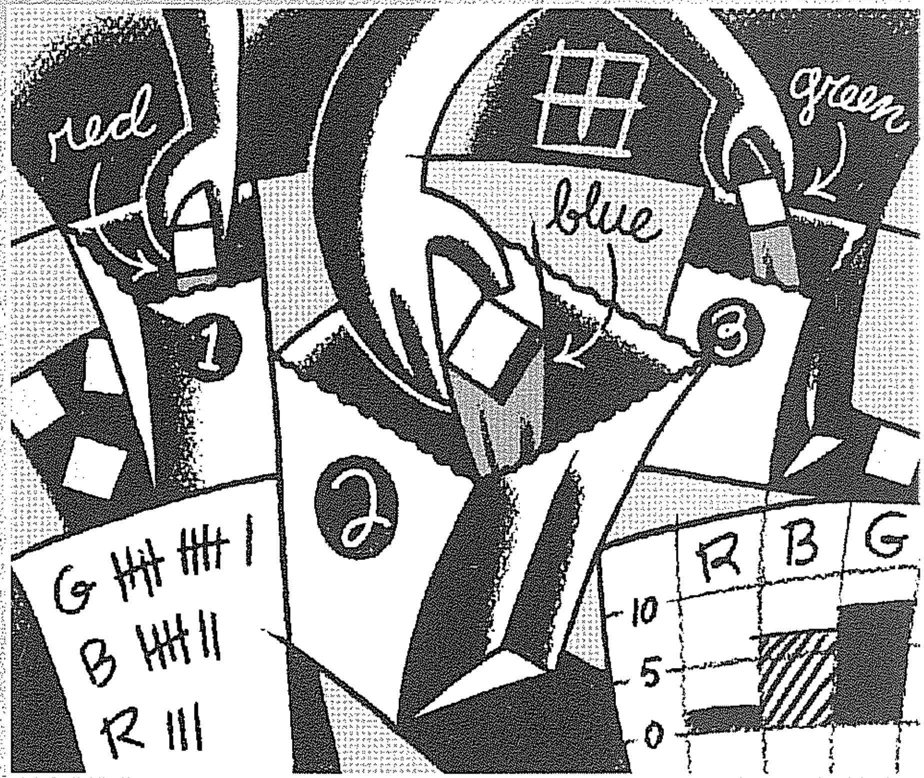
Unit VIII / Math and the Mind's Eye



Visual Encounters with Chance

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Visual Encounters with Chance

To the Teacher: Each activity will take several hours of class time, as students must conduct experiments, analyze and organize data, and reflect and write about what they discover. The activities provide an inrroduction to some of the big ideas surrounding chance, such as: making decisions and predictions under uncertainty; getting and using information frnm samples; experimental probability as compared to building theoretical models for probability experiments; and an introduction to some visual representations of data.

1 Sampling, Confidence and Probability

Samples are drawn from Hidden Sack in order to predict likely vs. unlikely proportions. Students' confidence in their predictions is examined. An area model for representing the results of a probability experiment is introduced. Comparisons between guesses, experimental probabilities and theoretical probabilities are made.

2 Identifying Like Traits by Sampling

This activity builds upon the experience of making decisions based on random samples that was begun in Activity 1. In addition, histograms are used to represent and to compare data samples. Histograms provide another convenient visual representation of data.

3 Experimental and Theoretical Evidence

The distribution of sums for rolling two dice is investigated using both experimental and theoretical evidence. The context is set within a game in which players attempt to find an optimal strategy to win.

4 Checker-A Game

Results from a binomial experiment with equally likely outcomes (odd and even rolls on a regular die) are compared to theoretical probabilities determined hy counting the possible sequences of 6 tosses of the die. This activity is a precursor for work on counting srrategies, Pascal's triangle and the binomial distribution.

5 Checker-B Game

Results from a binomial experiment with unequally likely outcomes (odd and even products of faces of two regular dice) are compared to theoretical probabilities. Comparisons are made between the Checker-A and Checker-B games to point out the differences between equally likely and unequally likely binomial experiments.

6 Cereal Boxes

Simulation by a probability experiment is a tool often used when a direct theoretical approach to a probability problem is inaccessible. The cereal box problem uses the "sample until" technique that frequently occurs in problems involving chance. Visual representation of data, such as median marks, line-plots and box-plots are introduced to get at the concepts of central tendency, range and variation.

7 Monty's Dilemma

A probability simulation in which this game can be played many times very quickly proves to be a powerful mechanism for understanding what the best strategy is—to stick or to switch.

Math and the Mind's Eye materials are intended for use in grades 4-9. They are written so teachers can adapt them to fir student backgrounds and grade levels. A single activity can be extended over several days or used in part.

A catalog of Math and the Mind's Eye materials and teaching supplies is available from The Math Learning Center, PO Box 3226, Salem, OR 97302, 503-370-8130. Fax: 503-370-7961.

Math and the Mind's Eye

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Unit VIII • Activity 1

Sampling, Confidence and Probability

O V E R V I E W

Samples are drawn from Hidden Sack in order to predict likely vs. unlikely proportions. Students' confidence in their predictions is examined. An area model for representing the results of a probability experiment is introduced. Comparisons between guesses, experimental probabilities and theoretical probabilities are made.

Prerequisite Activities

Previous work with fractions, ratios and percent will be helpful, as will the area model of fractions or Unit IV, Activities 3 and 10, which deal with fraction bars.

Materials

Colored tile (red, blue, green), accompanying grid activity sheets, paper sacks, calculators, tape, colored pens or pencils, butcher paper.

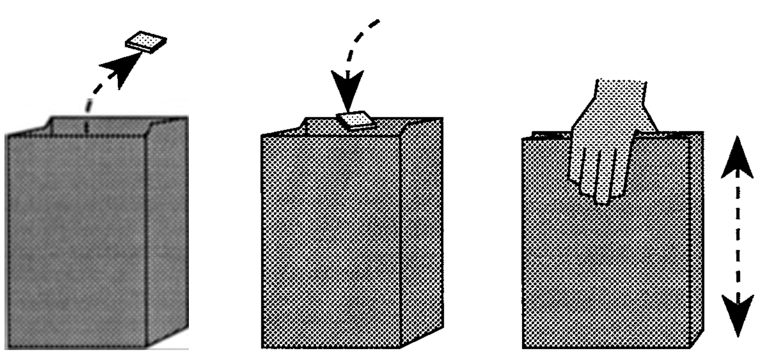
Actions

Part I: First Experiment

1. Prepare a paper sack that contains 12 red, 6 blue and 2 green tile for each group of four students and for yourself. Do not tell students the contents of these sacks or allow them to look at the contents until the very end of the activity.

2. Hold up your sack. Tell the students the sacks contain identical collections of 20 colored tile and their task is to predict the (approximate) number of each color without looking in the sacks. However, predictions must be based *only* on information gathered from the following sampling procedure:

Shake the sack. Draw one tile at a time from the sack; be sure to put that tile back in the sack before making the next draw. Demonstrate this procedure, shaking the sack before and after each draw.



Draw 1 tile from sack. — Return tile to sack. — Shake sack after each draw.

Comments

1. Brown lunch-size sacks work well. There is nothing special about the distribution 12-6-2 for the colors. You may use other distributions for the hidden sack. However, at least one color should be quite rare.

The major goal of this activity is to use sampling to predict the contents of the sacks. It is therefore important not to reveal the contents of the sack.

2. Be sure the class understands this sampling procedure. Each tile must be replaced before another is drawn. This ensures that draws are always made from the original contents of the sack. It is also *very* important to shake up the sack to mix the tile after each pull.

Note that this procedure is to be the only source of information about the contents of the sack.

Actions

3. Distribute the sacks and ask the students not to look inside. Ask each group to devise and then write down a plan for predicting the contents of its sack.

4. Tell the students, "Carry out your plan. Keep a record of your results so they may be shared with the class. Use your results to predict what is in your sack."

5. Pass out butcher paper and pens; ask each group to make a poster of its results. When they are ready, ask the groups to share their predictions and to tell how confident they are about them. Put up the posters around the room.

Comments

3. This is an example of a probability experiment. Each group is free to devise its own plan within the constraints of the sampling procedure in Action 2. Groups will have to decide how many draws to make and how to organize the information. Most teams will probably plan to pull a certain number of samples from the sack and keep a record. For example, some may decide to make exactly 20 pulls because there are 20 tile in the sack (but, of course, there is nothing special about 20).

4. Groups will likely record their results in different ways. Some may list the number of red, blue and green tile they drew. Others may use tallies or bar graphs. You may wish to discuss these methods (see also Action 6 for another representation of results).

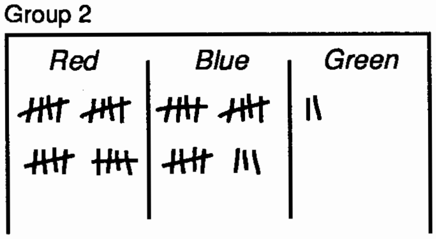
5. Groups 1, 2 and 3 below show some sample student responses.

Group 1

Red	Blue	Green
34	15	1

"Our group got mostly red. We feel pretty sure there are more than twice as many red tile as blue ones and there aren't very many green tile. Our prediction: 14 red, 5 blue, 1 green."

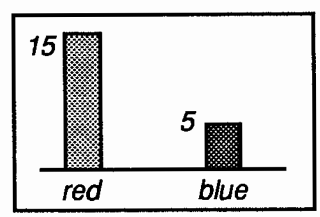
Group 2



"In 40 tries, we got red 20 times, blue 18 times and green twice. We decided to make a prediction by cutting our results in half, so we think the sack contains 10 red, 9 blue and 1 green."

"We feel pretty certain there are about as many red as blue, though we're not sure if there's more of one of these colors. we feel good about our guess for green—there can't be many green tile in the sack!"

Group 3



"Here is graph of our results. We got 3 times as many red as blue in 20 draws. So, our guess: 15 red and 5 blue tile are in the sack."

"We're not sure, though. Maybe we should have drawn more times. We do feel good about predicting more red than blue."

Actions

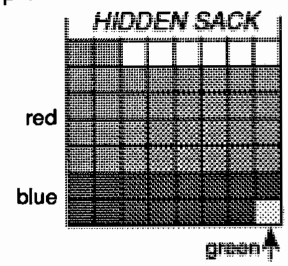
6. Tell the students that an area model can also be used to represent the results of a probability experiment. Display Transparency A and demonstrate how the results of this experiment can be pictured by shading the grid squares with appropriate colors. Ask the groups to make grid representations of their experimental results and to label them "Hidden Sack".

7. Ask the groups to attach the area model grids for their experiments to their posters. Pass out copies of Activity Sheet VIII-1-A and put up a transparency of it. Ask the groups to examine their results for the number of tile of each color and and to answer the questions on Activity Sheet VIII-1-A. Discuss.

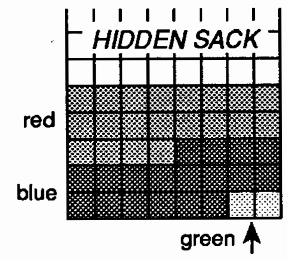
Comments

6. Some sample grid representations are pictured here.

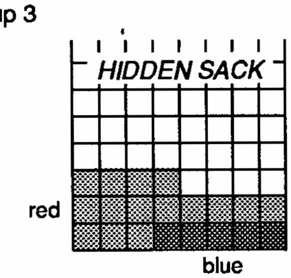
Group 1



Group 2



Group 3

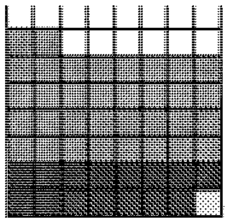


7. It is important that students have an opportunity to reflect about the confidence they have in their predictions. This confidence may be influenced by their observations of other groups' results. Variation in the results (which might be considerable if small numbers of draws were made) may cause groups to question their predictions. On the other hand, they may be very confident of some observations, such as "the sack contains more red tile than green ones" or "there are no yellow tile."

Actions

8. Ask the teams to compute the percent of each color that is on their grid. These percentages are examples of experimental probabilities. Have the teams post their probabilities on their posters. Record at the overhead the range of probabilities for each color on Transparency B.

Group 1

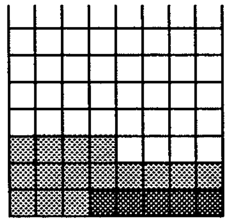


red = $\frac{34}{50}$ = 68%

blue = $\frac{15}{50}$ = 30%

green = $\frac{1}{50}$ = 2%

Group 3



red = $\frac{15}{20}$ = 75%

blue = $\frac{5}{20}$ = 25%

green = $\frac{0}{20}$ = 0%

Comments

8. Note that percentages are calculated out of the total number of trials for each group, which may be different sizes at this point. The range of experimental probabilities for each color spans from the lowest to the highest percentage obtained by the groups (such as 25% to 45% for blue).

Actions

9. Discuss the above percentages. How did the students obtain them? What do they indicate about the contents of the sack? What is a range of reasonable color mixtures in the sack? Are there any further observations about the contents students can make confidently?

Comments

9. The range of percentages gives an indication of likely and unlikely compositions in the sack. For example, if every group obtained more than 50% red, it is likely that more than half the tile in the sack are red and unlikely that only 6 of the tile are red. Similar statements can be made about each color. The students may become somewhat confident about making statements such as: "There are probably more that 10 red tile." "There are only a few green tile." "I'd be surprised if someone opened the sack and found more green tile than red ones!"

Thus, even though the exact number of each color is still unknown, one can begin to feel confident about a "range of reasonable" contents for the Hidden Sack. This might be compared with the students' confidence in Action 5.

Actions

10. Ask the groups to discuss the following question and then report back to the class: "What could be done to improve the experiment so the class can get a better idea of what is in the sack?"

Comments

10. Here are three possible suggestions: draw more samples; group all the class data together; have each group do the experiment the same number of times, so results are more uniform.

Discuss the advantages of drawing larger samples. In the extreme, a sample of only 1 or 2 tile would not give much information about the contents of the sack; 20 or 30 give a better picture. We should obtain an even better idea from 50 to 100 tile. The main idea is that larger samples are less likely to misinform us about the contents of the sack.

Continued next page.

Actions

Part II: Second Experiment

11. Have each group generate a sample of 40 draws from their sacks and make a grid paper diagram of their results. Have them also compute the percentage of red, blue and green tile that turn up in their sample. Label the grids "Hidden Sack—40 draws" and post the results next to their previous grid. Record the range of probabilities for each color on Transparency B. Discuss the results.

Comments

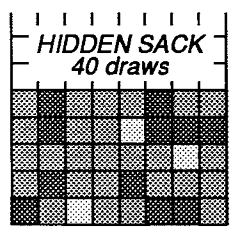
10. *Continued.* Also, discuss the advantages of having each group draw the same-sized sample from the sack. This gives a better basis of comparison across groups. If one group only drew 10 tile and another drew 50, the percentages of red, blue and green tile in their samples may be quite different. For example, a group that draws only 10 tile may get 8 red tile and 2 blue and not discover a green one.

11. There is nothing special about 40, except that $1/40 = .025$, so the experimental probabilities will be terminating decimals. The intent is to have each group draw a large, uniform sample size for comparison purposes (see Comment 10).

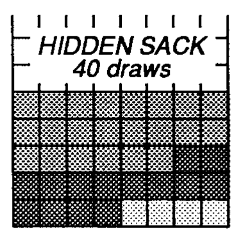
The range of probabilities from this (perhaps larger) sample can be compared to the initial range obtained in Action 8.

Some groups may color their grids as they go so as to display the sequence in which the tile were drawn. These grids are helpful for observing "runs" of colors and for representing randomness in a visual manner. You might discuss questions such as, "What was the longest run of red tile?" or "How many draws did it take before the first green tile appeared?"

Other groups may tally their draws and construct grids that show the draws of each color contiguously.



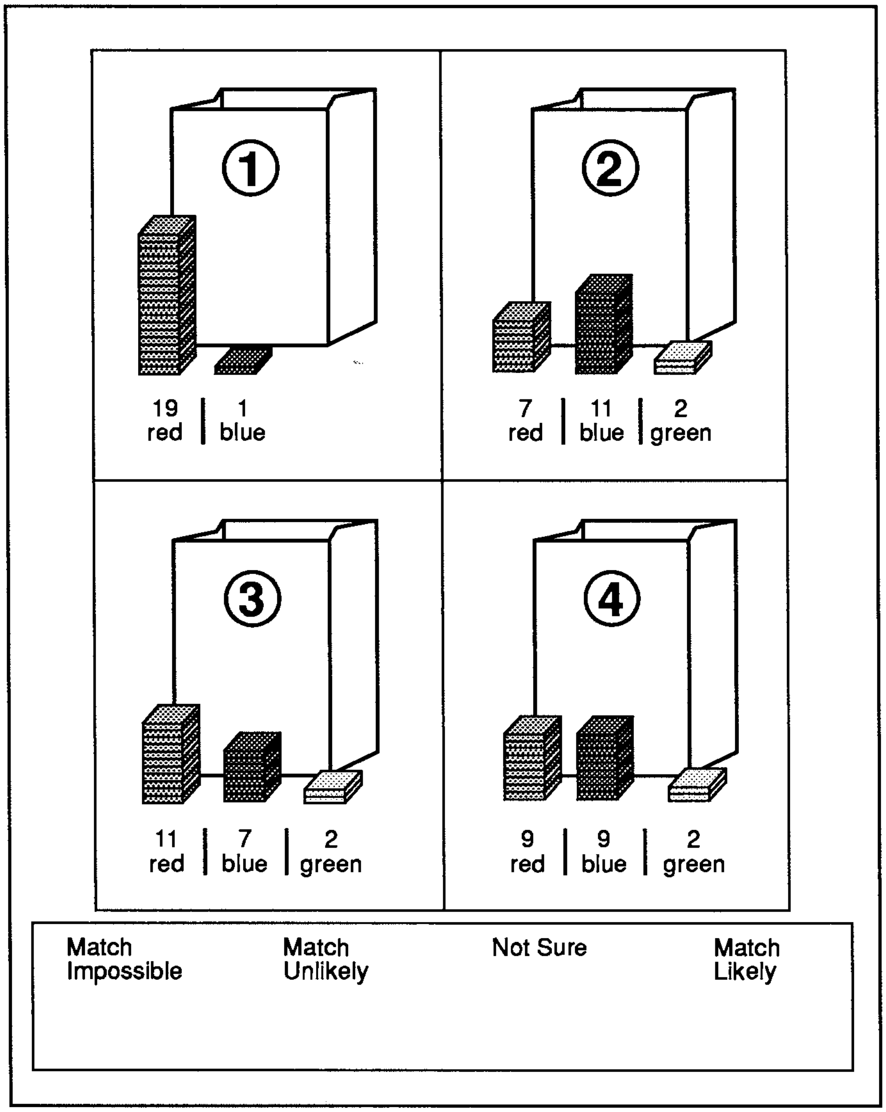
red: $\frac{26}{40} = 65\%$
blue: $\frac{11}{40} = 27.5\%$
green: $\frac{3}{40} = 7.5\%$



red: $\frac{22}{40} = 55\%$
blue: $\frac{14}{40} = 35\%$
green: $\frac{4}{40} = 10\%$

Actions

12. Put up Transparency C:



Discuss the following questions about each sack: "Is it possible the contents of this sack match the 'hidden' contents of the original sacks? Is it reasonable to think they match?"

Have the students decide where to place each sack. One possible placement:

Match Impossible 1	Match Unlikely 2	Not Sure 4	Match Likely 3
19 R, 1 B	*7 R, 11 B, 2 G*	*9 R, 9 B, 2 G*	*11 R, 7 B, 2 G*

Comments

12. The distinction between "impossible", "possible" and "likely" is an important idea in this action. Ask students to defend their position on the four sacks. The students may want to carry out an experiment with one or more of these sacks to check how close the drawn samples come to the contents of the Hidden Sack.

Actions

Comments

13. Distribute copies of Activity Sheet VIII-1-B and put up a transparency of it. Ask the groups to propose their own hypothetical sacks of 20 tile with color distributions that fall into each of the categories shown on the activity sheet. Solicit suggestions from the groups and record the results on Activity Sheet VIII-1-B at the overhead. Discuss. Ask the students to explain their reasoning.

13. Without knowledge of the exact contents of original sack, the samples drawn in Action 11 could have been drawn from a range of color distributions, some of which are more likely than others. It is important to be aware of these possibilities and to reflect about the likelihood of each.

One possible direction for the discussion is to invite students to express their tolerance for "possible" as opposed to "likely" and "impossible" contents for the original sack. Where will they draw the line? For example, a composition of 1 red, 1 blue and 18 green is possible, but highly unlikely. A composition of 10 blue and 10 green is definitely impossible, since there is evidence of red tile. There is a distinction between *mathematically* possible and *belief* in what is possible.

Some sample suggestions from students

Match Impossible	Match Unlikely	Not Sure	Match Likely
19 R, 1 B	*7 R, 11 B, 2 G*	*9 R, 9 B, 2 G*	*11 R, 7 B, 2 G*
20 B	*5 R, 14 B, 1 G*	*10 R, 9 B, 1 G*	*13 R, 5 B, 2 G*
10 B, 10 G	*10 R, 5 B, 5 G*		*11 R, 6 B, 3 G*

14. Ask each group to make its final choice for a sack from the "Match Likely" category of Activity Sheet VIII-1-B. Pass out copies of Activity Sheet VIII-1-C and ask the students to complete it.

14. Some choices for a Match Likely sack will already be listed on the transparency of Activity Sheet VIII-1-B. The groups might choose one of these or they might choose a different one of their own.

15. When the students have completed Activity Sheet VIII-3-C, reveal the contents of the sack. Discuss. Ask the students to compute the theoretical probabilities of each color (the percentages) in the Hidden Sack and to compare these to their experimental probabilities from the 40-draw experiment. How close was their experiment to the true probabilities?

15. It is important to emphasize with the students that there is a range of "right" answers for their Match Likely sacks. It doesn't matter if they are off a tile or two from the exact contents. The important part is to conduct an experiment that can come reasonably close to predicting the contents of the sack. For example, no group would predict 6 red, 10 blue and 4 greeen at this point, as their 40-draw data contradict this.

Activity Sheet VIII-1-A

Name ______________________________

1.What can you say for sure about the contents of the sack?

2. What else can you say about the contents of the sack?

3. At this point, what do you think is likely to be in the sack?

4. How confident are you of your answer to Question 3?

Name ____________________

Propose some 20 tile sacks that you feel could fit in each of these categories. List the contents—number of red, blue, green tile—for each sack you propose under the category.

Match Impossible	Match Unlikely	Not Sure	Match Likely

Name ____________________________

1. Your group has chosen a sack containing 20 tile that you believe matches the contents of “Hidden Sack”. Let’s call your sack the “Likely Sack” since you feel it is likely to produce a match. Now imagine drawing a sample of 40 tile from the Likely Sack. Based on your knowledge of what is in this sack:

a. How many times would you expect each color to be drawn?

Red ______ Blue ______ Green ______

Explain the reasoning you used to arrive at your answers.

b. Write the percentage of each color of tile in your Likely Sack.

% Red ______ % Blue ______ % Green ______

These percentages are your “theoretical probabilities” of selecting each color of tile from your Likely Sack.

Name ______________________________

2. Recall the percentages of Red, Blue and Green tile that you obtained from your 40-draws experiment (these may be attached to your posters). These were your "experimental probabilities". Use these and your answers from part 1 above to complete the chart below.

Contents of Your Likely Sack			(*from Part 1b*) Your Theoretical Probabilities (Likely Sack)			(*from Poster Grid*) Your Experimental Probabilities—40 Draws (Hidden Sack)		
R	**B**	**G**	**R**	**B**	**G**	**R**	**B**	**G**

3. Compare your "experimental" and "theoretical" probabilities. How close are they? Do you feel that your group's Likely Sack matches the contents of the Hidden Sack? Why or why not?

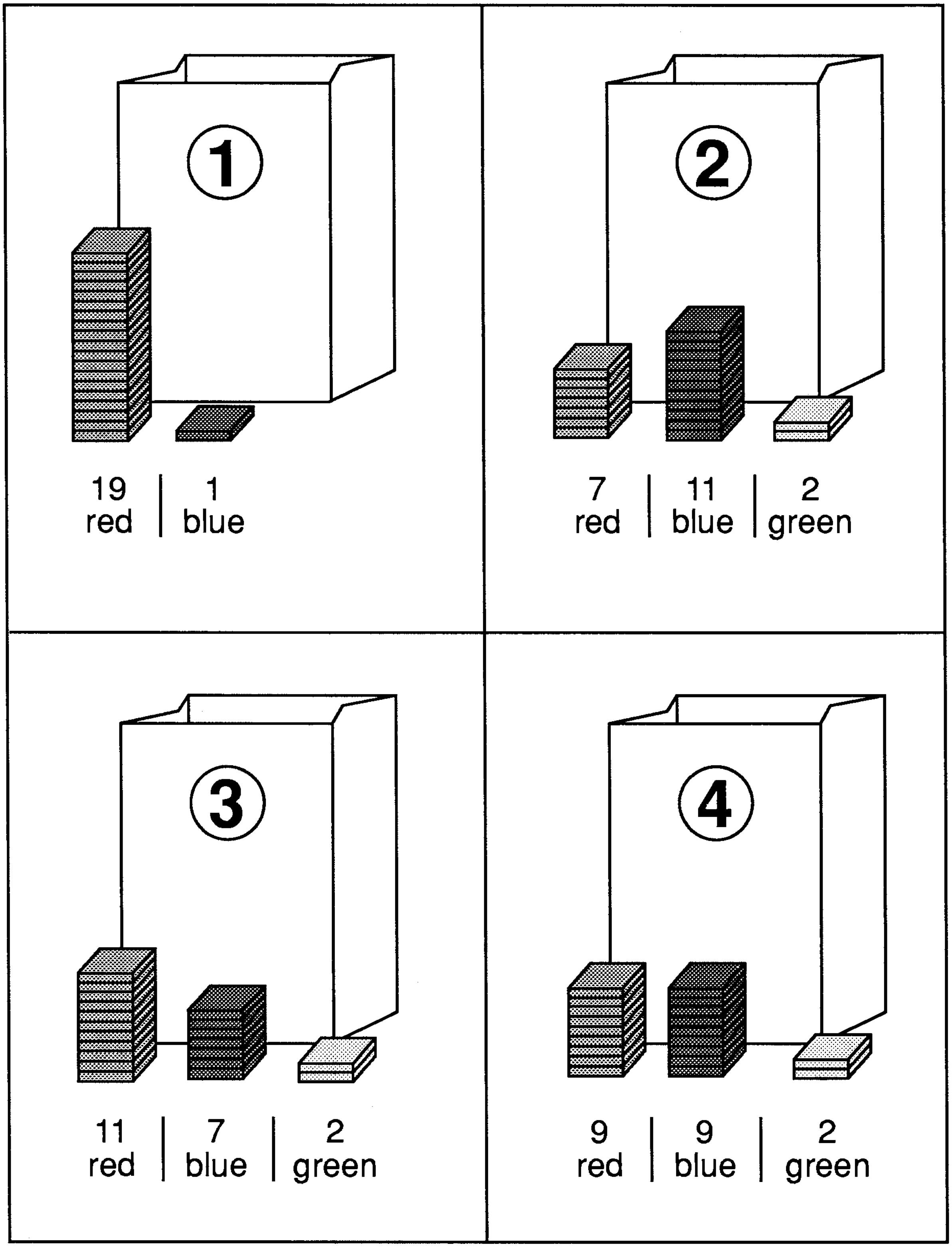
VIII-1 Master for Transparency A

Range of Experimental Probabilities: Hidden Sack

Red	Blue	Green

Range of Experimental Probabilities: Hidden Sack—40 Draws

Red	Blue	Green



Match Impossible	Match Unlikely	Not Sure	Match Likely

VIII-1 Master for Transparency C

Identifying Like Traits by Sampling

OVERVIEW

This activity builds upon the experience of making decisions based on random samples that was begun in Activity 1. In addition, bar graphs are used to represent and to compare data samples. Bar graphs provide another convenient visual representation of data.

Prerequisite Activities

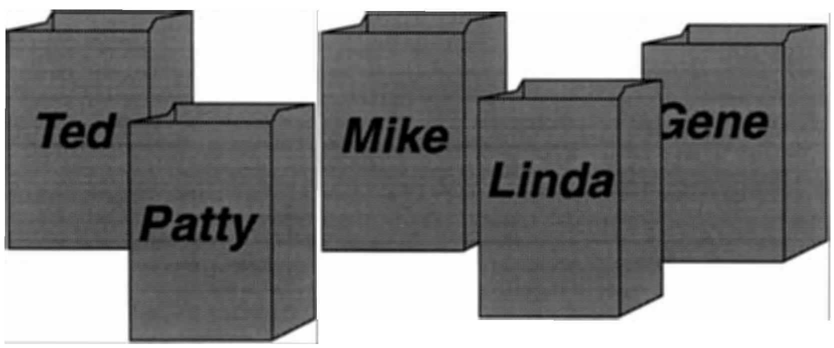
Experience translating among equivalent expressions—fractions, decimals and percent (Unit IV).

Materials

For each group of 4—a set of 5 paper lunch sacks with color tile contents (see Key); pens (red, blue, green), masking tape, butcher paper, calculators, activity sheets, transparencies.

Actions

1. Pass out copies of Activity Sheet VIII-2-A, *The Situation*, to the students and put up a transparency of VIII-2-A. Read *The Situation* with the students. Tell them each sack contains exactly ten colored tile. Each set of five sacks is the same, so every group is working on the same problem. Ask them to devise a plan of action in their groups.

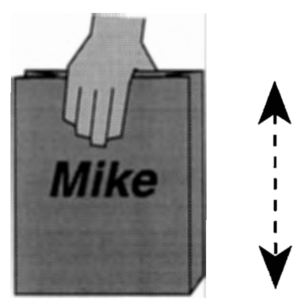


2. Pass out a set of the five paper sacks, with names on each sack, to each group. Ask the students to carry out their plans. Remind them: NO PEEKING!

Comments

1. Demonstrate the way information can be gathered by holding up a sack and pulling out one tile, then replacing it. At no time during this activity may the students look into any of the sacks (spoils the mystery), nor should you disclose the contents of the sacks until the very end of the activity. Students may ask whether all sacks have exactly ten tile, or whether two sacks are exactly the same, or whether the sacks are the same at each table. Assure them that every group has sacks with the same contents as the sacks of every other group.

2. It is *essential* that the students *shake the sacks between pulls*. (Students can find clever ways to pull tile and put them to one side in the sack. This defeats the purpose of the random trial experimentation.) Remind the students to organize their data in some way that can easily be shared with the other groups. This part of the activity will take some time.

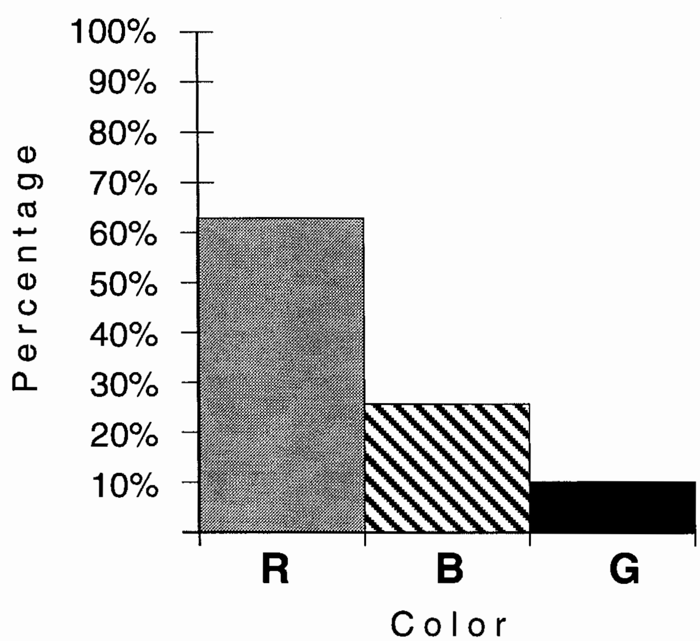


Actions

3. When groups are ready, give them a large piece of butcher paper and ask each group to make a poster showing the results of its experiment and its conclusion. Have each group name itself. (Each group is a scientific laboratory conducting an experiment, so each group will need to identify its laboratory with an appropriate name.)

4. Ask the groups to share the results of their data. Which two people do they think are related and why? How confident are they of their decision? Have the students present their posters, discuss their results and tape the posters to the wall.

5. Explain that one way to represent the results of this experiment is to make bar graphs of the results for each person. Show the students Transparency A (Sample Bar Graph), an example of a possible graph for a sack, and explain what the percentages mean. Pass out five copies of Activity Sheet VIII-2-B to each group. Ask the students to use colored pens to make bar graphs of their results for each of the five subjects.



Comments

3. Have the groups tape their posters to the wall of the room.

4. It is important to display the group results simultaneously so comparisons can be made of results across groups. Large pieces of butcher paper taped to the board work nicely for displaying and comparing group results. At this point (hopefully!) there will be disagreement among the groups as to which persons they think are related. This is fine. When we make a decision under uncertainty, we do the best we can to gather information in an unbiased way and then we make a decision informed by our data. There are not necessarily "right" or "wrong" answers.

5. Some groups may have already made a bar graph as part of their poster. The bars should be in percentages, so ask the students to convert from frequencies to percentages on Activity Sheet VIII-2-B if necessary. If their bars are already in percentages, the students can skip to Action 6 and make copies for themselves of their bar graphs on Activity Sheet VIII-2-C.

On Transparency B, for example, samples were drawn from one of the sacks and the results showed about 65% red, 25% blue and 10% green. Each group will need five copies of Activity Sheet VIII-2-B, one for each subject. The groups may need some refreshers on converting their frequency data into percentages for the bars. For example, if they got 13 red in 25 draws from one sack, $^{13}/_{25} = .52 = 52\%$ red for that bar. Students will need calculators at this point.

Actions

6. Pass out copies of Activity Sheet VIII-2-C to each student. Ask them to make a copy on this sheet of their experimental bar graphs for each subject and save them for comparison purposes later in this activity. Then ask each group to tape its experimental bar graphs (Sheets VIII-2-B) to the bottom of its posters.

7. After all the groups have completed their graphs, start a discussion. Is there any consensus in the class on who they think are related?

8. Solicit information from the class to see what they would like to do next with the experiment (see the Comments for possibilities).

Comments

6. You may wish to make several transparencies of Activity Sheet VIII-2-C for your own use, or student use, at the overhead.

7. It may be useful to move some of the graphs around during the discussion for comparison purposes. For example, the students may wish to put all the graphs together for one of the five people; or perhaps they might want to compare graphs for several people.

8. At this point, especially if there is a lot of disagreement among groups on who they think are related, there are several possible next steps. Your course of action will depend on the age and patience of your students and on the class' desire to find out *now* or to continue experimenting. One possibility is to leave the group results posted for several days and let class members try to decide what to do next.

The students may want to repeat the experiment. This is a particularly good idea if there were great differences in the number of tile that different groups drew out of the sacks. A group which drew only 10 tile might get very different results from a group which drew 50 tile—or 100 tile. If your students did Activity VIII-1, they know about the importance of sample size.

The students might want to pool the class' data for each sack. That is, rather than "competing" across groups, they may decide it is advantageous to "cooperate" and pool data.

Actions

9. When (and if) the groups and the teacher are ready to reveal the contents of the sacks, put up Transparency B (the Key). Pass out copies of Activity Sheet VIII-2-D and ask students to construct the theoretical bar graphs for each subject, based on the key. Start a discussion with the students. How do their theoretical and experimental bar graphs for the five subjects compare (Activity Sheets VIII-2-C and Activity Sheet VIII-2-D)?

10. Pass out copies of Activity Sheet VIII-2-E (Reflections on the Experiment). Ask the students to write their suggestions on this sheet. After the students have had an opportunity to reflect and write, start a discussion of their suggestions.

Comments

9. It is up to you and the class, whether or not you decide to reveal the actual contents of the sacks—decisions in real life often have to be made without "knowing for sure" whether we have made the "right" decision. Most often in "real-life" probability and statistics experiments, we don't get to look in the sacks. However, most students are going to be very anxious to look in those bags.

If you do decide to reveal the contents of the sacks, you may find students may have a lot to say at this point—especially if they want to defend their group's results that were different than the actual sack contents. (Some may even want to look in the sacks just to check against the Key).

You may want to make a master transparency of the theoretical bar graphs on Activity Sheet VIII-2-D for discussion.

10. This part of the activity might well be started at home. Individual students could write down their suggestions and then share them in their groups, before groups share with the whole class. The main idea is to have students reflect on and write about the design and the conduct of a probability and statistics experiment.

Even though our decisions must be made under uncertainty, if we reduce or eliminate sources of bias or error prior to conducting the experiment, we can be more confident in the decisions we make based on the data. This point can be emphasized with the students.

Name ___________________________

The Situation

Information from blood samples for five people—Ted, Patty, Mike, Linda and Gene—has been collected. The Bureau of Missing Persons has reason to believe that two of these five people are closely related and thus genetic information has been coded from the blood samples for each person.

This coded information has been represented by tile in five paper sacks. Two of these paper sacks have identical contents. The contents of any sack can only be revealed by pulling out one tile at a time, then replacing the tile in the sack and shaking the sack before drawing the next tile.

The Problem: Which two people are related?

Devise a plan in your group to gather data and answer this question. Some things you might want to address in your group plan are:

The way you will go about gathering data,

The amount of data you will gather,

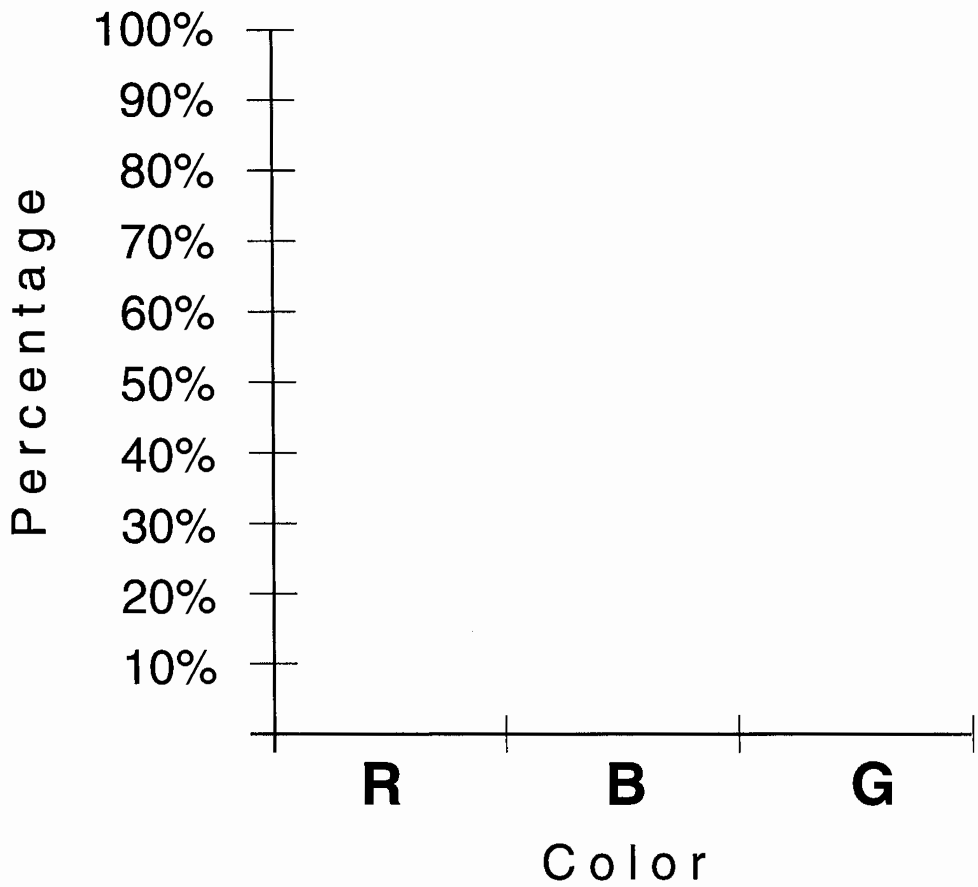
The organization of your data,

Ways to present your results to others.

Laboratory Name ______________________________

Experimental Bar Graphs

Subject's Name ____________________________________



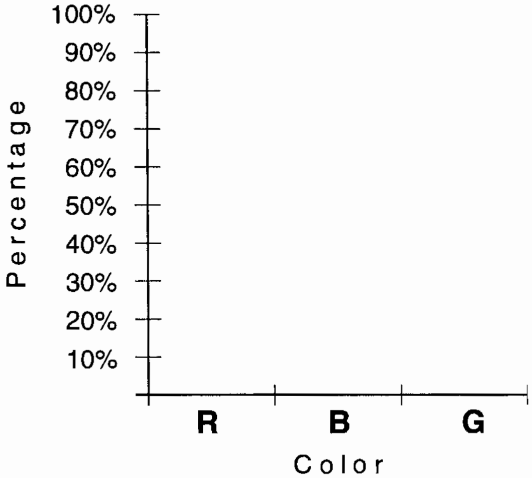
Total number of draws _________ from this sack.

Laboratory Name ______________________________

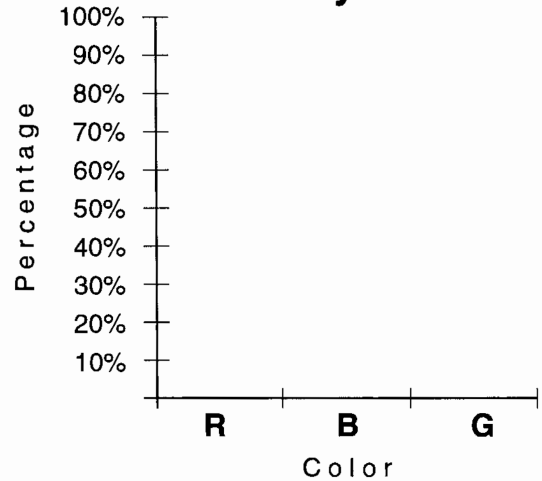
Name ______________________________

Experimental Bar Graphs

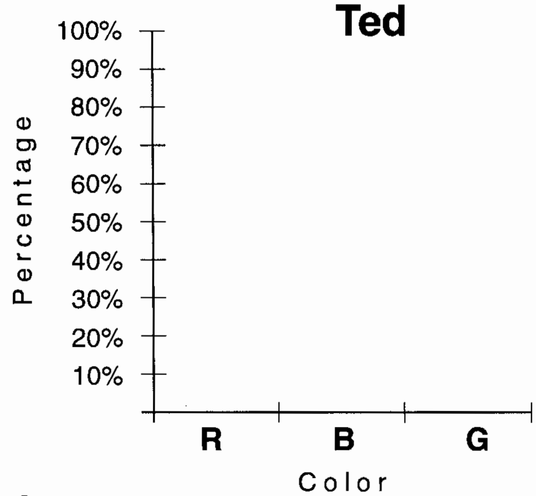
Gene



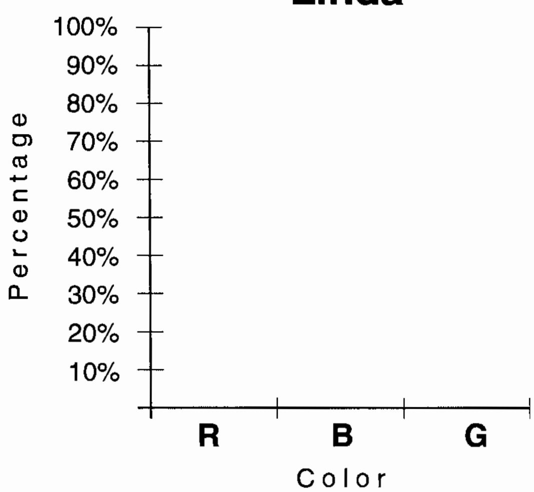
Patty



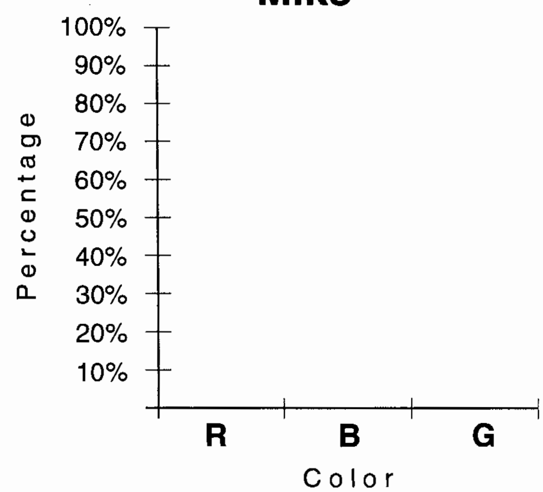
Ted



Linda



Mike

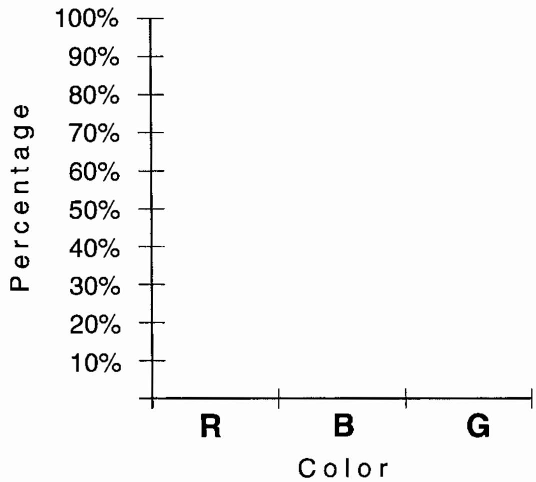


Laboratory Name ______________________________

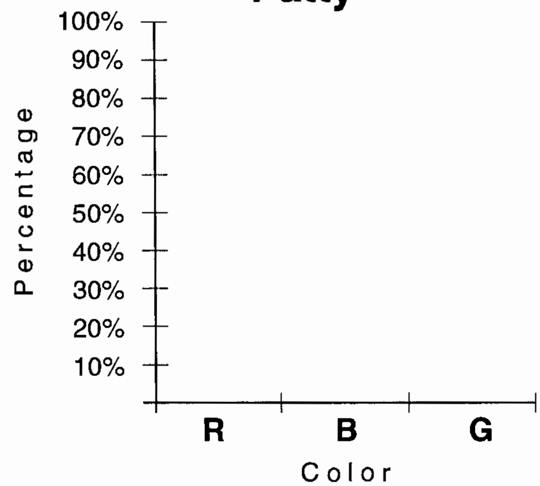
Name ______________________________

Theoretical Bar Graphs

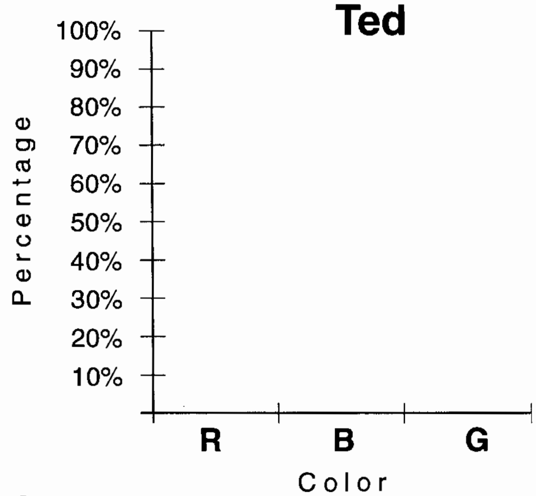
Gene



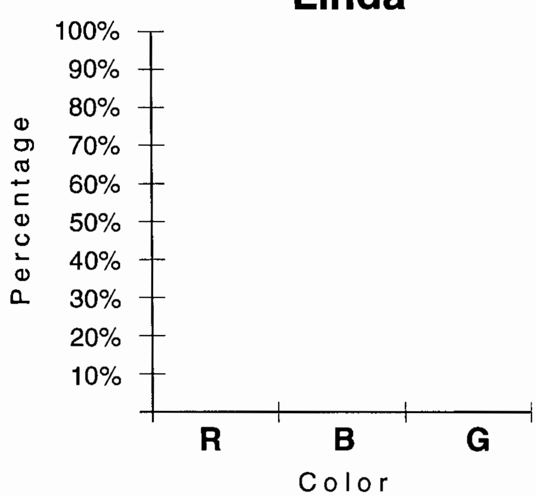
Patty



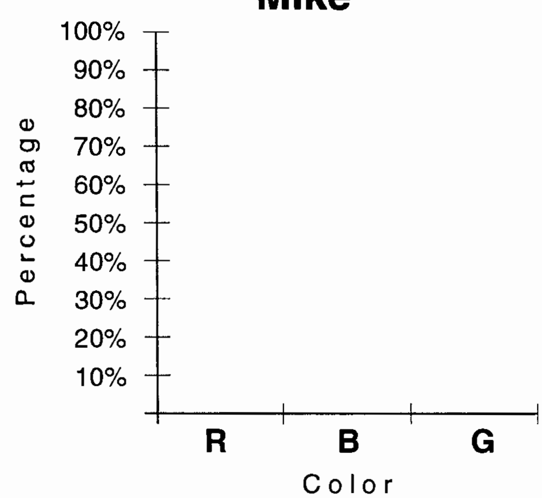
Ted



Linda



Mike

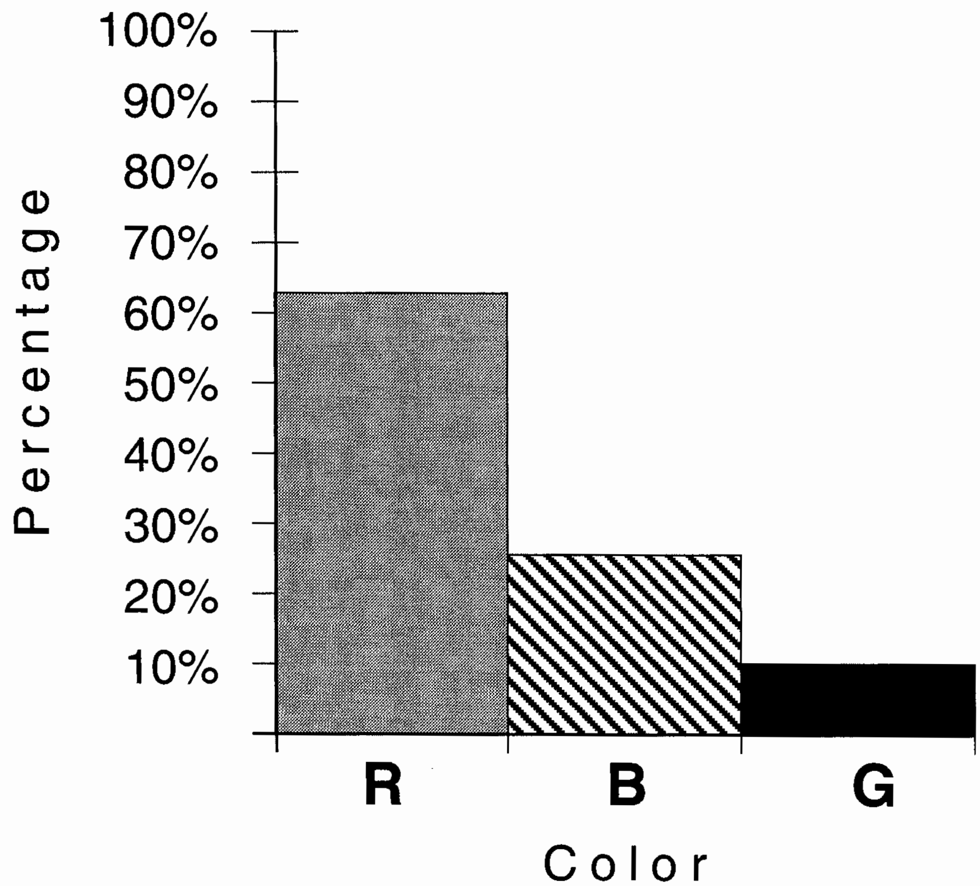


Name ______________________________

Reflections on the Experiment

What suggestions would you make to other classes who were doing this activity? What could they do to improve their experiment? Write your suggestions below in the form of a letter to another class.

Sample Bar Graph



The Key

	RED	BLUE	GREEN
Ted	7	3	0
Linda	6	2	2
Mike	4	3	3
Patty	7	2	1
Gene	6	2	2

Experimental and Theoretical Evidence

O V E R V I E W

The distribution of sums for rolling two dice is investigated using both experimental and theoretical evidence. The context is set within a game in which players attempt to find an optimal strategy to win.

Prerequisite Activities

Experience translating among equivalent expressions using fractions, decimals, percents.

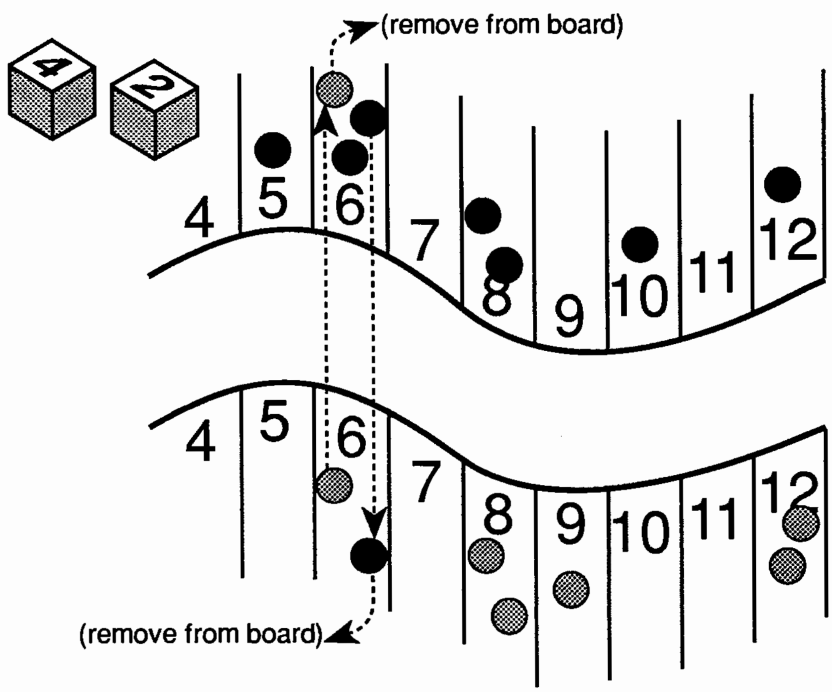
Materials

For each pair of students—2 regular 6-sided dice, preferably of 2 different colors; 24 small chips for counters, 12 of color A and 12 of color B; game board activity sheet, tape. Transparencies and other activity sheets.

Actions

Part 1: Experimental Evidence

1. Put up Transparency A, The River Crossing Game Board. Pass out the directions for The River Crossing game, Activity Sheet VIII-3-A, and read through the directions with the students. Demonstrate a move in the game, as shown below.



2. Give copies of Activity Sheet VIII-3-B (the game board) and 24 colored counters (12 each of 2 colors) to each pair of students. Ask them to prepare to play the game by placing their counters on the numbers in any way they want.

Comments

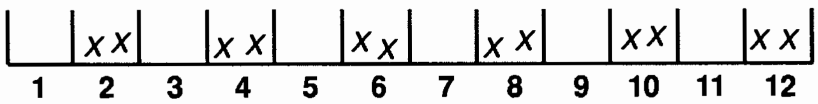
1. Check to see if the students understand the rules. One arrangement for the counters would be to place all of them on the number five, while another arrangement would be to spread them out evenly, one on each number.

Both players may move a counter across the river on a given toss if they still have a boat in that position. Remove the counters from the game board when they have crossed the river, in order to avoid confusion about which boats have crossed.

2. Students take opposite sides of the river to place their counters. Encourage students to place their counters so they have the best chance of getting all their boats across first. Students may wish to "shield" their counter arrangement from their opponent by holding a piece of paper or a notebook between them as they place their counters.

Actions

3. Pass out copies of Activity Sheet VIII-3-C, Counter Placements, and put up a transparency of Activity Sheet VIII-3-C. Before they begin to play, ask the students to indicate each of their own counter placements with an X on the line labeled "Counter Placement" under Game 1.



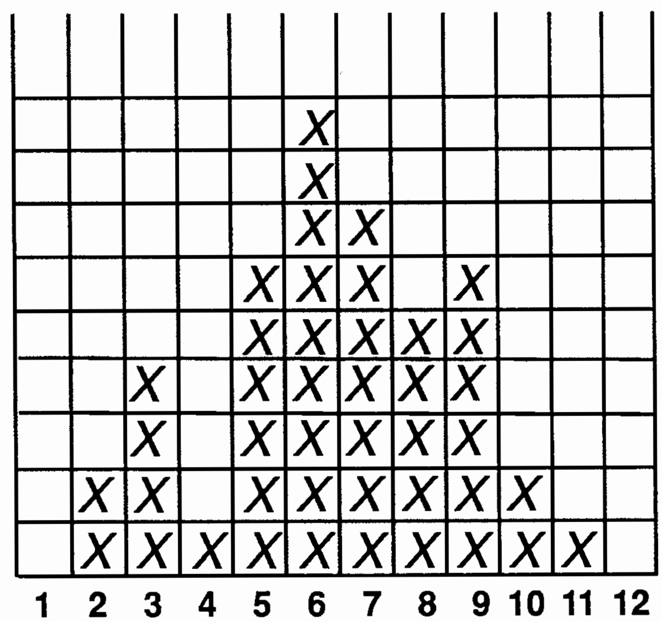
This player put all the boats on even numbers.

4. Pass out Activity Sheet VIII-3-D to the students and put up a transparency of Activity Sheet VIII-3-D. After each toss ask the students to make a tally mark, X, on the line plot on Activity Sheet VIII-3-D. Demonstrate this on the overhead.

Comments

3. The line plot of counter placements will help students keep a record of their strategy from one game to another.

4. This line plot provides a cumulative record for all the tosses made by a pair of students. As they play more games, the students can keep adding to this same plot. Students *must* make a tally mark *for every toss*, whether or not they were able to move a counter. After several games the line plot might look like this:



Actions	Comments
5. Pass out two dice to each pair of students. Ask the students to play the game several times. Remind them to keep a record of their tosses on the line plot (Activity Sheet VIII-3-D) and to record their counter placements prior to each game (Activity Sheet VIII-3-C).	5. It is useful to use dice of two different colors so students will be able to distinguish one die from the other. Students may wish to play the game several times to search for a "best" strategy. Each time they start a new game the students may change the placement of their counters. Students may need extra copies of Activity Sheets VIII-3-C or VIII-3-D.
6. Start a discussion on the students' strategies for counter placement. What is the best arrangement of the counters? Ask several students to come to the overhead and show their favorite counter placement on the game board. Ask each in turn why they think this is a good placement for the counters. Also ask them if they changed the placement of their counters from the first game to the second or later games and why.	6. There certainly are many possible "best" arrangements of the counters. Students may have strong beliefs about the best counter placement, even though they have contrary evidence from playing the game several times. For example, even after playing the game they may still believe it is best to have about the same number of counters on each number. Or, they might have a favorite number they wish to put extra counters on. After several games, we might expect a number of the students to put more counters towards the middle and fewer out at the extremes. The students will begin to realize that not all the numbers are equally likely to occur (if they don't already know this).
7. Ask the students to tape (or tack) the line plots for their tosses (Activity Sheet VIII-3-D) to the wall or board. Start a discussion about the graphs. What do the students notice? Do the graphs have anything in common?	7. Students will notice that the numbers in the middle come up more often. Ask them why this is so. They may also notice there is variation from one graph to another; for example, one pair may have tossed only one 2, while others may have tossed four or five 2s.
Part II: Theoretical Evidence 8. Ask the students how many different ways they can obtain each of the possible sums from tossing the two dice. Show them some examples on Transparency B. Pass out copies of Activity Sheet VIII-3-E and ask the students to complete it. Ask students to share their results for particular sums at the overhead.	8. For example, there is only one way to obtain a sum of 2, 1 on the first die and 1 on the second die. On the other hand, we can get a sum of 4 in 3 different ways: 1st Die / 2nd Die / Sum 1 / 3 / 4 2 / 2 / 4 3 / 1 / 4 The use of 2 different colored dice helps distinguish the outcome $1 + 3 = 4$ from the outcome $3 + 1 = 4$.

Actions

9. Put up a transparency of Activity Sheet VIII-3-F and pass out copies of Activity Sheet VIII-3-F. Ask students to fill in the grid of all possible sums. Tell the students this grid represents an area model for the possible pairs for tossing two cubical dice. Ask the students, "What part of the total area of the grid is occupied by each number?"

10. Tell the students that the fraction of the total area occupied by each number in the grid is called the "theoretical probability" that the number will occur as the sum when they toss two dice. Ask the students to complete Activity Sheet VIII-3-F by filling in the probabilities that are requested. Show them an example on Transparency VIII-3-F like the one below.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5			
3	4	5	6			
4						
5						
6						

There are 3 ways to get a sum of 4.
The probability is $\frac{3}{36} = \frac{1}{12} \approx .083$.

Comments

9. In our grid the sum of 4 takes up $3/36 = 1/12$ of the area of our model. You might shade in this area for the students, indicating that it represents the fraction $1/12$. This grid provides us with an area model for the probability of each of the possible sums.

10. If we assume that our dice are fair—that each number has the same chance (1 in 6) of coming up—then each of these 36 pairs also has an equal chance of occurring. We say that the "theoretical probability of getting a sum of 4 is 3 out of 36" since 3 of the 36 possible pairs add up to 4. We normally write this shorthand as Prob(4) = $3/36 = 1/12 \approx .083 = 8.3\%$.

Actions

11. Discuss the results. Ask the students "What does it mean to say that the probability that we roll a 4 is $1/12$? Or that the probability that we roll a 7 is $1/6$?" Ask the students "What are the assumptions about the dice in our grid model?"

12. Now that the students have built a theoretical model for The River Crossing, ask them to play the game one more time. This final play can be a whole class activity.

13. Start a discussion after this final game. What do the students think is the best placement of counters and why? Ask the students if they amended their choice for the "best" counter placements after seeing the area model for the theoretical probabilities. What did they change and why?

Comments

11. One interpretation of the statement Prob(4) = $1/12$ is that there are 3 outcomes in the 36 of our grid model that add to 4. Another interpretation of the statement Prob(4) = $1/12$ is that if a pair of dice were tossed many, many times, *we would expect about $1/12$ of the sums to be 4*. In this view, the theoretical probability is a model which gives us an informed estimate for the number of 4s we would expect to obtain in an actual experiment.

There are always assumptions in any probability model. In this case, we are assuming that the dice are *fair*, not weighted to certains numbers. We are also assuming that the dice do not influence each other, that is, they are *independent* of each other. The students may come up with other assumptions.

12. Ask the students to label their counter placement this time as "My final strategy" on Activity Sheet VIII-3-C. (Extra copies of Activity Sheet VIII-3-C might be needed). The final play of the game can be conducted as a sort of River Crossing "Bingo" in which the teacher, or one of the students, rolls and keeps track of each toss on a clean transparency of Activity Sheet VIII-3-D.

13. One important issue to bring up in the discussion is, "How have the students modified their game strategy during the process of obtaining first experimental and then theoretical information about the sums of two dice?"

Actions

14. Pass out copies of Activity Sheet VIII-3-G, page 1, and ask the students to complete it. Discuss the students' answers to question 3 in class. Ask students to share their solutions for question 3 at the overhead.

For students with some experience with decimals and percent: pass out copies of Activity Sheet VIII-3-G, page 2, and ask them to complete it. Discuss the results, or have the students write about their results, comparing the experimental and theoretical probabilities.

15. (Optional) Pass out copies of Activity Sheet VIII-3-H. Ask the students to work on this activity in teams outside of class over a period of time. Ask teams to post their results and conjectures on the wall. When appropriate, start a discussion based on the students' posters.

Comments

14. Activity VIII-3-G may make a good homework assignment and/or assessment activity. The purpose is to give students a chance to reflect on the process of learning about the distribution of dice sums, first from the experimental information gathered by actual plays of the game and then from the theoretical probability model. Activity VIII-3-F may be used to provide summary feedback to instuctors on students' understanding.

Question 3 is an extension of the two dice problem. Students will need to use percentages or ratios or fractions in some way to estimate the expected number of rolls of a given sum in 100 rolls. There are numerous solution methods here. One way might be to use the theoretical probability in the area model to help estimate the number of rolls. For example, we might expect about $1/12$ of the 100 rolls to come up 4, about 8 rolls.

15. This activity might take a week or so. All the sums from 3 to 18 are possible. There are 216 possible triples from tossing three dice. You might remind the students that we built a convenient area model for the results from tossing two dice and then ask the students how they could build a model of all the results for tossing three dice.

Name ______________________________

The River Crossing*

Directions

This is a game for two players. Each player is given twelve counters representing boats to be placed on the numbers along the bank of a river. The arrangement of the counters (boats) is completely up to the player.

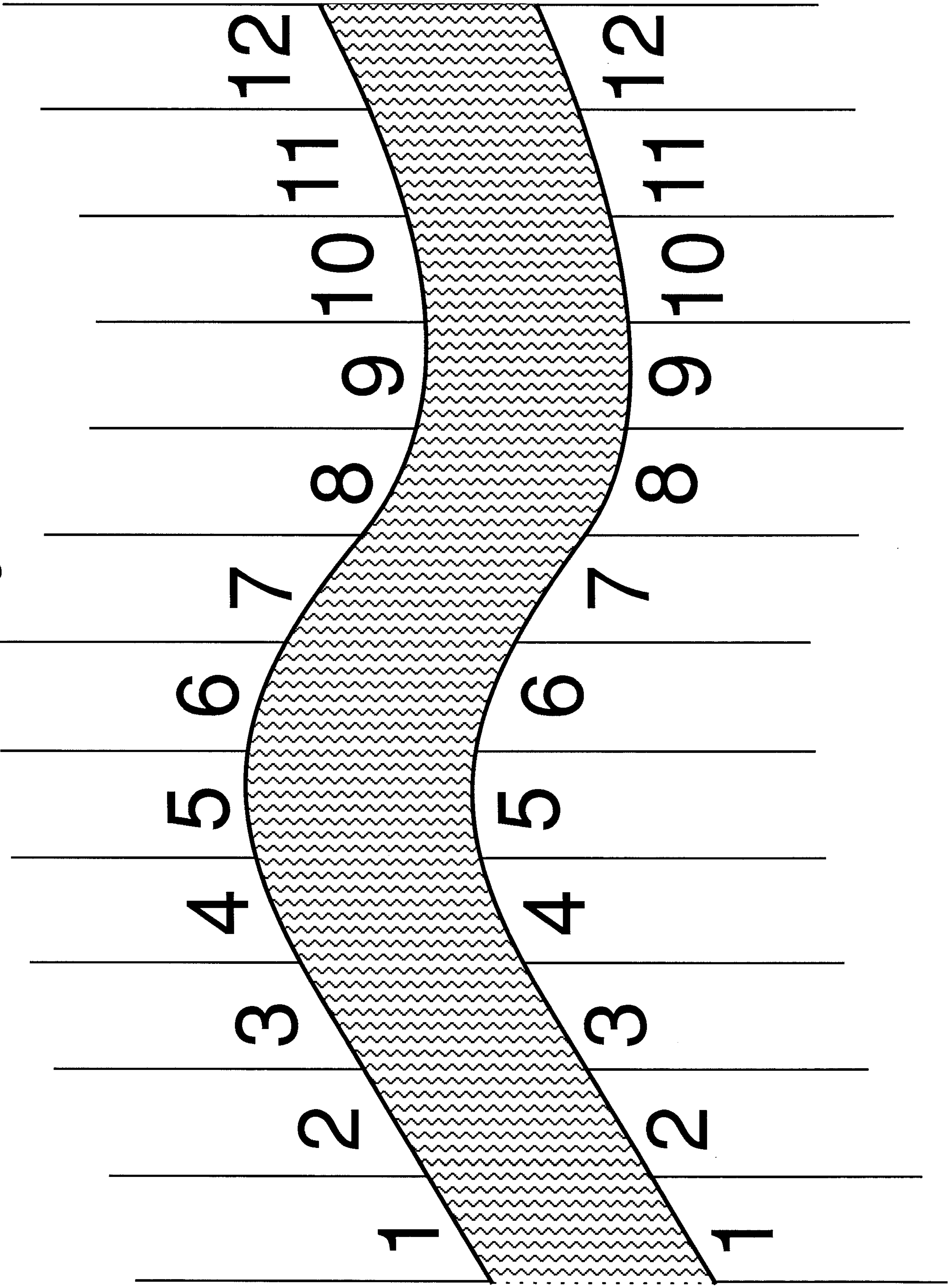
The Players take turns throwing two fair dice. On each roll the sum of the two upturned numbers is determined. If either player has a counter in that position, they may move it across the river to the other side and then remove the counter. Play continues until one player removes all twelve counters from the board.

The Problem

What is the best arrangement for the counters?

* This activity was adapted from a problem in *Mathematics Activities from Poland* by Jerzy Cwirko-Godycki.

The River Crossing Game Board



Name ______________________________

Counter Placements for River Crossing

Game 1

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an* ***X.***

Game 2

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an* ***X.***

Game 3

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an* ***X.***

Game 4

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an* ***X.***

Game 5

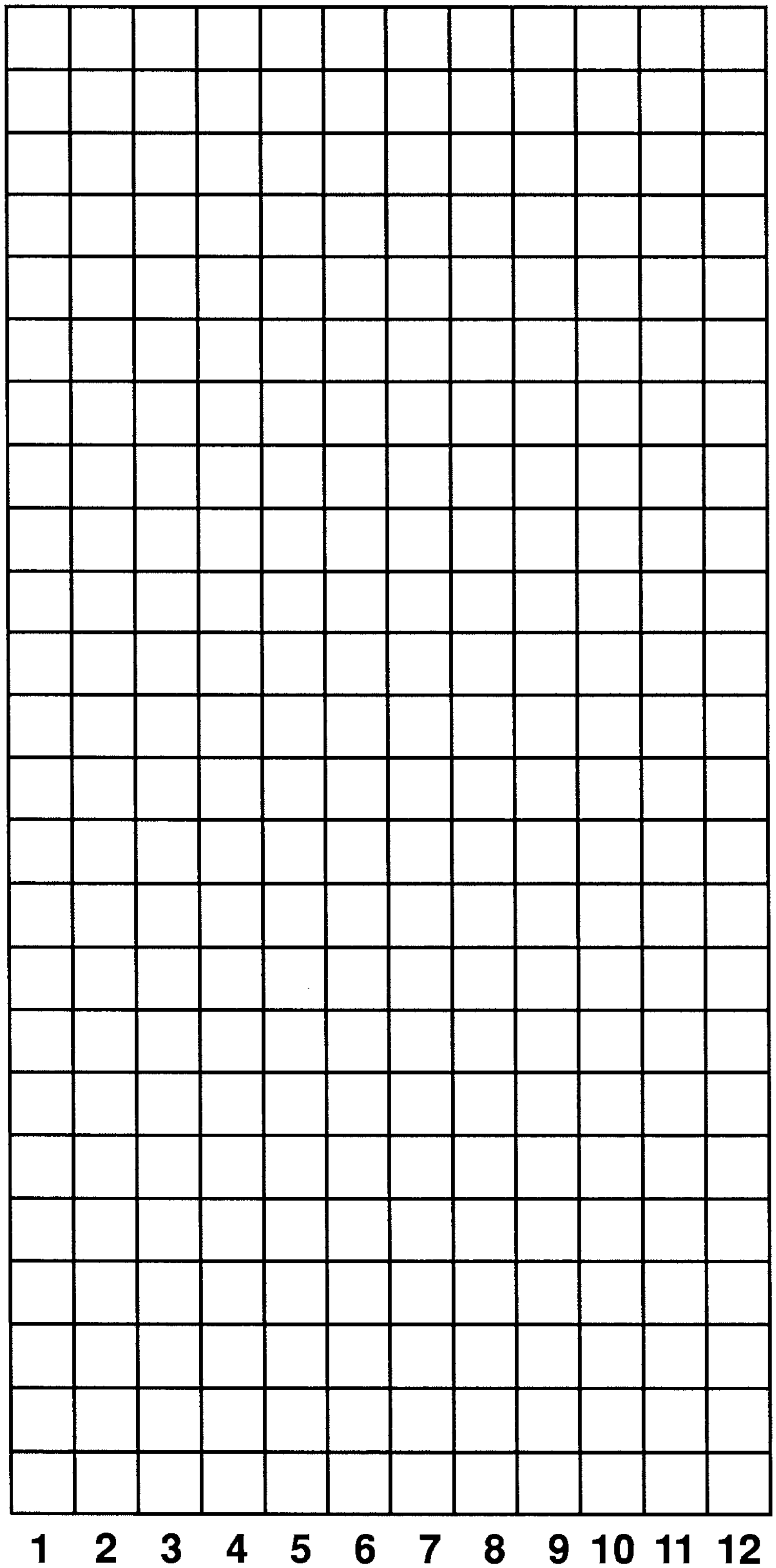
1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an* ***X.***

Name ____________________________

Activity Sheet VIII-3-D

Line Plot for Sums of Two Dice



Name ____________________________

Possible Sums For Tossing Two Cubical Dice

1st Die	2nd Die	Sum

Name ______________________________

Area Model For Possible Sums From Two Cubical Dice

2nd die

+	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
1st die 3						
4						
5						
6						

Theoretical Grid

Theoretical probabilities of outcomes from tossing two cubical dice

Prob(2) =

Prob(3) =

Prob(4) =

Prob(5) =

Prob(6) =

Prob(7) =

Prob(even sum) =

Prob (multiple of 3) =

Prob(8) =

Prob(9) =

Prob(10) =

Prob(11) =

Prob(12) =

Prob(odd sum) =

Prob(number ≥ 5) =

Prob (number < 7) =

Name ______________________________

Reflection on The River Crossing Activity

1. Did you change the placement of your counters after playing a few River Crossing games? Explain why or why not.

2. Did you change the placement of your counters after making the area model of all the outcomes for the sum of two dice? Explain why or why not.

3. Suppose you tossed a pair of dice 100 times. How many times would you expect to get

a sum of 6?	a sum of 1?
a sum of 11?	a sum greater than 7?
an odd sum?	a sum less than 7?

Explain how you determined your answers to these questions.

4. Was there anything that surprised you in The River Crossing Game? Explain.

5. Write a note to a friend on the back of this sheet. Explain The River Crossing Game to your friend Tell your friend what you think is the best placement for the 12 counters. Explain why you think this is the best placement.

Name ______________________

6. Get your experimental data from the plot of the tosses (Activity Sheet VIII-3-D). Calculate the percentage of time each possible sum occurred on your graph. For example, if you made 52 tosses and got a 2 three times, your percentage of twos would be $3/52$ = .057 = 5.7% of the time. Do this for each of the possible sums from 2 to 12. Then compare your experimental percentages to the theoretical percentages of each sum from Activity Sheet VIII-3-F. Fill in the chart below.

SUM	Experimental Percentage	Theoretical Percentage
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Name ____________________________

River Crossing Extension

1. Suppose that you tossed three cubical dice and crossed a boat with each sum of the three dice. What would the river dock look like for a three dice game? Draw a picture.

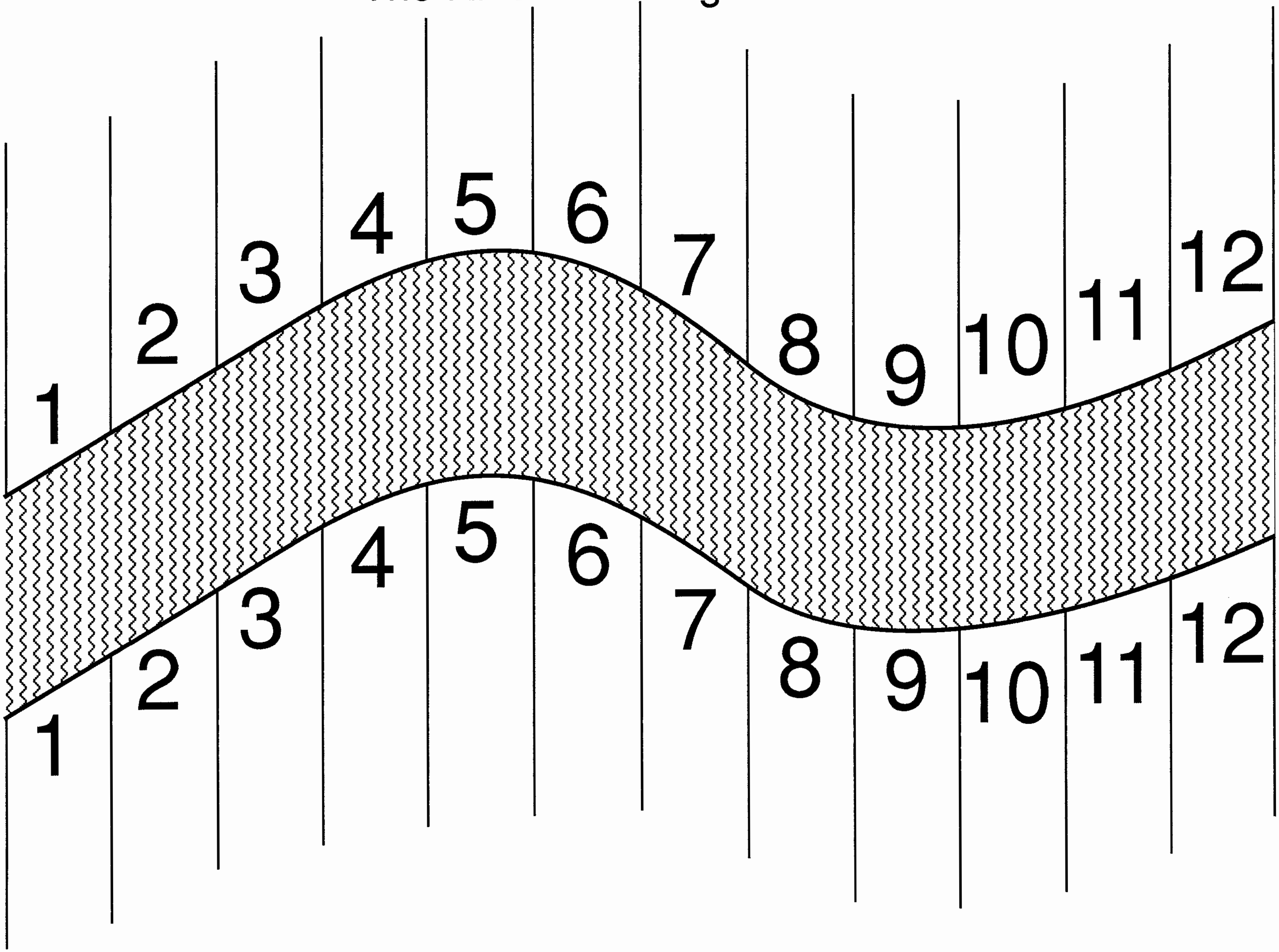
2. If you had 20 counters (boats) to place on this dock, where would you place them? Indicate below.

Counter Placement

3. (Challenge) How many different ways can you get each possible sum from tossing three cubical dice? For example, you can obtain a sum of four in three different ways,

1st Die	2nd Die	3rd Die	Sum
1	1	2	4
1	2	1	4
2	1	1	4

The River Crossing Game Board



Possible Sums For Tossing Two Cubical Dice

1st Die	2nd Die	Sum

Checker-A Game

O V E R V I E W

Results from a binomial experiment with equally likely outcomes (odd and even rolls on a regular die) are compared to theoretical probabilities determined by counting the possible sequences of 6 tosses of the die. This activity is a precursor for work on counting strategies, Pascal's triangle and the binomial distribution.

Prerequisite Activities

Unit VIII, Activity 3 is recommended. Familiarity with decimals, percents, bar graphs, and experimental and theoretical probability.

Materials

For each student—a regular 6-sided die, the Checker Game board and a small chip for a game marker. Transparencies, activity sheets, masking tape.

Actions

Part I: Experimental Data

1. Pass out a copy of Activity Sheet VIII-4-A, the Checker Game Board, a 6-sided die and a chip to each student. Put up Transparency A, the Checker-A Game. Explain the game to the students. Even tosses move **Left**, odd tosses move **Right**. Make a transparency of Activity Sheet VIII-4-A, the game board, and play the game once with the students. Let them roll the die while you move the chip on the game board at the overhead.

2. Ask the students: "Which square do you think the chip will land on at the finish? Why?" Solicit student opinion.

3. Groups of 4 or 5 students can work together on this activity. Pass out copies of Activity Sheet VIII-4-B. Ask the students to imagine playing the game 100 times. On Activity Sheet VIII-4-B, ask the students to record their guesses for the number of times the chip will land on each square if they play the game 100 times. After they make their guesses, ask them to compare their guesses with other members of their group. Ask for several volunteers to share their guesses and their reasoning with the class.

Comments

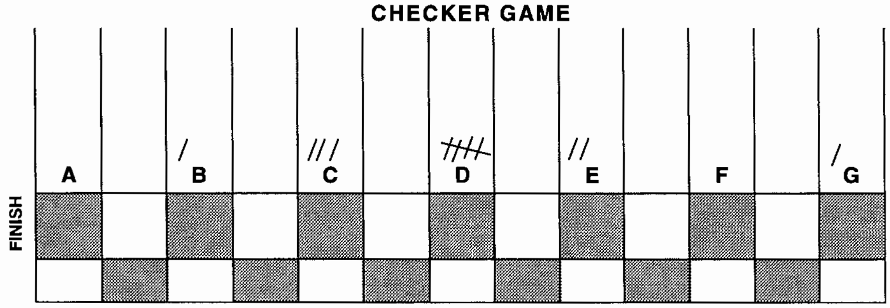
1. A play consists of six tosses. **Left** and **Right** are determined from the **Start** square on the board. That is, if a player moved **Right** 6 times in a row, the chip would end up at square G. You will need to make several transparencies of the game board.

2. Students may have a favorite square with no other reason for choosing it. Also, some students may think all squares have an equal chance of winning, while other students may think that some squares have more chances to win than others.

3. Students may notice there is only one path each to squares A and G, while there are many more routes to some of the interior squares. They may also notice that the board is symmetric, that is, there are the same number of paths to B as to F and so forth. A transparency of Activity Sheet VIII-4-B is helpful.

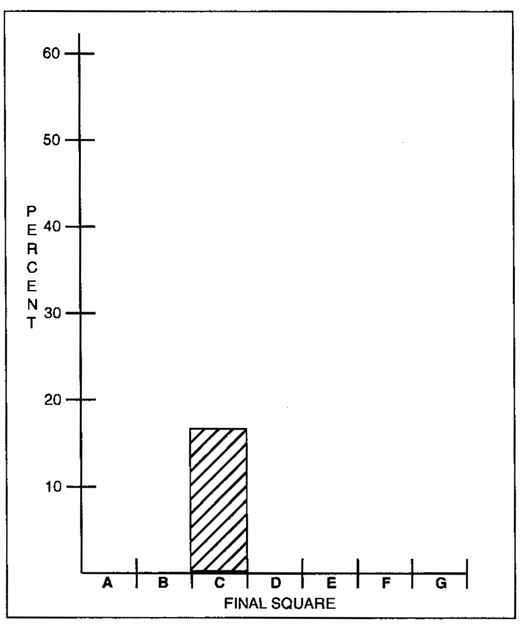
Actions

4. Ask the students to play the game and complete Activity Sheet VIII-4-B. Each time they end at a lettered square, ask the students to make a tally mark (/) on the game board in the column above that square.



5. When the students have completed the 100 trials in their groups, ask them to use their data to calculate the experimental probabilities of landing at each square and to record these at the bottom of Activity Sheet VIII-4-B.

6. Pass out a copy of Activity Sheet VIII-4-C to each group of students and put up a transparency of Activity Sheet VIII-4-C. Ask the students to make a graph of the results of their 100 trials and to tape their results to the wall.



Comments

4. Each group will need a total of 100 plays of the game. The students can use their tallies from the game board to record their totals on Activity Sheet VIII-4-B.

5. You may need to remind students that the "experimental" probability for each square can be calculated from (# occurrences)/100, where 100 = # trials. Thus, if they landed on the C square 15 times, P(C) = $^{15}/_{100}$ = .15 = 15%.

6. You might demonstrate by filling in the bar for one of the outcomes at the overhead, using a group's data for that square. At the left is an example from a group which landed on the C square 17 times in 100 trials. (Keep copies of the graphs so students can compare them to the Checker-B graphs in the next activity.)

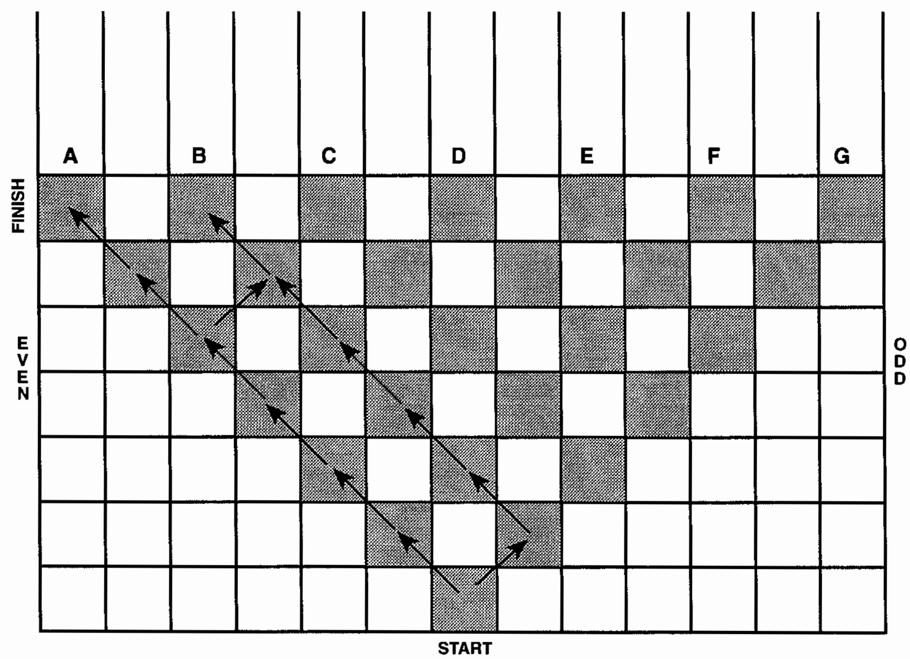
Actions

7. Discuss the results of the experiment with the students. Were they surprised by the results? Do the results make sense to them? How did the experimental results compare with the predictions they made before they did the experiment?

Part II: Developing a Theoretical Model

8. Ask the students how many different paths there are on the checker board to reach each of the lettered squares. Draw several examples of paths on a transparency of the game board.

When the students have finished, ask for volunteers to share their results. Ask the students if they found any patterns as they counted the paths to the squares.

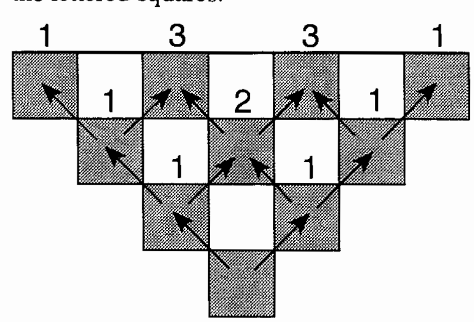


Comments

7. It is important to get students to compare their experimental results to their original predictions. Students can also be asked to compare their individual results to their group's total results. Was there any variation in their individual results? What are the similarities or differences between their individual results (their tallies) and the group's total results? What are the similarities or differences between the groups' bar graph results posted on the wall? How would they account for any similarities or differences?

8. In the figure at the left, we see there is only one path to get to the square labeled A—we must go left (toss **Even**) six times in a row. However, there are many paths to some of the other squares. Two paths to square B are given. There are many patterns that can arise as students look for paths. The students may notice there is symmetry in the paths, that is, there is also only one path to square G, the number of paths to square B is the same as the number of paths to square F, and so forth. You may need to pass out extra copies of Activity Sheet VIII-4-A (the game board) as the students count the paths. You may find that a sequence of E's and O's is a useful notation to represent paths and keep them straight. For example, EOEEOE represents an even-odd-even-even-odd-even sequence of tosses and is a path that ends at square C.

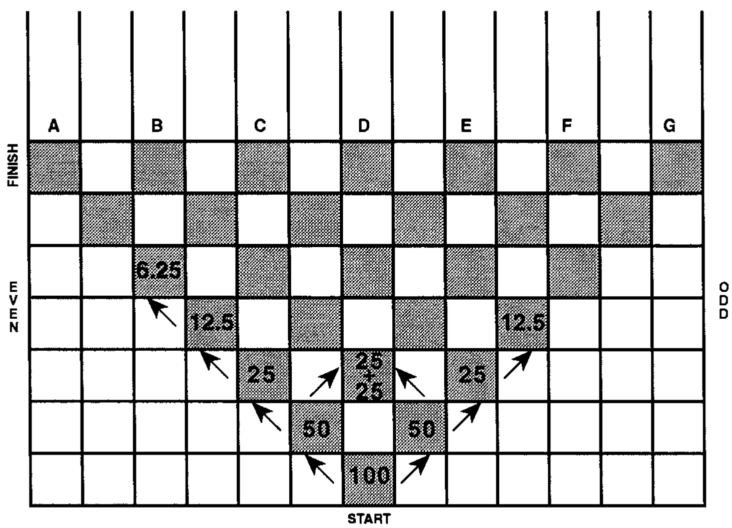
You might suggest to the students that they keep track of the number of paths that land at the intermediate squares, as below. These are the numbers of Pascal's triangle. There are 64 different paths altogether that end at the lettered squares.



Actions

9. Ask the students how they might determine the "theoretical" probability of landing on each square in the Checker-A Game. Have them discuss this in their groups first and then share their ideas with the whole class.

10. Put up a transparency of the game board. Write the number 100 on the Start square? Ask the students: If 100 chips were used in the starting square and we rolled a die for each one, about how many of them would we expect to move to the left? to the right? Now if we rolled again, about how many of those would we expect to move to the left? to the right?

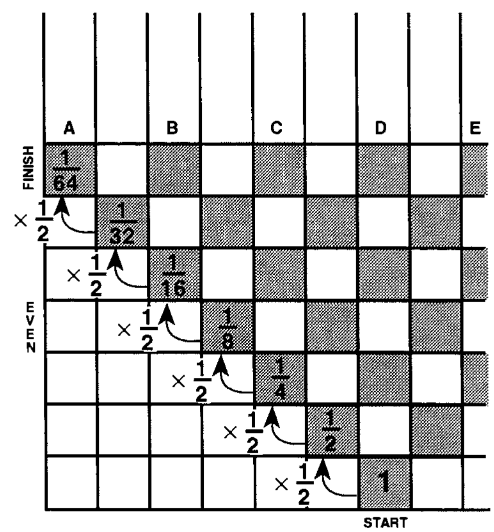


Comments

9. Students might suggest that the paths from Action 8 can be used to calculate theoretical probabilities. For example, there are 6 different ways to land on square B: EEEEEO, EEEEOE, EEEOEE, EEOEEE, EOEEEE and OEEEEE. There are 64 different paths, in total, from start to finish in the checkerboard. So, the theoretical probability for landing on square B is:
$P(B) = 6/64 = .09375$, or about 9.3%.

However, it is *very important* to point out this theoretical probability model *assumes each of the 64 paths has an equal chance of occurring*. Every probability model carries assumptions with it.

10. This also might help students to find theoretical probabilities for each door. Students can continue to split the number of chips in half at each stage, as shown below. 50% move to the left; then, of those that moved left, 50% would be expected to move left again, so there would be 25% of the chips on the second left hand square and so forth. This method begins to introduce probability trees and multiplying sequential probabilities. ($1/2$, $1/2 \times 1/2$, $1/2 \times 1/2 \times 1/2$, and so forth). In this way we can see that the probability of any one particular path of six moves in the Checker-A game is $1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 \times 1/2 = 1/64$. (This approach will also extend to the Checker-B problem in Activity 5).



Actions

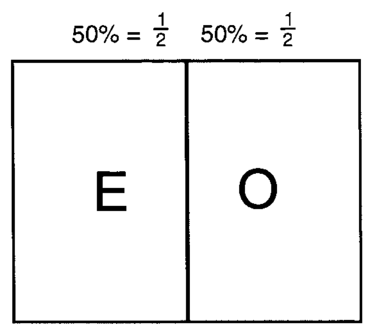
11. Pass out copies of Activity Sheet VIII-4-D and have the students complete it. Ask the students to compare the theoretical probabilities they obtained for each square to the experimental probabilities on their bar graphs. How close were their experimental results to the theoretical probabilities? Were there any surprises? Discuss.

12. Ask the students, "Can you make an area model for the theoretical probabilities of landing on the squares?"

Comments

11. The experimental results may vary quite a bit from group to group and also may deviate from what the theoretical probabilities predict. However, the general shape of most students' graphs will support the theory. The squares in the middle have a higher probability of occurring and those at the extremes are less likely to occur.

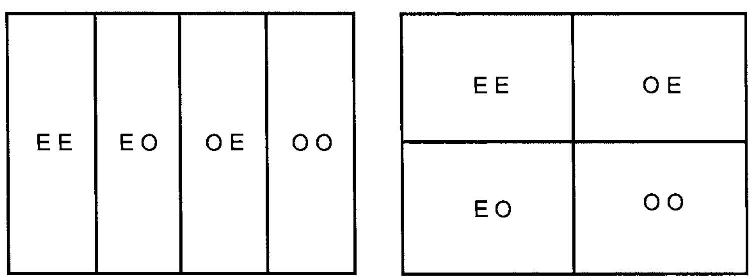
12. You might suggest the students start with a simpler problem with smaller sized game boards. Suppose the game were only one toss of a die. For example, draw the diagram below on the overhead.



This model represents 50% odd and 50% even for one toss of the die.

Then ask the students to build an area model for the odd-even sequences from two tosses of the die, for a game that was two steps long. To the left are two possible models.

Each area is $\frac{1}{4}$, or 25%

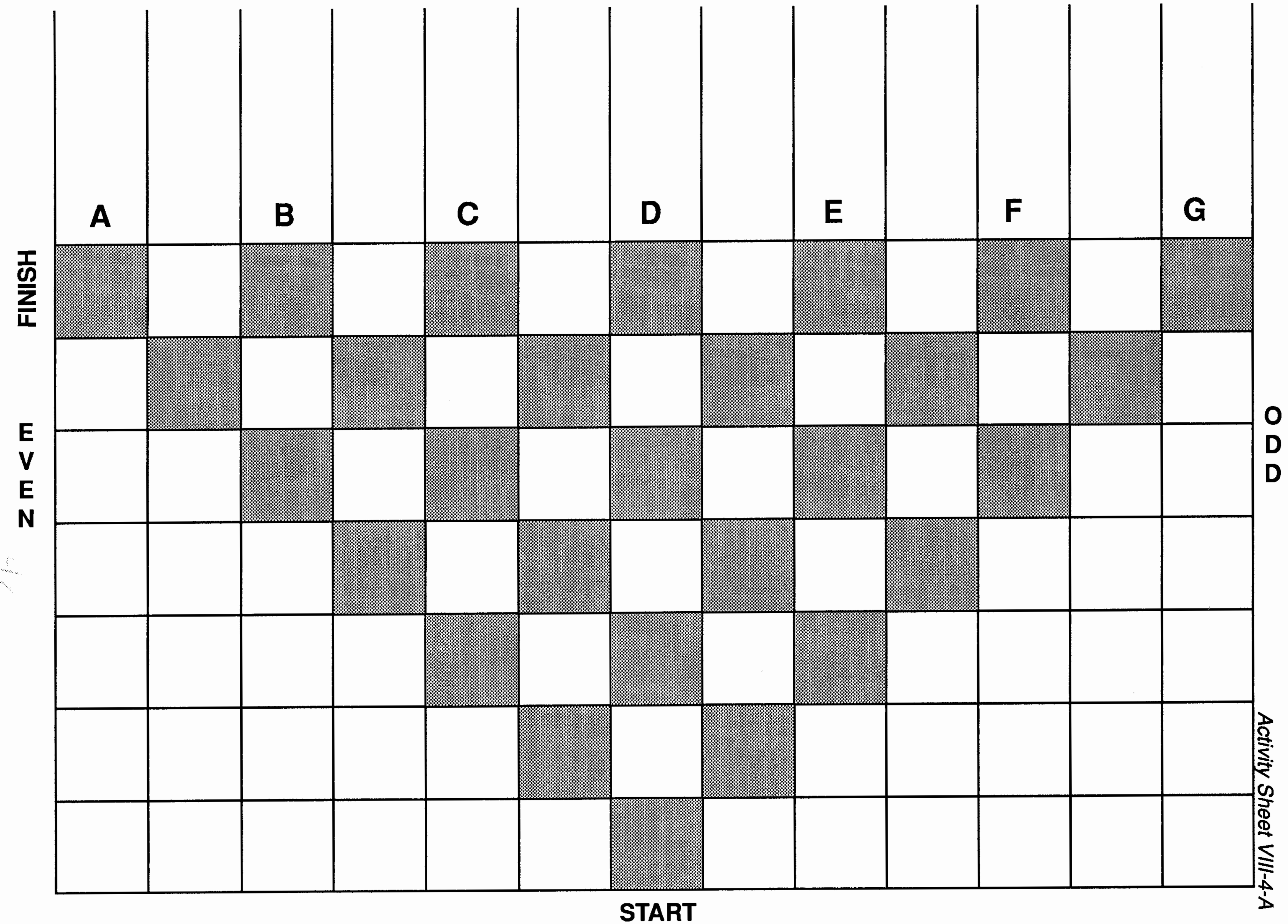


Each area is $\frac{1}{8}$, or $12\frac{1}{2}$%

The second model allows us to build the next size game by cutting each rectangle in half. Ask the students what this model would look like for the four toss, then five toss and, finally, the six step game of the Checker-A activity. The Grid Blank may be helpful for some students.

Your students may make an area model like Transparency B. You might also share the posters from Transparencies C and D. Students can make similar posters of their own area models for the Checker-A game on grid blanks. The posters on Transparencies C and D were made from the area model on Transparency B by writing in the letter of the final square for each sequence of O's and E's to get Transparency C, and then by shading each final square to get Transparency D.

CHECKER GAME



Name ______________________________

Checker-A Game

1. Imagine playing the Checker-A Game 100 times. Make some predictions: How many times do you think the marker would end up at square A? at square B? etc. Record your predictions here.

Ending Square:	A	B	C	D	E	F	G
Your Guesses:							

Compare your guesses with those of your groupmates. Were there any general observations about the game that prompted your answers?

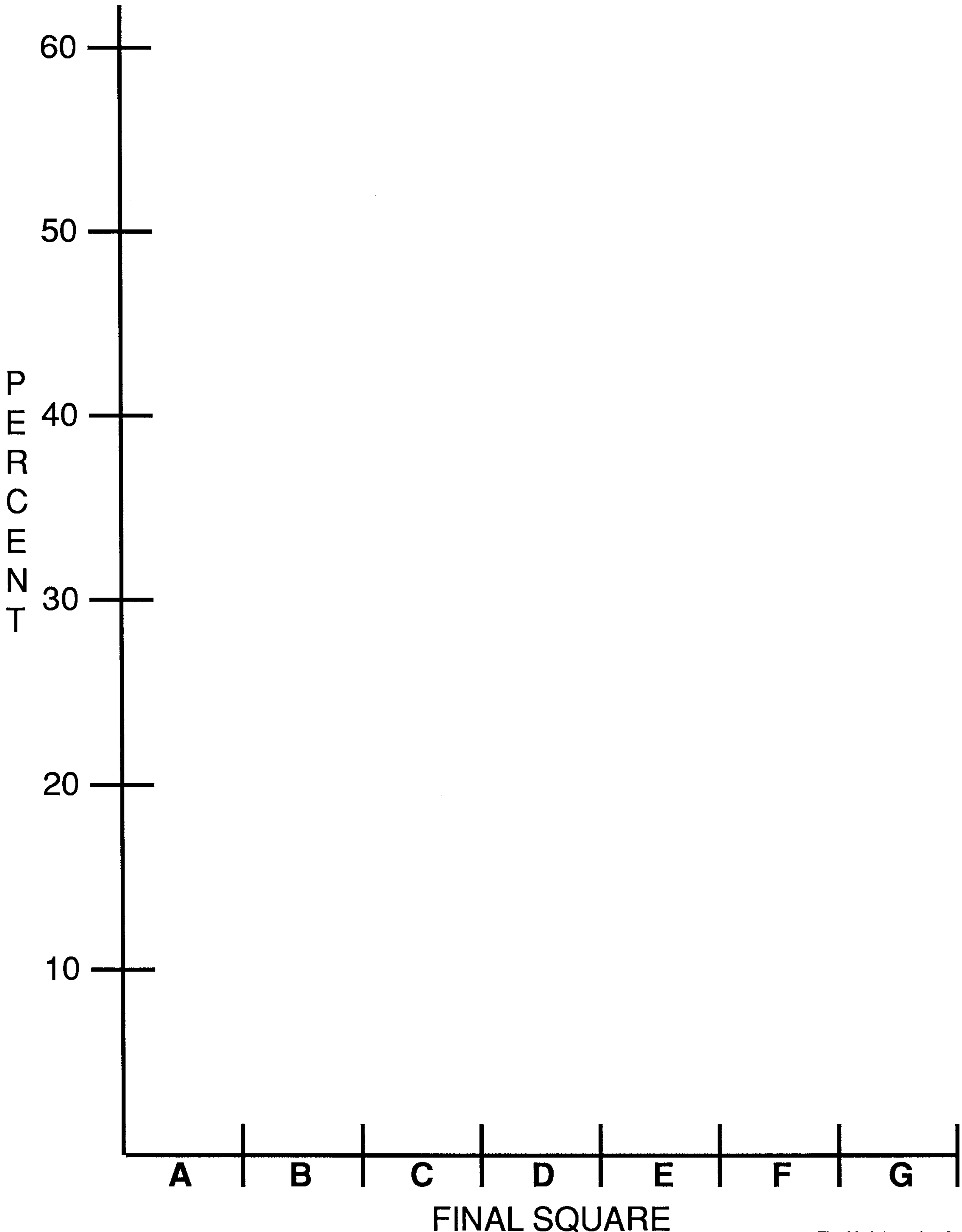
2. Each person in the group now plays the game a number of times. With tally marks (on the game board) keep track of the number of times the marker ends at each of the lettered squares. Put the results obtained by each person into the chart below and complete the chart. Your group will need a total of 100 plays.

	Ending Square							
Player's Name	A	B	C	D	E	F	G	Total Plays
1								
2								
3								
4								
5								
Group Total								100

3. Use your group's results from the chart to calculate the "experimental" probabilities for landing on each of the lettered squares. Write your results below.

P(A) = P(B) = P(C) = P(D) =

P(E) = P(F) = P(G) =


60
50
40
30
20
10
PERCENT
A
B
C
D
E
F
G
FINAL SQUARE

Name ______________________________

Comparison of Experimental and Theoretical Probabilities for the Checker-A Game

Final Square	Experimental Probability	Theoretical Probability
A		
B		
C		
D		
E		
F		
G		

The Checker-A Game

Begin with a marker on the START square. A play of the game consists of tossing a regular die 6 times.

For each toss of an EVEN number, move the marker one square diagonally to the LEFT.

For each toss of an ODD number, move the marker one square diagonally to the RIGHT.

After six tosses, the marker will be on one of the squares marked A, B, C, D, E, F or G.

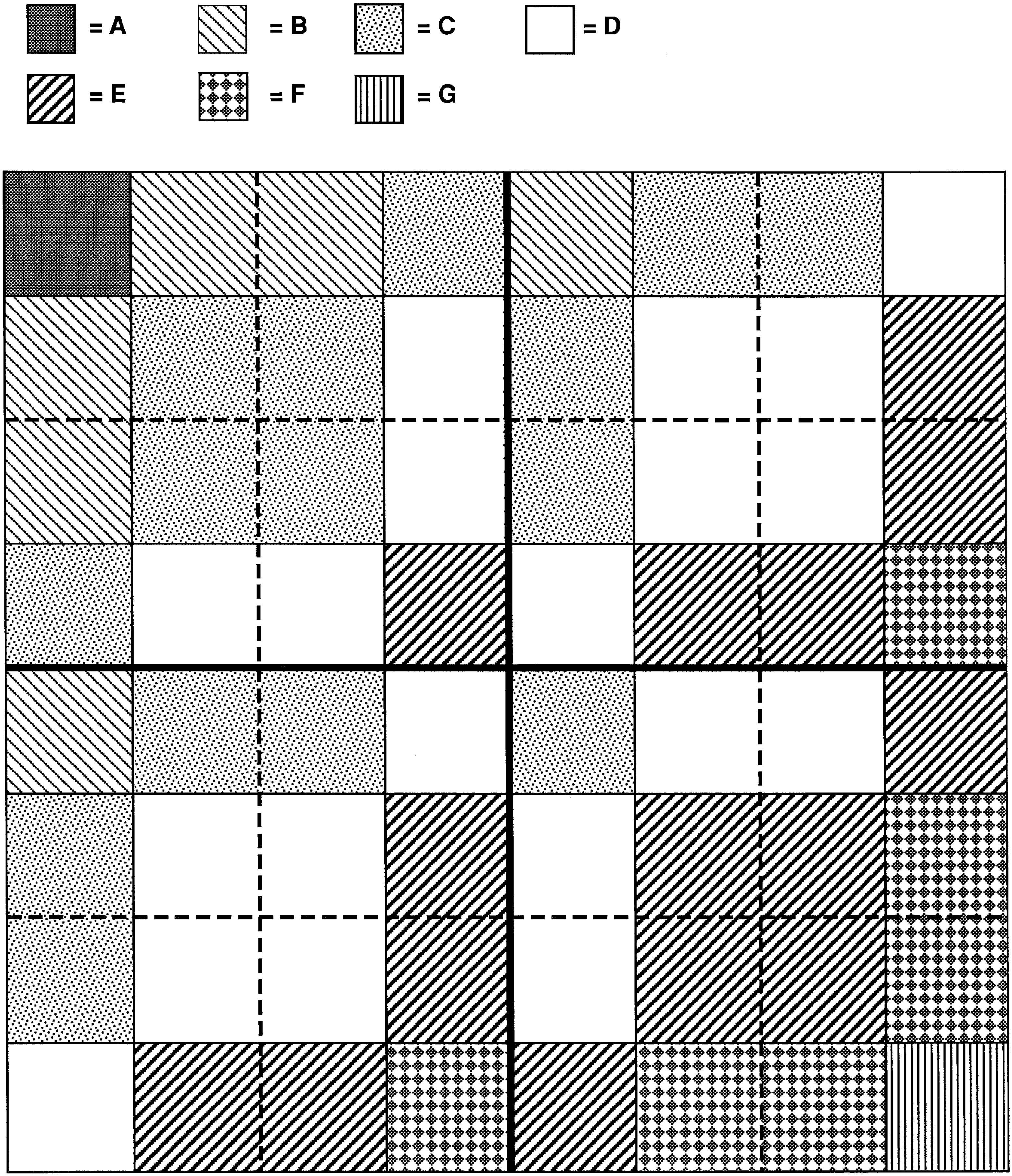
Which square would you choose to be the winner? Why?

E E E E E E	E E E E O E	E E O E E E	E E O E O E	O E E E E E	O E E E O E	O E O E E E	O E O E O E
E E E E E O	E E E E O O	E E O E E O	E E O E O O	O E E E E O	O E E E O O	O E O E E O	O E O E O O
E E E O E E	E E E O O E	E E O O E E	E E O O O E	O E E O E E	O E E O O E	O E O O E E	O E O O O E
E E E O E O	E E E O O O	E E O O E O	E E O O O O	O E E O E O	O E E O O O	O E O O E O	O E O O O O
E O E E E E	E O E E O E	E O O E E E	E O O E O E	O O E E E E	O O E E O E	O O O E E E	O O O E O E
E O E E E O	E O E E O O	E O O E E O	E O O E O O	O O E E E O	O O E E O O	O O O E E O	O O O E O O
E O E O E E	E O E O O E	E O O O E E	E O O O O E	O O E O E E	O O E O O E	O O O O E E	O O O O O E
E O E O E O	E O E O O O	E O O O E O	E O O O O O	O O E O E O	O O E O O O	O O O O E O	O O O O O O

VIII-4 Master for Transparency B

EEEEEE A	B	B	C	B	C	C	D
B	C	C	D	C	D	D	E
B	C	C	D	C	D	D	E
C	D	D	E	D	E	E	F
B	C	C	D	C	D	D	E
C	D	D	E	D	E	E	F
C	D	D	E	D	E	E	F
D	E	E	F	E	F	F	OOOOOO G

 Master for Transparency C



VIII-4 Master for Transparency D

VIII-4 Grid Blank

Checker-B Game

O V E R V I E W

Results from a binomial experiment with unequally likely outcomes (odd and even products of faces of two regular dice) are compared to theoretical probabilities. Comparisons are made between the Checker-A and Checker-B games to point out the differences between equally likely and unequally likely binomial experiments.

Prerequisite Activities

Unit VIII, Activities 3 and 4, *The River Crossing* and the *Checker-A Game*.

Materials

For each student—a pair of regular 6-sided dice, the Checker Game board and a small chip to play the game. Transparencies, activity sheets, masking tape.

Actions

Part I: Experimental Data

1. Pass out to each student: a copy of Activity Sheet VIII-5-A, the Checker Game board, a pair of 6-sided dice and a chip. Put up Transparency A, the Checker-B Game. Explain the game to the students. Even tosses move **Left**, odd tosses move **Right**. Put up a transparency of Activity Sheet VIII-5-A, the game board, and play the game once with the students, letting them roll the dice while you move the chip on the game board at the overhead.

2. Ask the students, "On which square do you think the chip will land at the finish?" Solicit student opinions.

Comments

1. A play consists of six tosses. **Left** and **Right** are determined from the **Start** square on the board. That is, if a player moved **Right** 6 times in a row, the chip would end up at square G. You will need to make several transparencies of the game board.

2. Students may believe the center square is the most likely outcome, as occurred in the Checker-A Game. It is also possible that some students may think that all squares have an equal chance of winning. Other students may think that some squares have more chances to win than others.

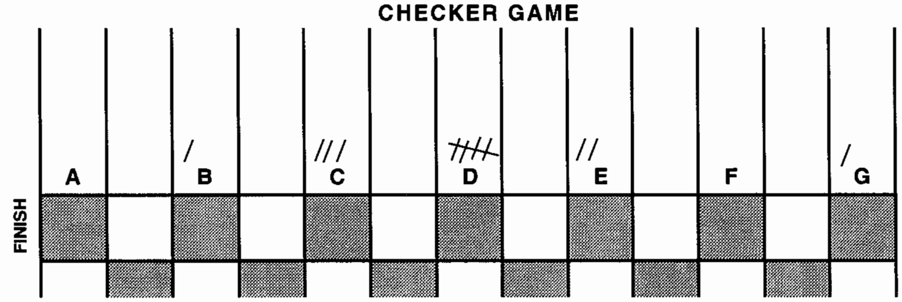
A question may come up during the discussion about whether **Odd** and **Even** products "are fair". That is, students may suspect that odd and even products are not equally likely to occur in this game. If students wish to get more information about the number of each type of product, you might first want to skip to Action 8 (Part II, Action 8–10) and then come back to gather the game data after doing the product analysis. This can be done in either order: (1) first the game data and then the product analysis for odd and even or (2) first the product analysis and then the game data.

Actions

3. Pass out copies of Activity Sheet VIII-5-B. Ask the students to imagine playing the game 100 times. On Activity Sheet VIII-5-B, ask the students to record their guesses for the number of times the chip will land on each square when they play the game 100 times. After they make their guesses, ask them to compare their guesses with other members of their group. Ask for several volunteers to share their guesses and their reasoning with the class.

4. Ask the students to play the game and complete Activity Sheet VIII-5-B. Each time they end at a lettered square, ask the students to make a tally mark (/) on the game board in the column above that square.

CHECKER GAME



5. When the students have completed the 100 trials in their groups, ask them to use their data to calculate the experimental probabilities of landing at each square and to record these at the bottom of Activity Sheet VIII-5-B.

Comments

3. Students may notice that there is only one path to squares A and G, while there are many more routes to some of the interior squares. They may also suspect that although the board is symmetric (there are the same number of paths to B as to F, and so forth), not all the paths are equally likely to occur in this game, as they were in the Checker-A Game. Groups of 4 or 5 students can work together on this activity. A transparency of Activity Sheet VIII-5-B is helpful.

4. Each group will need a total of 100 plays of the game.

5. You may need to remind the students that the "experimental" probability for each square can be calculated from (# occurrences)/100, where 100 = # trials. Thus, if they landed on the C square 15 times, *n* P(C) = $^{15}/_{100}$ = .15 = 15%.

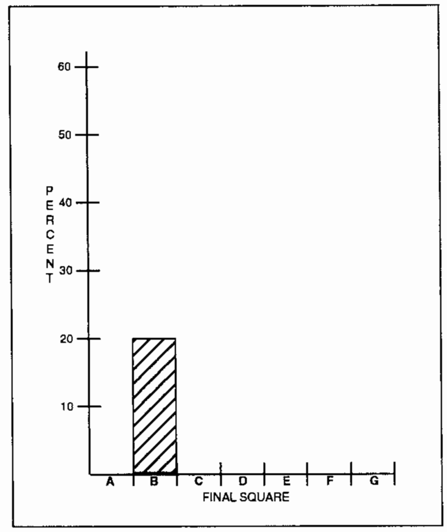
Actions

6. Pass out a copy of Activity Sheet VIII-5-C to each group of students and put up a transparency of Activity Sheet VIII-5-C. Ask the students to make a graph of the results of their 100 trials and to tape their results to the wall.

7. Discuss the results of the experiment with the students. Were they surprised by the results? Do the results make sense to them? How did the experimental results compare with the predictions they made before they did the experiment? How do the results of the Checker-B Game differ from those of the Checker-A Game?

Comments

6. You might demonstrate by filling in the bar for one of the outcomes at the overhead, using a group's data for that square. Below is an example from a group which landed on the B square 20 times in 100 trials.



7. It is important to get students to compare their experimental results to their original predictions. Students can also be asked to compare their individual results to their total group results. Was there any variation in their individual results? What are the similarities or differences between their individual results (their own tallies) and the group total tally results? What are the similarities or differences between the groups' results posted on the wall? How would they account for any similarities or differences?

It is also important to encourage the students to compare the two checker games. Are there any similarities, any differences, between the results of the two games? How would they account for these similarities and/or differences?

Actions

Comments

Part II: Analysis of the Product of Two Dice

8. Ask the students, "If two dice are rolled, do you think that even products are as likely to occur as odd products? How could we find out?" Discuss.

8. If students have completed actions 1–7 and have played the game, they will know that odd and even are not equally likely to occur in the Checker-B Game. Students might suggest gathering some data on odd and even tosses, to get experimental probabilities for odd and even products. Do this if they suggest it. Ask them how many trials they want to consider for their experiment.

Some students may recall the grid model that was used for the sum of two dice in The River Crossing (Activity VIII-3) and suggest using that approach to see if odd and even products are as likely to occur.

9. Remind the students about the grid model that was used to analyze the River Crossing Game. Pass out a copy of Activity Sheet VIII-5-D to each of the students and put up a transparency of Activity Sheet VIII-5-D. Fill in a few of the squares on the transparency, as below. Then, ask students to complete Activity Sheet VIII-5-D.

2nd die

1st die: X	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6			
3	3	6	9			
4						
5						
6						

Theoretical Grid

9. If the grid model did not arise in the discussion in action 8, remind the students of the approach to the River Crossing Problem. You might put up the transparency of that grid from Activity 3 or just sketch the grid for the sums of two dice on the overhead.

2nd die

1st die: X	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

10. Ask students, "Based on the grid model, what is the theoretical probability that we get an **Even** product from rolling two dice? What is the theoretical probability of an **Odd** product?" Start a discussion on the results of Activity Sheet VIII-5-D with the students. Ask for volunteers to fill in their results at the overhead.

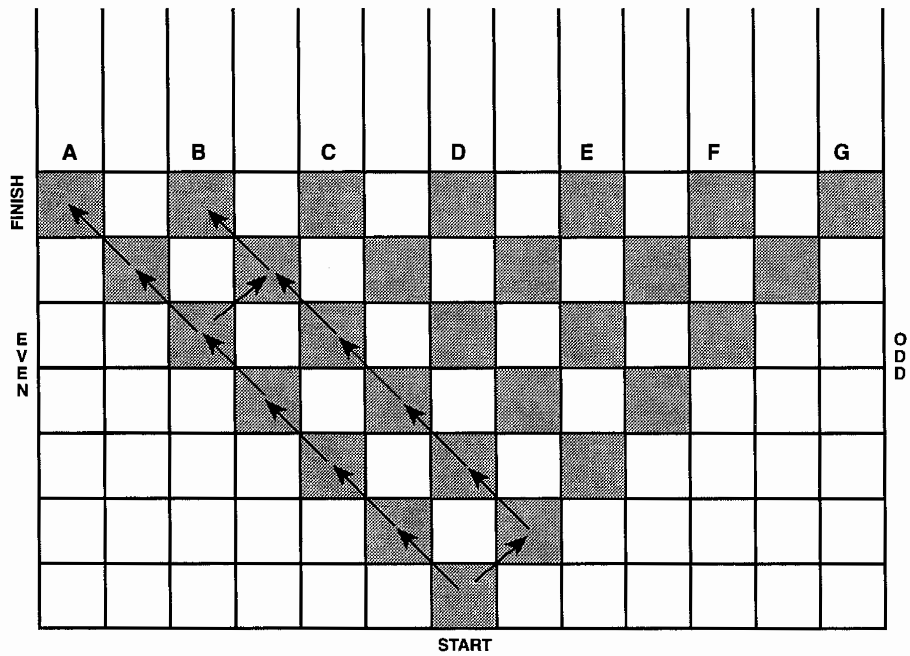
10. If the students collected experimental data on **Odd** and **Even** products of two dice, ask them to compare their experimental results to the theoretical probabilities for **Odd** and **Even** products based on the grid model.

Actions

Comments

Part III: Developing a theoretical model

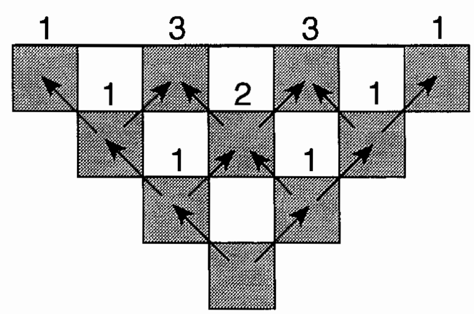
11. Ask the students how many different paths there are on the checker board to reach each of the lettered squares. Draw several examples of paths on a transparency of the game board.



When the students have finished, ask for volunteers to share their results. Ask the students if they found any patterns as they counted the paths to the squares.

11. In the figure at the left we see there is only one path to get to the square labeled A—we must go left (toss **Even**) six times in a row. However, there are many paths to some of the other squares. Two paths to square B are given. There are many patterns that can arise as students look for paths. The students may notice there is symmetry in the paths, that is, there is also only one path to square G; the number of paths to square B is the same as the number of paths to square F, and so forth. You may need to pass out extra copies of Activity Sheet VIII-5-A (the game board) as the students count the paths. You may find that a sequence of E's and O's is a useful notation to represent paths and keep them straight. For example, EOEEOE represents an even-old-even-even-odd-even sequence of tosses and is a path that ends at square C.

You might suggest to the students that they keep track of the number of paths that land at the intermediate squares, as below. These are the numbers of Pascal's triangle. There are 64 different paths altogether that end at the lettered squares.



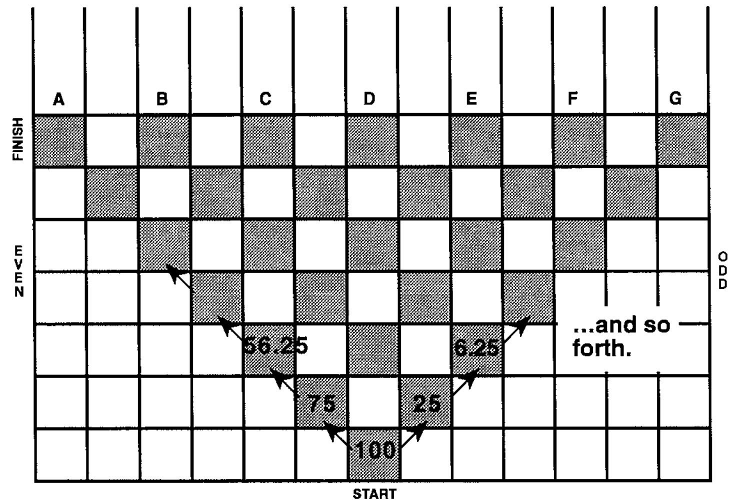
If you have done Checker-A with your students, this part is a review.

12. Ask the students how they might determine the "theoretical" probability of landing on each square in the Checker-B game. Have them discuss this in their groups first and then share their ideas with the whole class.

12. It might help to recall the discussion about theoretical probabilities from the Checker-A Game. Of course, in the Checker-B Game the paths to the ending squares do not all have an equal chance of occurring, like they do in the Checker-A Game. Turning left and turning right at each juncture do not have an equal chance of occurring, since Prob(**Even**) = .75 and Prob(**Odd**) = .25 in the Checker-B Game. This particular question might be a good one for the class to think about for a while. You might use it as a homework problem.

Actions

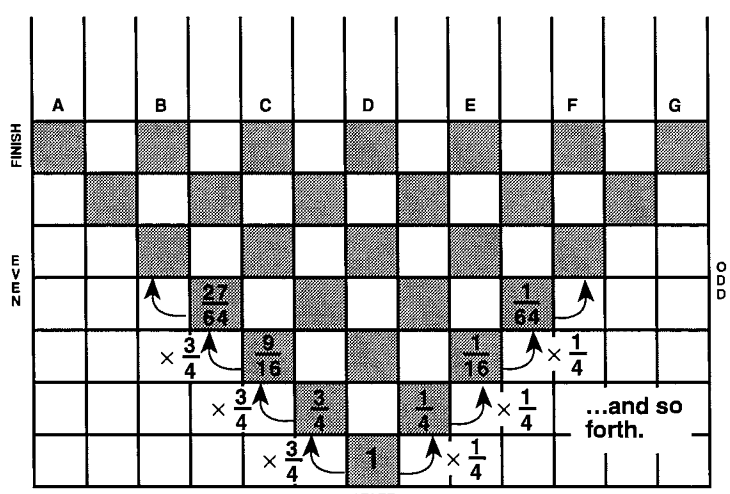
13. Put up a transparency of the game board. Write the number 100 on the Start square? Ask the students: If 100 chips were used in the starting square, and we rolled dice for each one looking at products, about how many of them would we expect to move to the left? to the right? Now if we rolled again, about how many of those would we expect to move to the left? to the right?



Comments

13. Wherever our chip is on the board, when we toss an even product, we move left. We expect $\frac{3}{4}$ of the chips to move left to the next level on the game board. Similarly, $\frac{1}{4}$ will move to the right, as the chance of an odd product at any time is $\frac{1}{4}$. So, if we started with 100 chips, we would expect 75 of them to go left and 25 to go right. At the next level $\frac{3}{4}$ of the 75, or 56.25, of the chips would go left the second time. This may help the students get started in developing a theoretical model for the Checker-B Game.

14. Pass out copies of Activity Sheet VIII-5-E, ask students to work in their groups to complete it. (Make several transparencies of Activity Sheet VIII-5-E.) Ask the students to compare the theoretical probabilities they obtained for each square to the experimental probabilities on their graphs. How close were their experimental results to the theoretical probabilities? Were there any surprises? Ask for volunteers to share their results and their reasoning.



14. It might help students to use successive products to find the probability of a particular path occurring. We expect $\frac{3}{4}$ of the chips to move left after the first toss and then $\frac{3}{4}$ of those to move left again on the second toss. That is, $\frac{3}{4}$ of $\frac{3}{4}$, or $\frac{9}{16}$, of the chips would move left on the first two tosses.

From the Checker-A Game recall that one way to represent a path is with a sequence of letters like EOEEOE. This path would end up at square C. The chance of getting path EOEEOE is then $\frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4}$, or $(.75)^4\ (.25)^2 = .02$, approximately. We write P(EOEEOE) = .02. Of course, this only gives us the probability of one particular path to square C in the Checker-B Game. We have to consider all the possible paths to square C. Students may discover that paths in the Checker-B Game with the same number of right and left turns have the same probability of occuring. For example, EOEEEO will also land on square C and P(EOEEEO) = $(.75)^4\ (.25)^2$ also.

Actions

15. Pass out copies of Activity Sheet VIII-5-F and ask the students to complete it, reflecting on their experience with the two Checker Games.

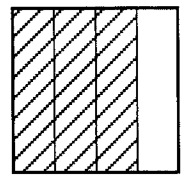
16. (Optional) Challenge: Ask your students, "Can you devise an area model for the theoretical probabilities of landing on the squares in the Checker-B Game?"

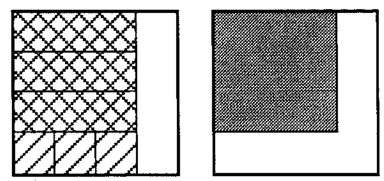
Comments

15. This activity sheet may make a good extended homework activity.

16. This is a much trickier problem than in the Checker-A Game where we obtained a nice pattern for an area model and were able to make posters of it. Students may need some time to try various things with this challenge. For example, here is an area model of the first toss being an even product and then the second toss also being an even product (¾ of ¾).

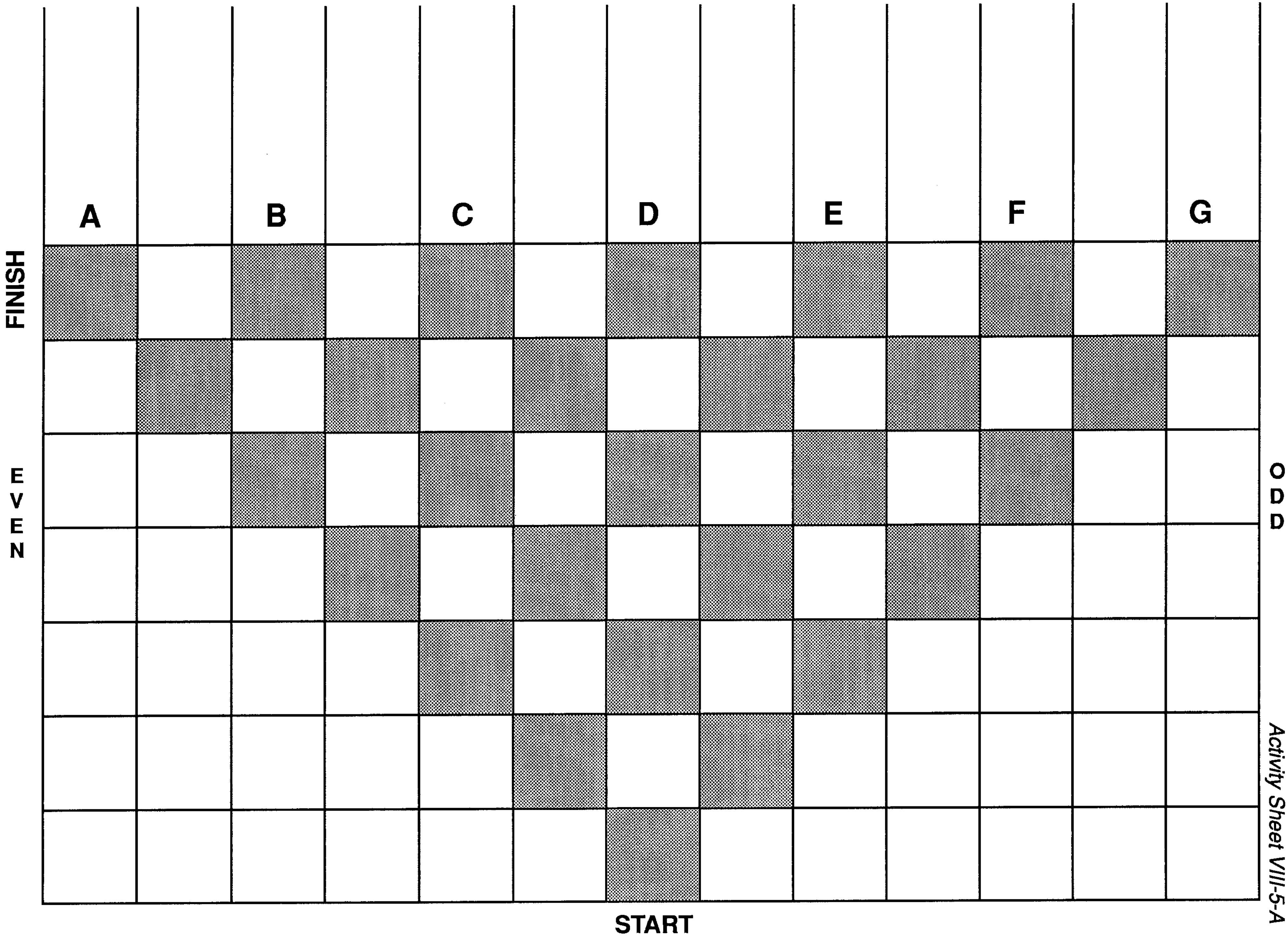
75%





75% of 75%

CHECKER GAME



Name ____________________________

Checker-B Game

1. Imagine playing the Checker-B Game 100 times. Make some predictions: How many times do you think the marker would end up at square A? at square B? etc. Record your predictions here.

Ending Square:	A	B	C	D	E	F	G
Your Guesses:							

Compare your guesses with those of your groupmates. Were there any general observations about the game that prompted your answers?

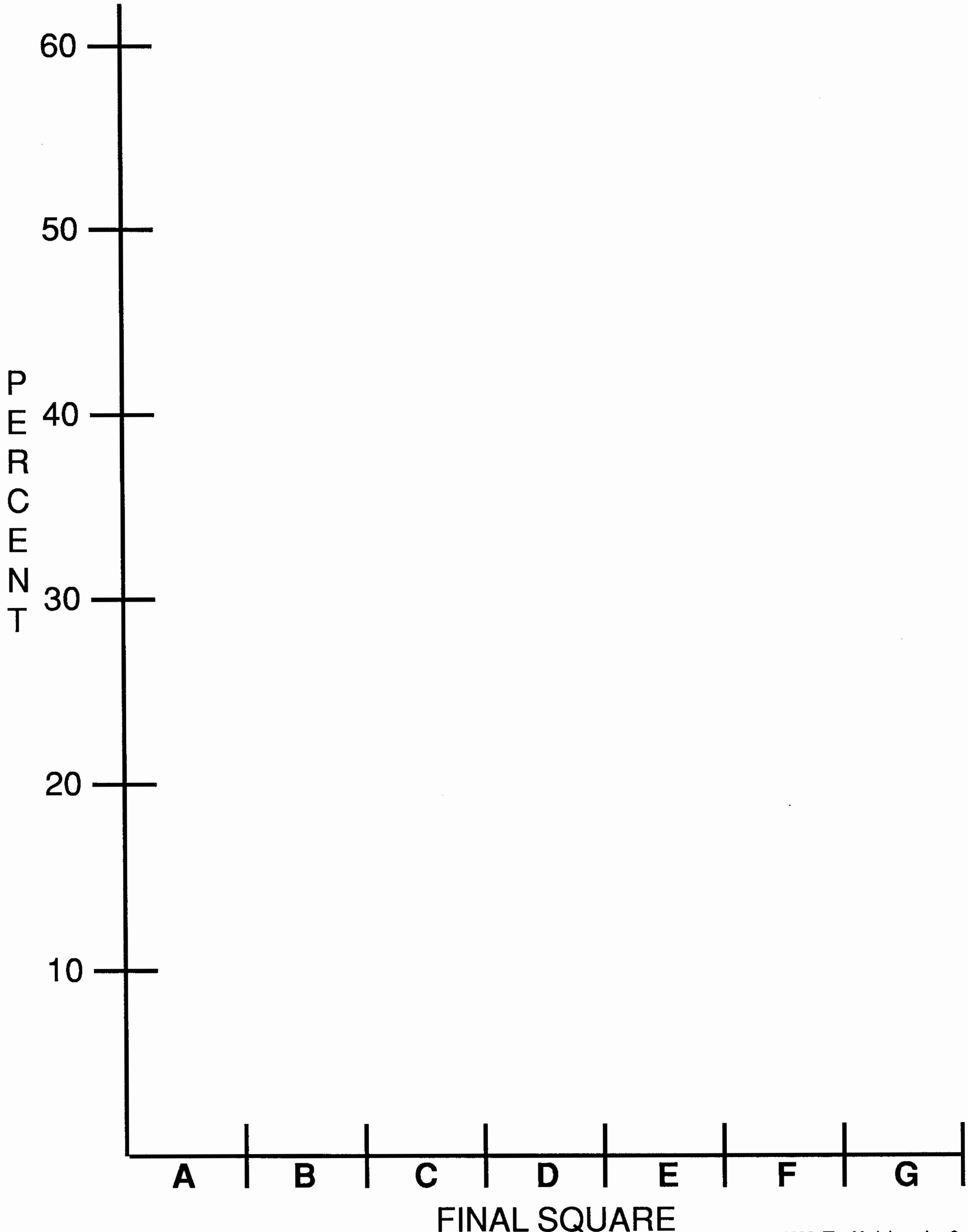
2. Each person in the group now plays the game a number of times. With tally marks keep track of the number of times the marker ends at each of the lettered squares. Put the results obtained by each person into the chart below and complete the chart. Your group will need a total of 100 plays.

	Ending Square							
Player's Name	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**Total Plays**
1								
2								
3								
4								
5								
Group Total								**100**

3. Use your group's results from the chart to calculate the "experimental" probabilities for landing on each of the lettered squares. Write your results below.

P(A) = P(B) = P(C) = P(D) =

P(E) = P(F) = P(G) =


60
50
40
30
20
10
PERCENT
A
B
C
D
E
F
G
FINAL SQUARE

Name ______________________________

Grid Model for Possible Products From Two Cubical Dice

1. Complete this grid.

2nd die

X	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

1st die

Theoretical Grid

2. Use the grid to find these probabilities of products from tossing two cubical dice

Prob(even product) =

Prob(odd product) =

Prob(12) =

Prob(6) =

Prob(number > 12) =

Prob(number≤ 9) =

Prob(perfect square) =

Prob(multiple of 3) =

Activity Sheet VIII-5-E

Name ____________________________

Comparison of Experimental and Theoretical Probabilities for the Checker-B Game

Final Square	Experimental Probability	Theoretical Probability
A		
B		
C		
D		
E		
F		
G		

Name ____________________________

Reflection on the Checker-A and Checker-B Games

1. What differences would you expect in the results of 1000 plays of the Checker-A and Checker-B Games? Explain.

2. On which square would you bet in the Checker-A Game? Why? On which square would you bet in the Checker-B Game? Why?

3. Although the list of possible paths to a finish square is the same for both the Checker-A and B games, the probability that a path occurs is not the same for both games. Write a note to a friend explaining how you determined the theoretical probabilities of landing on each finish square in the Checker-B game and compare that to the theoretical probability for each square in the Checker-A game.

The Checker-B Game

Begin with a marker on the START square. A play of the game consists of tossing a pair of regular dice 6 times.

After each toss, find the product of the two top faces of the dice.

If the product is an EVEN number, move the marker one square diagonally to the LEFT.

If the product is an ODD number, move the marker one square diagonally to the RIGHT.

After six tosses, the marker will be on one of the squares marked A, B, C, D, E, F or G.

Unit VIII • Activity 6

Cereal Boxes

O V E R V I E W

Simulation by a probability experiment is a tool often used when a direct theoretical approach to a probability problem is inaccessible. The cereal box problem uses the "sample until" technique that frequently occurs in problems involving chance. Visual representation of data, such as median marks, line-plots and box-plots are introduced to get at the concepts of central tendency, range and variation.

Prerequisite Activities

Previous work with probability experiements, such as Unit VIII, Activity 3, is helpful.

Materials

A regular 6-sided die for each pair of students; transparencies and activity sheets for this activity. Note: students may also want calculators.

Actions

1. Put up Transparency A and read through the statement of the cereal box problem with the students. Ask students if they have any questions about the problem and clarify as necessary. Ask the students to think first to themselves about how many boxes they would have to buy. Pass out copies of Activity Sheet VIII-6-A. Ask the students to write down at the top of Activity Sheet VIII-6-A their own best guess for the number of boxes needed and to write down their reason for guessing that number of boxes.

2. Ask the students to share their guesses with the other members of their group. Then ask the students to share their guesses with the class. Record their guesses on Transparency B. Ask students if they have a reason for their guesses. Ask the students, "Is it possible to get all 5 stickers in exactly 5 boxes? Is it likely?"

Comments

1. Students will ask questions such as: Are there the same number of each sticker? Is there one sticker that is harder to get than the other ones? As such questions arise during this activity, discuss with the students the assumptions we may need to make as we solve such a problem. For example, we have no reason to believe any one sticker is harder to get, so let's assume all the stickers have the same chance of being in any one box. Also, we may want to assume the boxes are well mixed—that any store will have a random mix of all the stickers, rather than an oversupply of one or two stickers. These are the types of assumptions we make when we try to model a real world problem with a probability experiment. It is important to sensitize students to such assumptions and to discuss them as students raise questions.

2. Guessing the answer to a probability experiment is one way to get the students hooked on the problem. When they have invested some thought and energy by guessing, they are beginning to make it "their" problem. Guessing can also be used to build up intuition in probability problems. Students will improve with experience.

There likely will be a wide range of guesses for the number of boxes that need to be opened. It is not unusual for students' guesses to vary from the tens to the hundreds of boxes. If a student guesses exactly 5 boxes, ask the student if they think the chances are good for getting the 5 different stickers in the first 5 tries.

Actions

3. Ask students, "Since we cannot actually open all the cereal boxes in the classroom, how could we create an experiment to pretend to open boxes and check for the stickers?" Ask students to discuss this in their groups.

4. Ask the groups to share their ideas for creating an experiment to open the boxes and check for stickers.

5. Hold up a regular 6-sided die and ask the students, "Could we use this die to mimic opening the cereal boxes and checking for stickers?" Solicit student ideas and discuss with the students (see Comments). Tell the students that this type of process is called a *simulation* of the experiment with real cereal boxes.

Comments

3. You might remind the students that the "pretend experiment" should have the same assumptions as a real experiment in which we could open actual boxes and check. For example, there should be an equal chance for each sticker and the stickers should be evenly mixed throughout the boxes being opened.

4. Some groups may suggest putting pieces of paper with the numbers 1 to 5 in a bag, or using colored chips, and pulling them out. Other students might suggest rolling a die or using an equal area spinner. The idea of "sampling until" arises here. In the first box we open, we always get a new sticker. However, when we try for the second sticker, or a third sticker, we might just get the first one over again before we get a new one. Each time we try for a new sticker, we have to "sample until" we get one we don't already have.

Students may question whether the chips or numbers need to be put back in the bag after they are pulled out. (This question constantly arises in sampling problems: are we sampling with replacement or without replacement?) Ask the students how they can best assure that on each try each sticker will have exactly the same chance of occurring. Since our assumption is that each sticker has an equal chance of occurring each time, students may see that they have to replace the chips, or numbers, or whatever, to mimic opening real cereal boxes for this experiment.

5. The students may have already suggested rolling a die and letting the numbers 1 to 5 represent the 5 different stickers. We can keep track of which numbers have come up (the stickers we have collected) and the number of times in total we had to roll (the number of boxes we had to open) to get all 5 of the stickers. Since the die has 6 sides and there are only 5 stickers, the students may suggest that sixes just be ignored.

Actions

6. Pass out a regular 6-sided die to each pair of students. Put up Transparency C and demonstrate several rolls, tallying and crossing off the numbers that come up. Ask each pair of students to do one simulation trial of buying the cereal boxes by rolling the die until all the numbers from 1 to 5 have appeared. Ask the students to keep track of their rolls on Activity Sheet VIII-6-A, Trial 1.

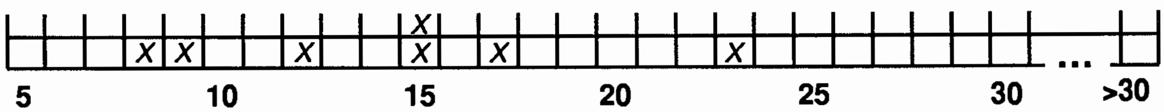
Trial	Tallies	Check off	# Opened
1	HHT HHT HHT III	~~1~~~~2~~~~3~~~~4~~~~5~~	18
2	HHT I	~~1~~2~~3~~~~4~~5	

Comments

6. One trial of the complete experiment consists of rolling the die until all the numbers 1 through 5 have appeared at least once. Sixes can be ignored and do not count as a roll. A tally is made for every roll (except sixes) and as a new number comes up it is crossed off. For example, at the left a pair of students took 18 rolls to obtain all 5 different stickers in their first trial experiment. In their second trial experiment, they have thus far rolled 6 times (6 tallies) and have gotten stickers 1, 3 and 4.

7. As each pair finishes their first trial, ask them to come to the overhead and record on Transparency D the result of the trial by placing an X on the line plot on the number of boxes they needed to open.

7. A sample of trials from a few pairs of students might look like this plot. Don't show Transparency D on the overhead screen until each pair has recorded its results.



8. Ask the students, "Now that we have some experimental data, how many boxes do you think you would have to open to collect all 5 stickers? Has anyone changed their mind from their first guess? How so?" Discuss.

8. You might raise such questions as: What do you think is the least number you'd have to buy? What is the greatest number you'd have to buy? Theoretically one might have to buy a very large number of boxes. Do the students think it would take a very large number or not?

9. Ask the students, "What could we do to get a better idea about how many boxes of cereal someone would expect to buy in order to collect all the sticker prizes?"

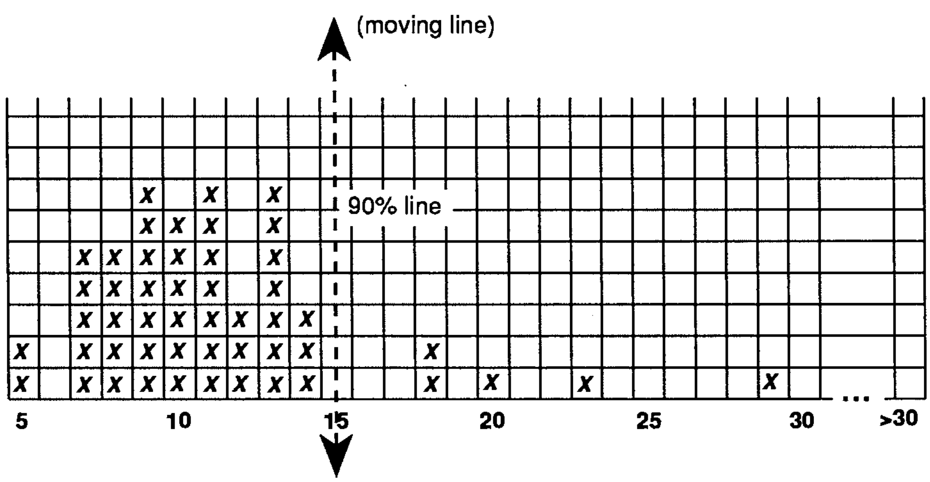
9. The students will probably suggest gathering more trials, especially if they have worked with other probability experiments. There is a lot of variation in a small number of trials of this experiment. One trial per pair is unlikely to give reliable or satisfactory information to the students.

10. Have each pair of students conduct the complete experiment of collecting the stickers at least 10 times, taking turns tossing the die and recording the results on Activity Sheet VIII-6-A. As they finish collecting their results ask them to come to the overhead and record their data on the line plot on Transparency D.

10. A total of about 50 or more trial experiments will yield a fairly representative data set for this simulation. Pairs of students should perform the experiment at least 10 times each (more times if you need more data) so they experience the potentially wide variation in results from one trial to the next.

Actions

11. Start a discussion with the students on the results of the experiment. Suppose one of their friends wanted to collect all the stickers? What would they tell their friend? How many boxes might the friend expect to buy to feel fairly certain of getting all the stickers? About how many would you have to buy to feel "90% certain" that you get all 5 stickers? How about if we wanted to feel 80% certain? 50% certain?



Comments

11. Two big ideas that may surface in this discussion are the notion of "likely range of values" and the concept of average. Students may use the graph to point out the most frequent number of boxes (called the mode). Also, students might suggest finding the average in the sense of the "mean" by summing all the values on the graph and dividing by the total number of trials. They will need calculators if they wish to pursue finding the mean. Finding the mean is a bit tedious with all the data in this experiment The median is actually a more convenient way to find an average (see Action 12) in this experiment.

The idea of "likely spread" can first be addressed visually by choosing a range of values on the graph that includes most of the trials. For example, in the sample cereal graph below, an interval of likely values occurs between 5 and 14. The values outside of this range didn't occur very often. The numbers 23 and 29 are far away from the cluster of other results and for this reason are called "outliers" in the experiment.

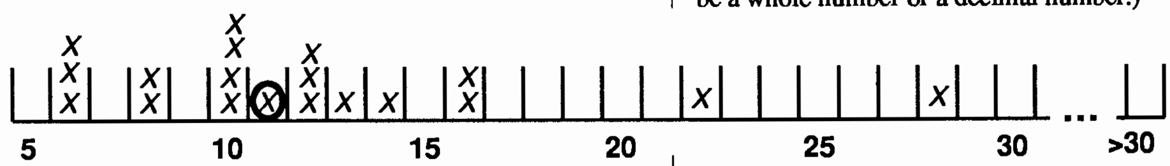
You may want to use a ruler (a clear one is best) to make a moving line on the graph of the students' data on Transparency D. For example, if we put a line through 15 on the graph on the left, 45 of the 50 pieces of data fall to the left of that line. Thus, 90% of the students' trials in this case took less than 15 boxes to collect all 5 stickers. We could say that according to our data we could feel 90% certain of collecting all 5 stickers in 14 boxes or less. (This is like a 90% experimentally generated confidence interval.) Ask students to say where the line should be drawn to feel 80% certain, to feel 50% certain. (80% certain would be 13 or less on the graph below and 50% certain would be 10 or less.)

Actions

12. **Concept of median.** Tell the students it is often convenient to find the middle point of data that is nicely ordered from smallest to largest, as our cereal data is on the graph. Put up Transparency E and tell the students this is some data from another cereal box experiment. Ask the students to count the number of trials and to find the median, the middle number of all these trials. Ask students to share their answers.

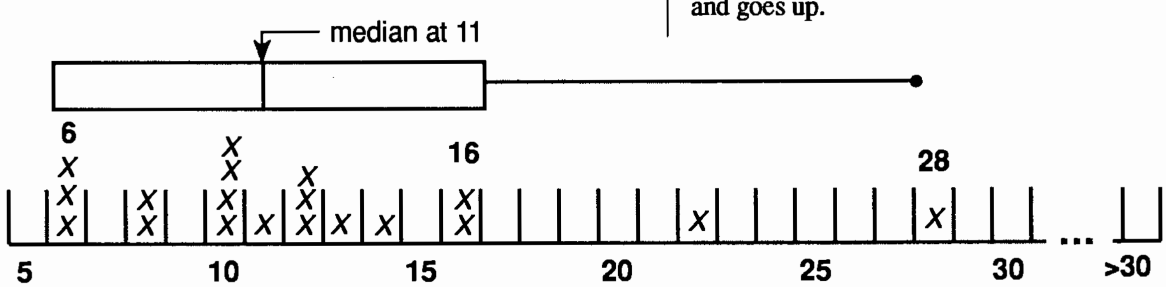
Comments

12. This middle point is called the median and is one way to get an "average" of numerical data. There are 19 pieces of data in the sample on Transparency E. The middle number of these 19 numbers occurs at the 10th piece of data, which in this case is 11 cereal boxes (circled below). For this data the median is not the most frequent (modal) number (which is 10 in this case), although the median is close to the mode in this case. (Note: if there were 20 pieces of data, then the median would occur at the midpoint of the 10th and 11th pieces of data. This might be a whole number or a decimal number.)



13. **Concept of Box-Plot.** Tell the students that the more likely outcomes from a data set can be "boxed" to provide another type of visual picture of the data. A whisker (or whiskers) is then put on the box to indicate the whole range of the data. Sketch this box-and-whisker for the data on Transparency E. Draw a vertical bar at the median value of 11. Tell the students this is a 90% box-and-whisker plot.

13. This particular box-and-whisker is called "a 90% box-and-whisker plot" since 90% (or very close to it) of the outcomes from this experiment are within the box. These are the most likely 90% of the outcomes. The whisker stretches out to 28 so as to include all the data. The number 28 would be considered an outlier in this data, since it is more than a box length (the box length is 10, ranging from 6 to 16) away from the box. In some experiments it makes more sense to pick the middle 90% of the data and give the box two whiskers. For the sticker collection data, the box really only stretches one way, since the data starts at 5 and goes up.



Actions

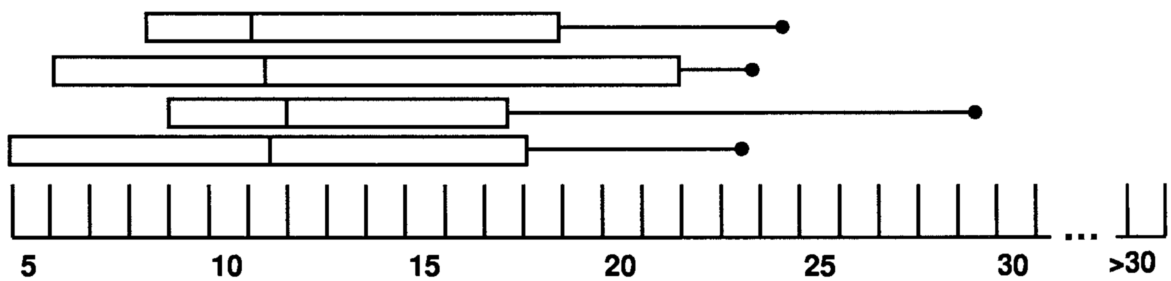
14. Pass out copies of Activity Sheet VIII-6-B. Ask students to put their own data together with at least one other pair's data (so at least 20 trials) and to plot it on Activity Sheet VIII-6-B. Ask them to place a line at the 90% certain mark for their data, then make a 90% box-plot and put in the median value. When the students have finished, ask them to sketch their box-plots (each with its median) at the overhead on Transparency F. Discuss the results. What do the students notice about the box-plots?

Comments

14. Students may notice some variability in the length of the box-plots. They may also notice most of the median values are close to each other. So, while there is some variability from group to group, there is also consistency in the box-plots; for example, they overlap a lot.

You may want to make several copies of Transparency F so each group can share its box-plot and median at the overhead.

A set of box-plots over the same number line, as below, provides a quick way to compare the results of several experiments.



Actions

15. Pass out copies of Activity Sheet VIII-6-C and ask the students to complete it. Start a discussion with the students on their responses to the activity sheet. Have them share their guesses during the discussion; also ask them to describe the experiment they devised and their results.

Comments

15. This activity might be used as a homework assignment. Line-plots contain a record of each individual piece of data and also the shape of the data (flat, mound-shaped, u-shaped, l-shaped, etc.), but take some time to construct. Box-plots show the range of the data, the likely values and the median. However, the individual values are lost and even the size of the data sample might not be apparent. Students' guesses will be much closer this time than they were the first time.

Name ______________________________

1. How many boxes will have to be opened to collect all 5 bike racing stickers? What do you think?

   a) My best guess is ______________ boxes?

   b) The reason I guessed this number of boxes is:

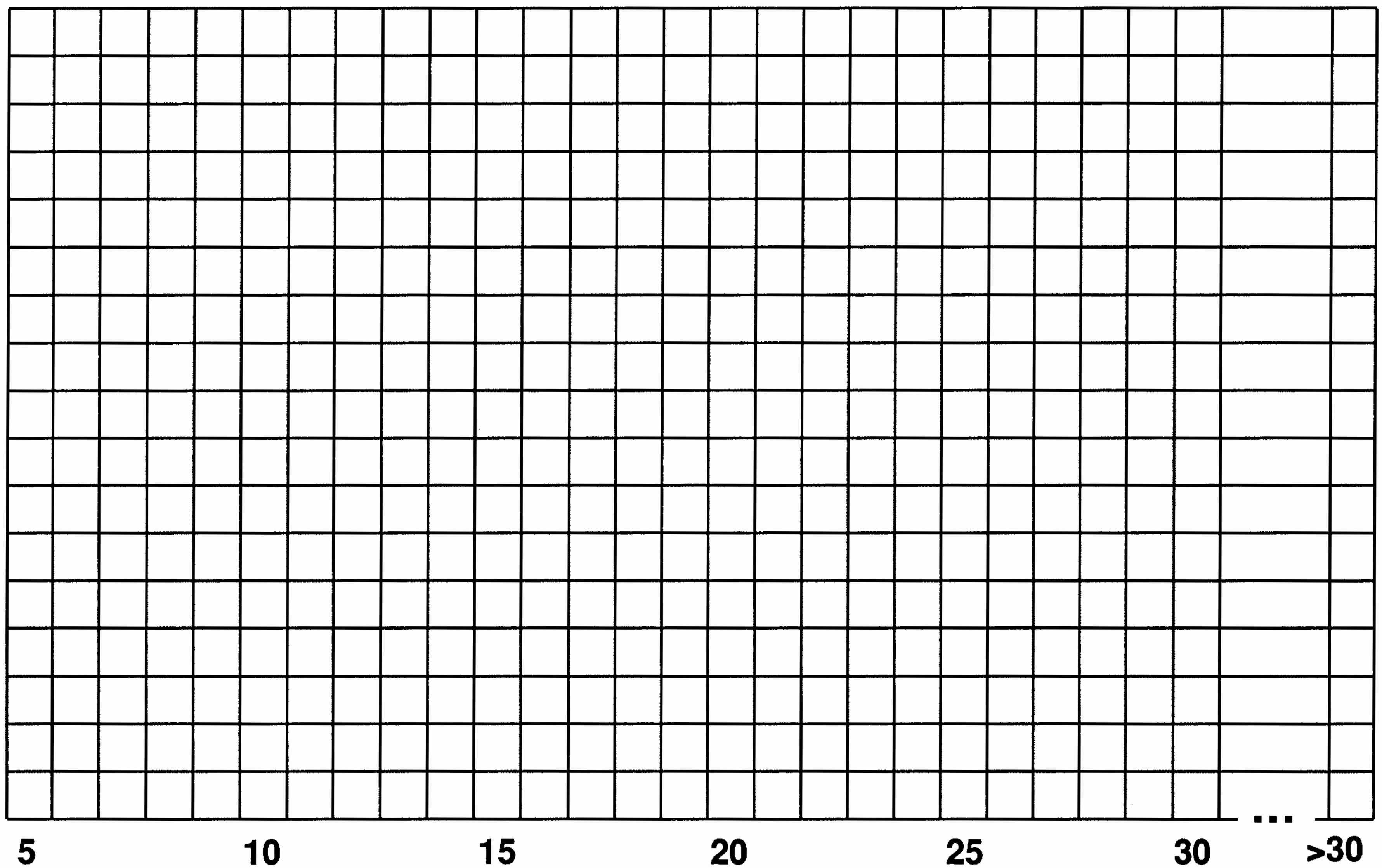
2. Record your data from simulating the cereal box experiment on this chart.

Trial	Tallies	Check off	# Opened
1		1 2 3 4 5	
2		1 2 3 4 5	
3		1 2 3 4 5	
4		1 2 3 4 5	
5		1 2 3 4 5	
6		1 2 3 4 5	
7		1 2 3 4 5	
8		1 2 3 4 5	
9		1 2 3 4 5	
10		1 2 3 4 5	

Name ____________

Activity Sheet VIII-6-B

Cereal Box Line-Plot



The median of this data is ______.   The total # of trials is ______.

People will probably need to buy at least_______boxes.

Name ____________________________

1. Now that you have completed the cereal box activity, how many boxes would you need to buy to collect all 5 stickers? Explain your reasoning.

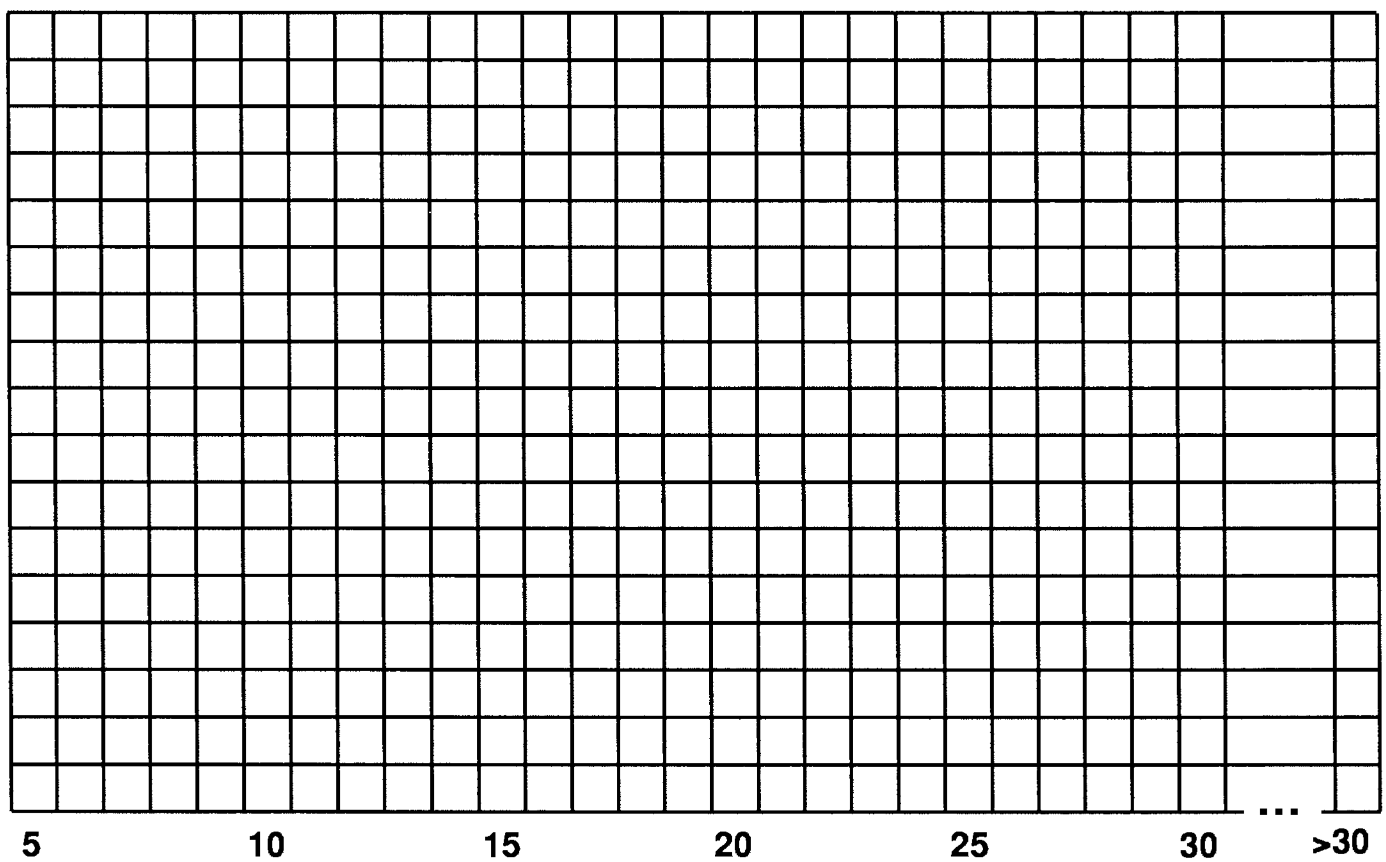
2. We have used both line-plots and box-plots to visualize the data from the cereal box simulation experiment. What are some advantages of each of these types of plots? What are some disadvantages? Explain.

Name ______________________________

Activity Sheet VIII-6-C
Page 2

3. Suppose there were 6 stickers in the cereal boxes. How many boxes would you need to buy to collect all 6? ________(your guess).

Devise an experiment to test the number of boxes you would have to buy to collect 6 stickers. Collect some data based on your experiment. Show the results of your data below. Make a 90% box-plot for your data. How did your results compare to your guess above?



Cereal Boxes

General Mills company once included bike racing stickers in their boxes of Cheerios. There were 5 different stickers. Each box contained just 1 of the 5 stickers.

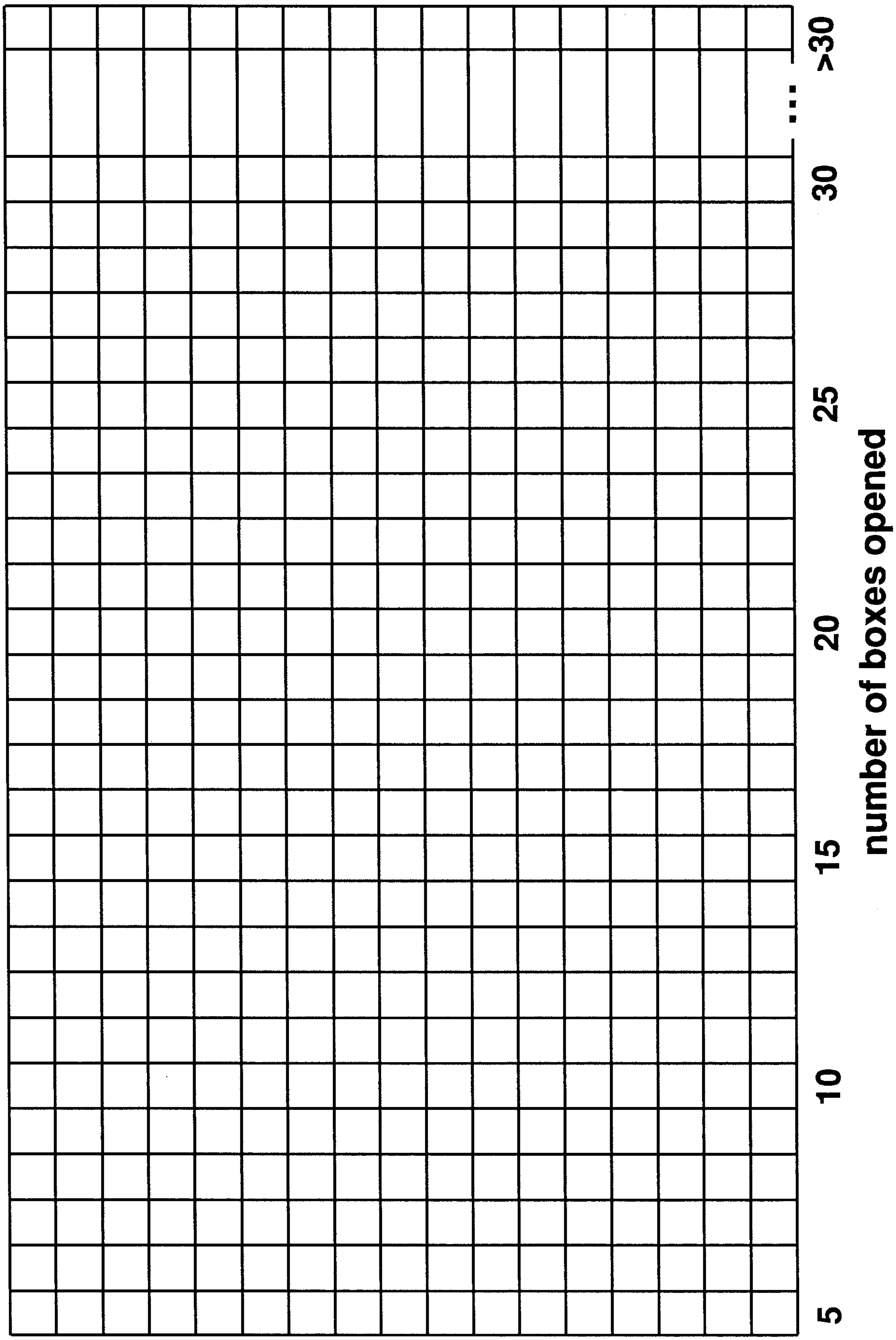
How many boxes of Cheerios would you expect to have to buy in order to collect all 5 stickers?

Make a guess. Write it down.

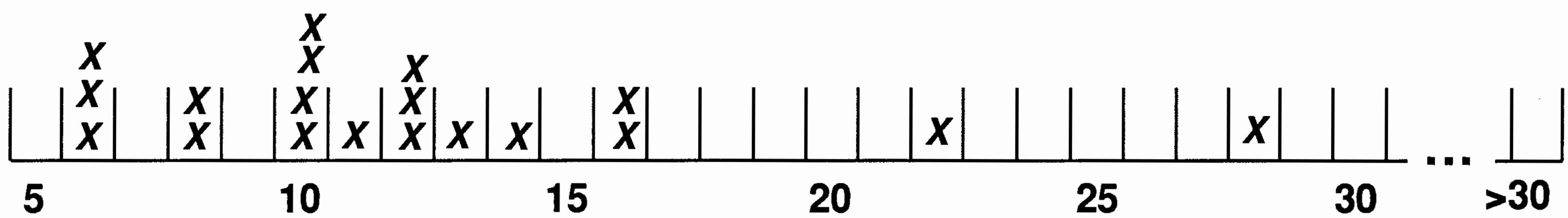
Guesses for the number of cereal boxes needed to collect all the stickers.

Trial	Tallies	Check off	# Opened
1	______	1 2 3 4 5	______
2	______	1 2 3 4 5	______
3	______	1 2 3 4 5	______
4	______	1 2 3 4 5	______
5	______	1 2 3 4 5	______
6	______	1 2 3 4 5	______
7	______	1 2 3 4 5	______
8	______	1 2 3 4 5	______
9	______	1 2 3 4 5	______
10	______	1 2 3 4 5	______
11	______	1 2 3 4 5	______
12	______	1 2 3 4 5	______
13	______	1 2 3 4 5	______
14	______	1 2 3 4 5	______
15	______	1 2 3 4 5	______

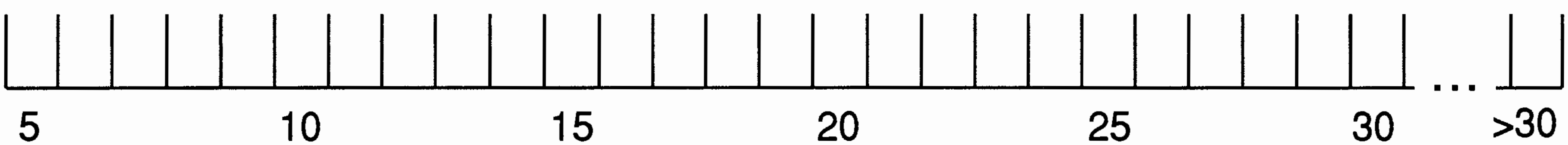
Cereal Box Line-Plot



Cereal Box
Sample Line-Plot



Box-Plots
of Student Data



Monty's Dilemma

OVERVIEW

A probability simulation in which this game can be played many times very quickly proves to be a very powerful mechanism for understanding what the best strategy is—to stick or to switch.

Prerequisite Activities

Previous work with probability simulations, such as Unit VIII, Activity 6, is helpful.

Materials

A bobby pin and a die (or coin) for each student; transparencies and activity sheets for this activity.

Actions

1. Put up Transparency VIII-7-A, Monty's Dilemma, and explain the game to the students. Ask the students to discuss in their groups whether they think it best to stick or switch. Solicit student opinion. Who thinks it is best to stick and why? Who thinks it is best to switch and why?

2. Put up Transparency VIII-7-B, Monty's Strategies, and explain each strategy to the students.

3. Ask the students, "How could we devise an experiment to play Monty's game many times using each of the three strategies to see how often we win with each strategy?"

Comments

1. At first it is important the students understand how the game is played. Monty reveals one of the doors they haven't chosen—always a gag prize—and then asks them if they would like to stick with their original door or switch to the other (unopened) door.

Students may feel it makes no difference whether you switch or stick, after Monty shows you a door. The feeling is: there are now only two doors left in the game, so what difference could it make, your chances are 1 out of 2, whether you switch or not.

2. At this point we modify the original game slightly by introducing a third strategy. The Stick and Switch strategies are the same as before. The Flip strategy is introduced because this emphasizes that after a door is opened, if we start over and randomly choose (say by flipping a coin) one of the two unopened doors, our chances of winning should definitely be 50%, or 1 out of 2. Students may feel there is no difference at all among the three chances, that it is ½ for all three strategies.

3. Students may suggest various possibilities with coins, dice or spinners. We need a way to simulate each of the three strategies—Stick, Flip and Switch.

Actions

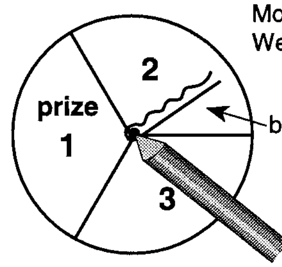
4. Put up a transparency of Activity Sheet VIII-7-A. Explain that just for convenience the prize is behind Door 1. Demonstrate the Stick strategy using the bobby pin spinner.

5. Demonstrate the Flip strategy using the bobby pin spinner and then tossing a die (or flipping a coin) to rechoose a door after Monty reveals a gag gift.

6. Demonstrate the Switch strategy using a bobby pin spinner.

Comments

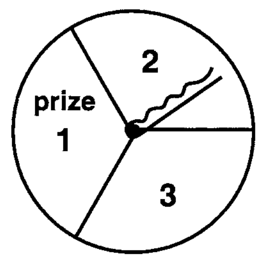
4. In the Stick strategy, we spin and, say, we get Door 2 as below. Monty shows us Door 3 (the gag prize). We stick, the prize is behind Door 1, so we lose on that turn.

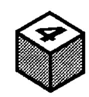


(Hold the bobby pin in place with the point of a pencil.)

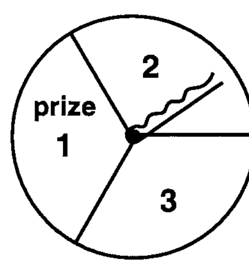
5. In the Flip strategy, we spin and, say, we get Door 2 as below. Monty will show us Door 3, a door with a gag prize behind it. Then, we will roll a die (or flip a coin) to decide whether we stick with Door 2 or switch to Door 1. Suppose that for odd tosses we stick with Door 2, for even tosses we switch to Door 1. If we roll a 4, as below, we switch to Door 1 and we win on that turn, as the prize is hidden behind Door 1.

Monty shows door 3. We get an **Even** toss—we Switch to door 1, we win!





6. In the Switch strategy, we spin and, say, we get Door 2 as below. Monty will show us Door 3, the gag prize. We switch, so we switch to Door 1. The prize is behind Door 1, so we win on that turn. Of course, if we spin and get Door 1, Monty could show us either Door 2 or Door 3 (both are gag prizes). We switch, so we would lose on that turn.



Monty shows door 3. We Switch from door 2 to door 1—we win!

Actions

7. Ask the students if they understand how to use the spinner and the die (or coin) to simulate playing the game with each strategy. Play the game several times more at the overhead if there are still questions needing to be clarified.

8. Pass out a bobby pin and a copy of Activity Sheet VIII-7-A and Activity Sheet VIII-7-B to each student. Ask each of the students to collect data for 100 trials of *each* of the three strategies and to record their results on Activity Sheet VIII-7-B. Put up a transparency of Activity Sheet VIII-7-B.

9. Ask the students to post their results for each strategy at the overhead on Transparency VIII-7-C or on the board if more room is needed. Start a discussion. Which of the three strategies gave the best chance of winning the most turns? Why did this happen? Can the students explain what the chances are of winning the game under each of the three strategies?

Comments

7. (No comments.)

8. Students can play the Stick strategy 100 times, then the Flip strategy 100 times, then the Switch strategy 100 times. Each time they can record whether they won or lost by putting a tally mark on Activity Sheet VIII-7-B. It is *very important that the students actually physically carry out* the simulation in each case, because in the process of doing this simulation, they will understand exactly what the chances are of winning under each of the three strategies and why. Teachers who have not done this activity before should also do this simulation of each of the three strategies for themselves, so they understand the chances of each strategy, too.

It may be a better use of time to ask the students to complete their simulations as a homework activity and bring the results to class the next day. This is a great opportunity to involve parents in the problems and in the collection of the data. Students can explain the game to their parents and ask them to help gather the data.

9. The data should point overwhelmingly to the Switch strategy as the best strategy for winning the most games. Most of the Stick strategy data from the students will show less than ½ chance of winning, most of the Flip strategy will show near ½ chance of winning and most of the Switch strategy data will show considerably more than ½ chance of winning. There may be a student or two who obtains data different than this, but most students will obtain data as we have described.

Students will probably come up with this reasoning themselves after doing to simulation. If necessary, help it along with good questioning.

Continued next page.

Actions

Comments

9. *Continued.* How can we win in the Stick strategy? Only if we chose the right door the first time. Our chances of doing that should be 1 out of 3, so probability $\frac{1}{3}$ of winning.

How can we win in the Flip strategy? We have to flip the coin or roll the die to get the right door. Our chances of winning here really are 1 out of 2, or probability $\frac{1}{2}$ of winning. The difference between this and the Stick strategy is that we make a new choice, randomly, between two doors after one is shown to us. In other words, we use the information about the gag prize door. In the Stick strategy, we never actually use the information, we just stay where we are.

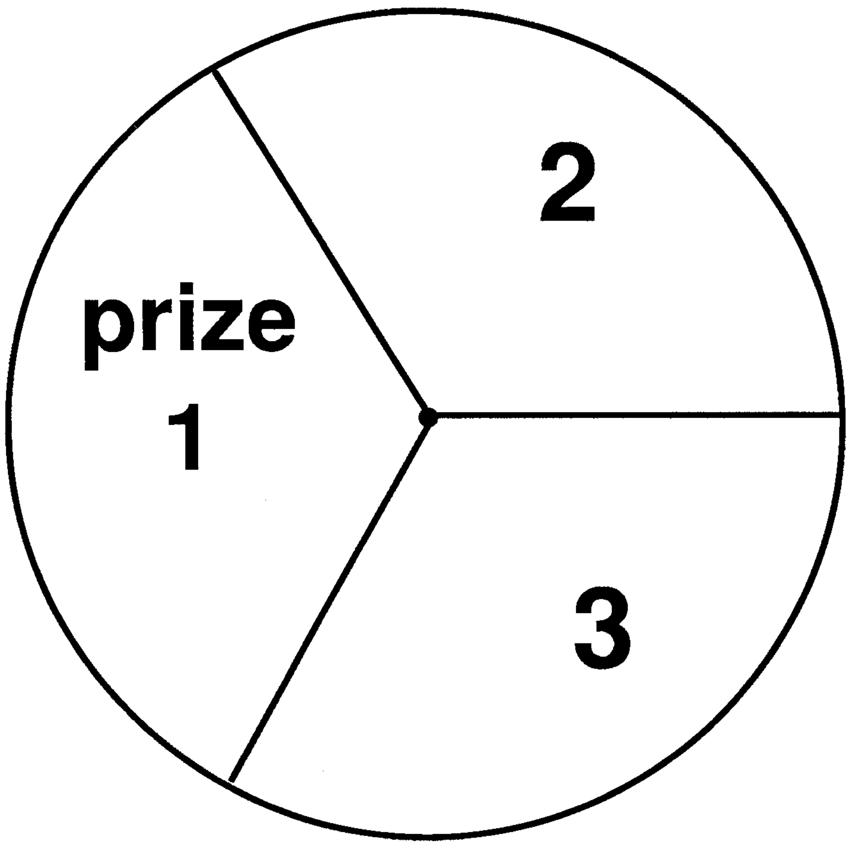
How can we win in the Switch strategy? Well, actually, the students may say that the real question is, how can we lose? The only way we can lose is if we actually picked the door with the prize behind it in the first place. If we pick Door 1 (the prize on our spinner model), Monty will show us either Door 2 or Door 3; we switch away from Door 1, we lose. However, if we pick Door 2 or Door 3 the first time, then Monty shows us Door 3 or Door 2 (the other one), we switch—to Door 1, the prize. So, the only way we lose is if we pick Door 1 first—so the probability of losing in the Switch strategy is $\frac{1}{3}$, thus the probability of winning is $\frac{2}{3}$!

10. Pass out copies of Activity Sheet VIII-7-C to the students and ask them to complete the writing reflection on the sheet.

10. Very often students do not completely understand why each of the three strategies in Monty's Dilemma *does* have a different probability of winning until they have actually carried out the spinner simulation and seen that it must be $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{3}$, respectively. The power of a simulation technique for solving a probability problem is quite evident in this activity.

(Note: More information about this game can be found in the April 1991 issue of the *Mathematics Teacher*, Vol. 84, No. 4, pp. 252–256.)

Name ____________________________



Name ____________________ Activity Sheet VIII-7-B

Tallies and Totals for Each Strategy

	WIN	LOSE
STICK	Total:	Total:
FLIP	Total:	Total:
SWITCH	Total:	Total:

Name ______________________________

Write a note to a friend explaining what the chances are of winning under each strategy in Monty's Dilemma. Tell your friend anything that surprised you about this game.

Monty's Dilemma

There is a TV game show in which the contestant is asked to choose one of three doors. Behind one of the doors is a whopping big prize and behind the other two doors are gag prizes.

After the contestant chooses one of the doors, the game show host reveals what is behind one of the other two doors, always showing a gag gift. Then the contestant is presented with the following dilemma:

> ***Would you like to keep the door you chose or switch to the other (still veiled) door?***

Monty's Strategies

Let us pose a mathematical (probabilistic) problem from this dilemma.

Which of these three strategies is most likely to lead the contestant to the winning door?

1) Just stay put and keep the original door you chose, after the door to the gag prize is opened.

(STICK strategy)

2) Choose again by randomly selecting a door from the remaining two closed doors.

(FLIP strategy)

3) Choose again by switching from the door you chose to the other closed door.

(SWITCH strategy)

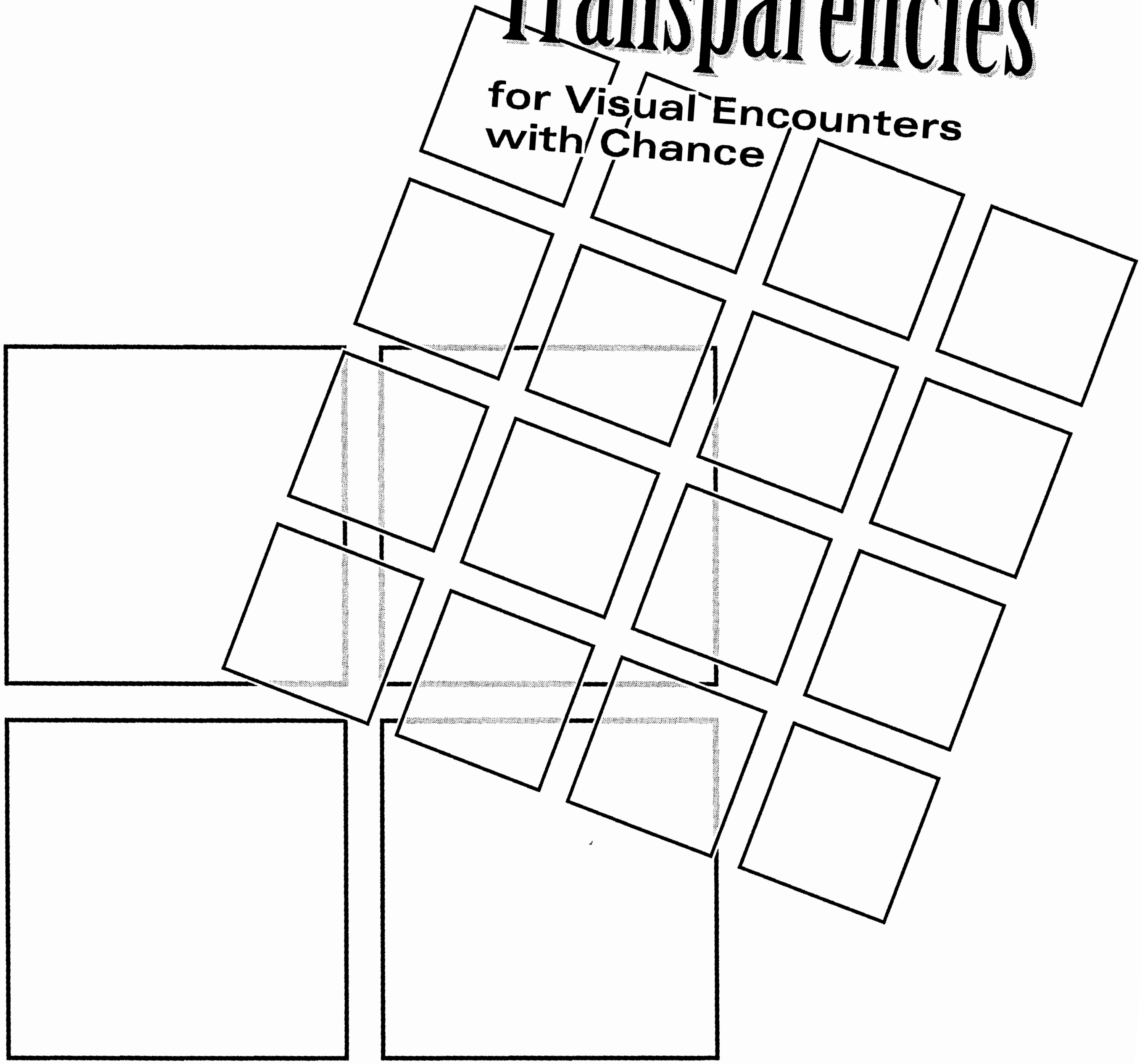
Results of 100 Trials for Each Strategy

STICK		FLIP		SWITCH	
Won	Lost	Won	Lost	Won	Lost

Unit VIII / Math and the Mind's Eye

Transparencies

for Visual Encounters with Chance



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Catalog #MET8

Name ____________________________

1.What can you say for sure about the contents of the sack?

2. What else can you say about the contents of the sack?

3. At this point, what do you think is likely to be in the sack?

4. How confident are you of your answer to Question 3?

Name ______________________________

Propose some 20 tile sacks that you feel could fit in each of these categories. List the contents—number of red, blue, green tile—for each sack you propose under the category.

Match Impossible	**Match Unlikely**	**Not Sure**	**Match Likely**

Name____________________________

1. Your group has chosen a sack containing 20 tile that you believe matches the contents of "Hidden Sack". Let's call your sack the "Likely Sack" since you feel it is likely to produce a match. Now imagine drawing a sample of 40 tile from the Likely Sack. Based on your knowledge of what is in this sack:

a. How many times would you expect each color to be drawn?

Red ______ Blue ______ Green ______

Explain the reasoning you used to arrive at your answers.

b. Write the percentage of each color of tile in your Likely Sack.

% Red ______ % Blue ______ % Green ______

These percentages are your "theoretical probabilities" of selecting each color of tile from your Likely Sack.

2. Recall the percentages of Red, Blue and Green tile that you obtained from your 40-draws experiment (these may be attached to your posters). These were your "experimental probabilities". Use these and your answers from part 1 above to complete the chart below.

Contents of Your Likely Sack			(*from Part 1b*) Your Theoretical Probabilities (Likely Sack)			(*from Poster Grid*) Your Experimental Probabilities—40 Draws (Hidden Sack)		
R	**B**	**G**	**R**	**B**	**G**	**R**	**B**	**G**

3. Compare your "experimental" and "theoretical" probabilities. How close are they? Do you feel that your group's Likely Sack matches the contents of the Hidden Sack? Why or why not?

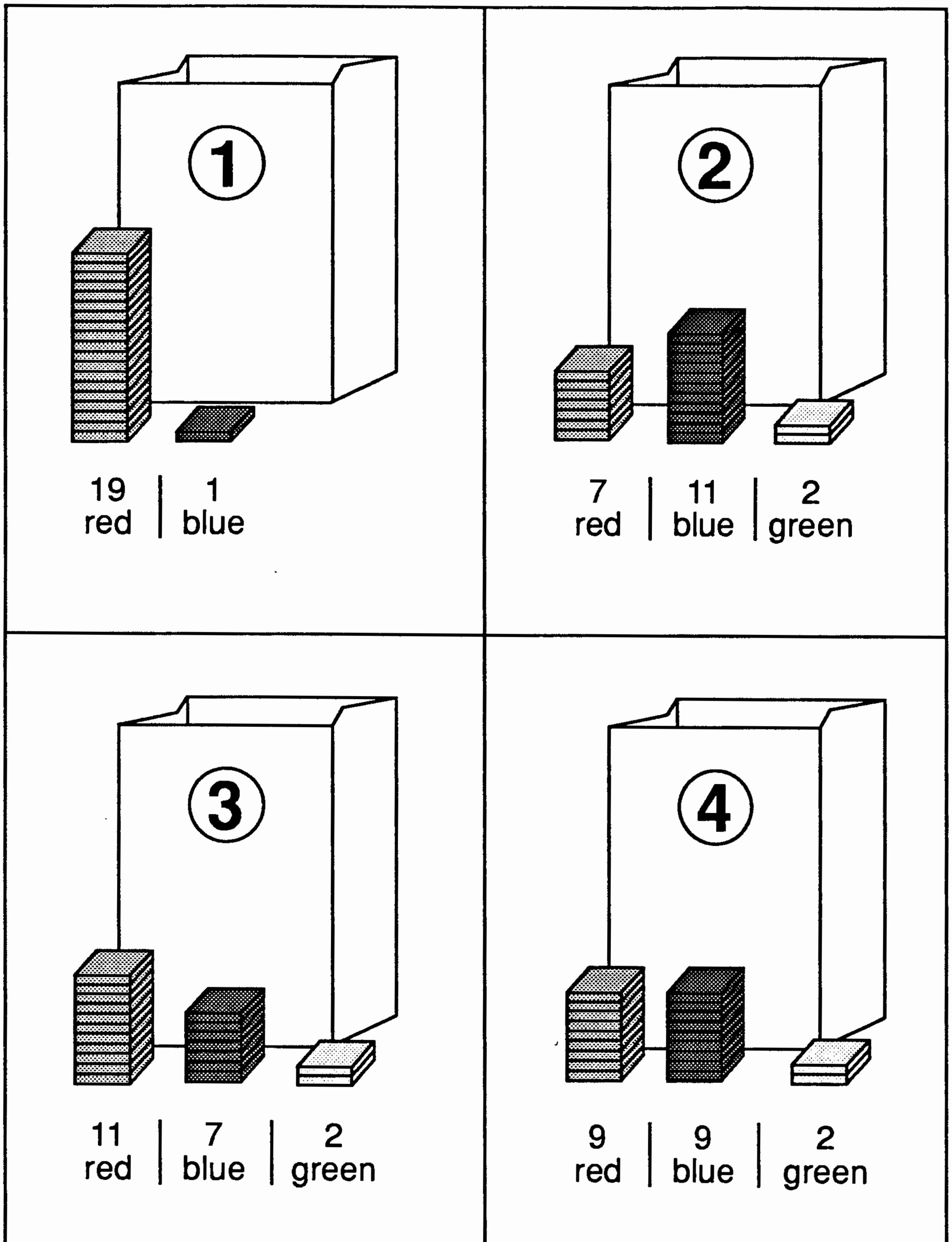
VIII-1 Master for Transparency A

Range of Experimental Probabilities: Hidden Sack

Red	Blue	Green

Range of Experimental Probabilities: Hidden Sack—40 Draws

Red	Blue	Green



Match Impossible	Match Unlikely	Not Sure	Match Likely

Name ____________________________

The Situation

Information from blood samples for five people—Ted, Patty, Mike, Linda and Gene—has been collected. The Bureau of Missing Persons has reason to believe that two of these five people are closely related and thus genetic information has been coded from the blood samples for each person.

This coded information has been represented by tile in five paper sacks. Two of these paper sacks have identical contents. The contents of any sack can only be revealed by pulling out one tile at a time, then replacing the tile in the sack and shaking the sack before drawing the next tile.

The Problem: Which two people are related?

Devise a plan in your group to gather data and answer this question. Some things you might want to address in your group plan are:

The way you will go about gathering data,

The amount of data you will gather,

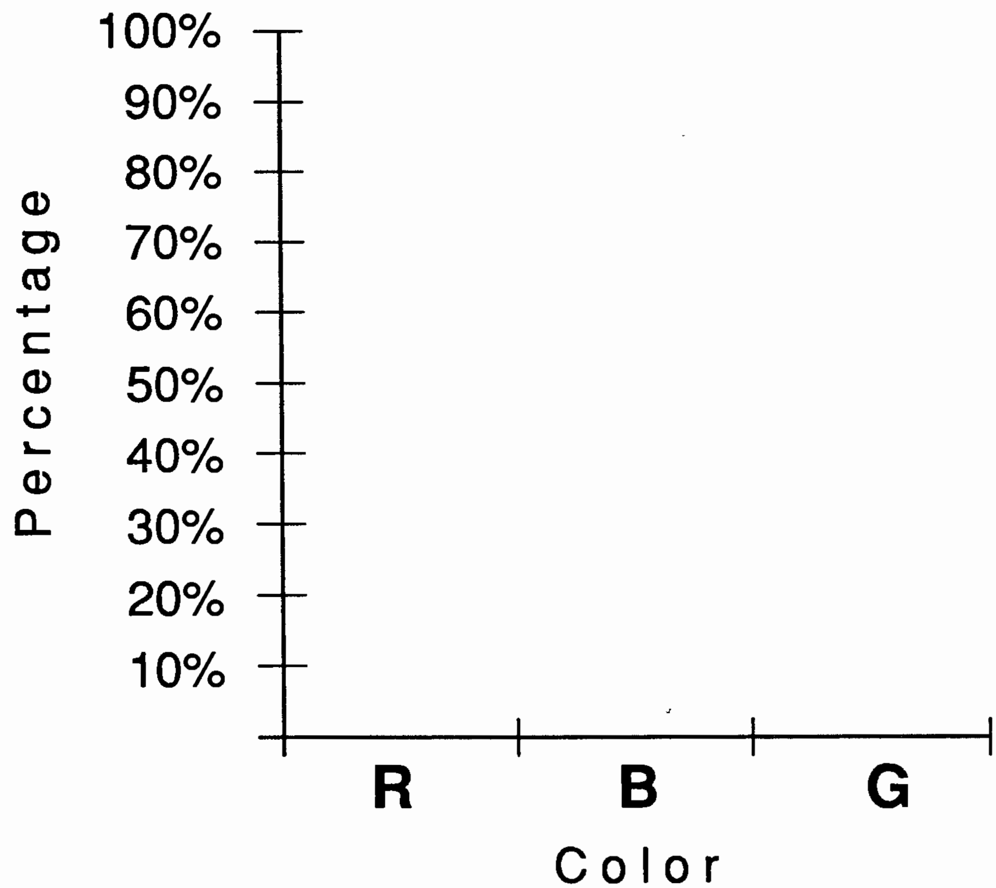
The organization of your data,

Ways to present your results to others.

Laboratory Name ____________________________

Experimental Histogram

Subject's Name ____________________________



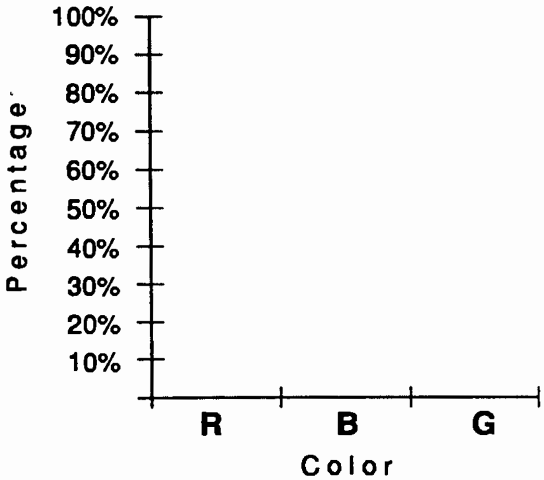
Total number of draws ________ from this sack.

Laboratory Name ______________________________

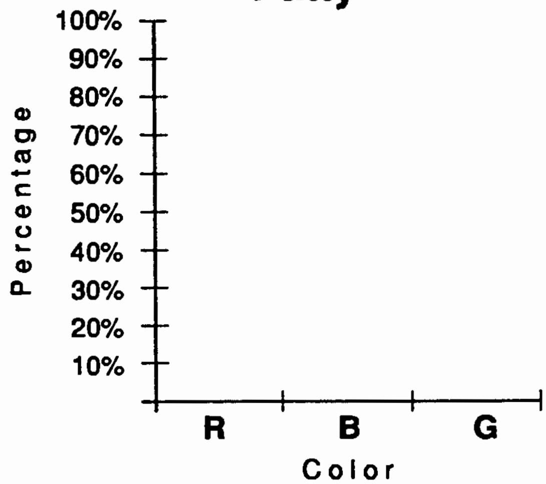
Name ______________________________

Experimental Histograms

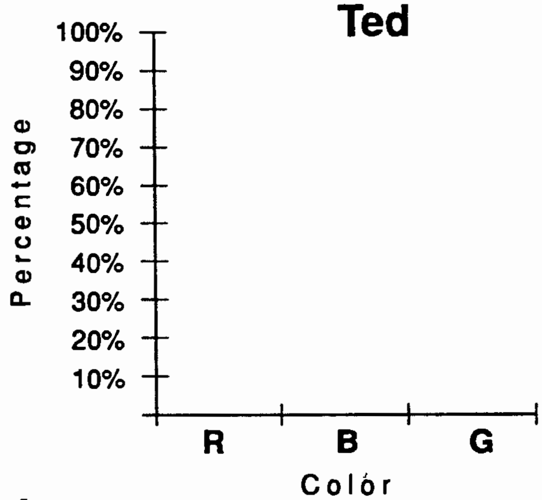
Gene



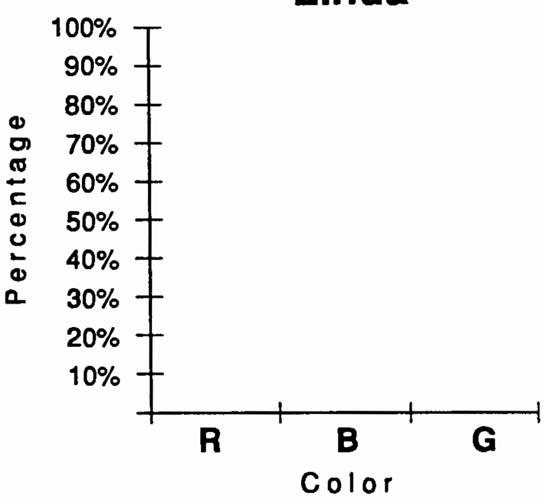
Patty



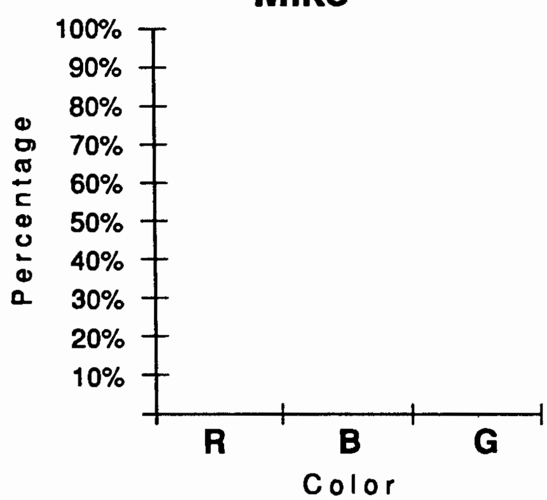
Ted



Linda



Mike

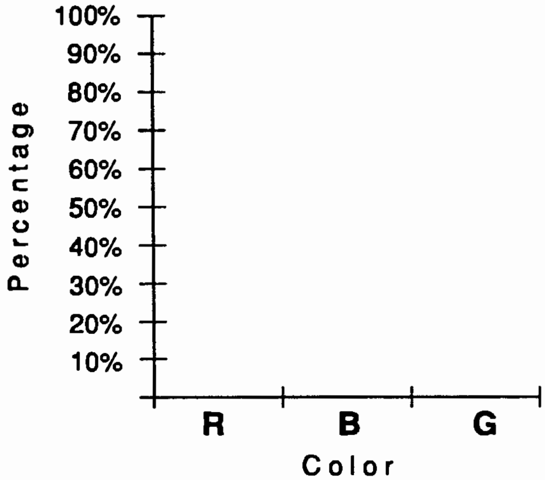


Laboratory Name ______________________________

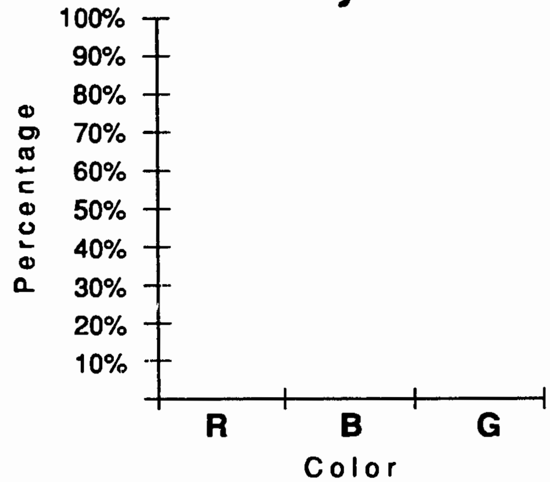
Name ______________________________

Theoretical Histograms

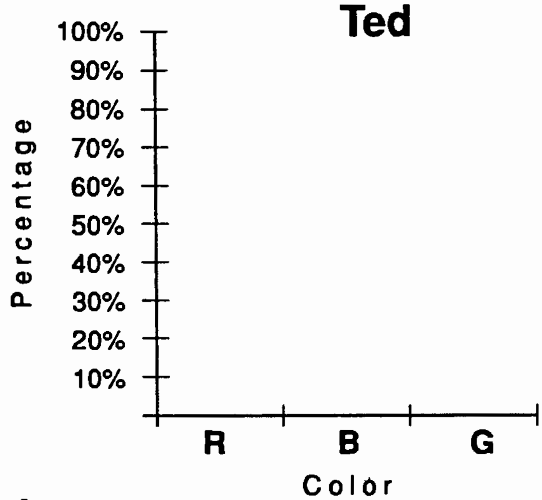
Gene



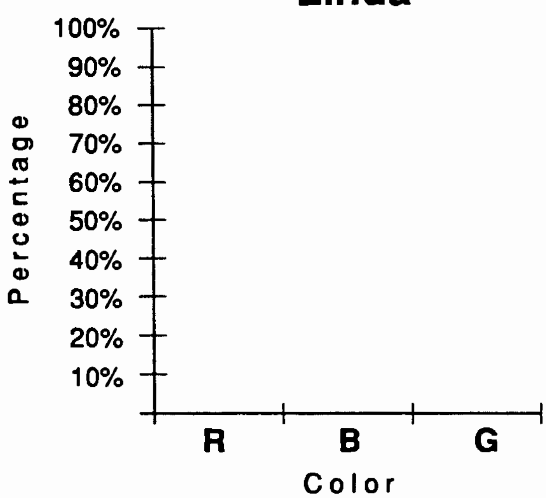
Patty



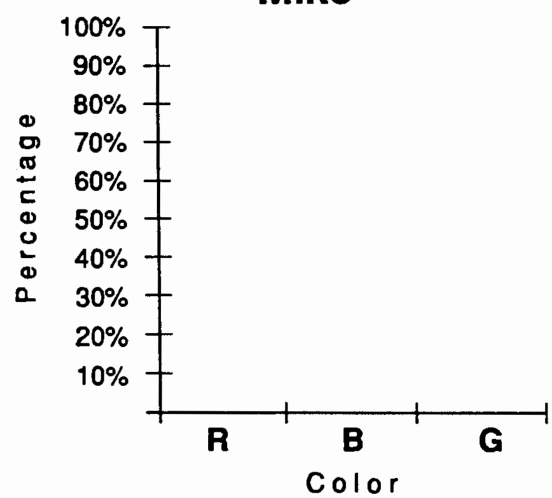
Ted



Linda



Mike

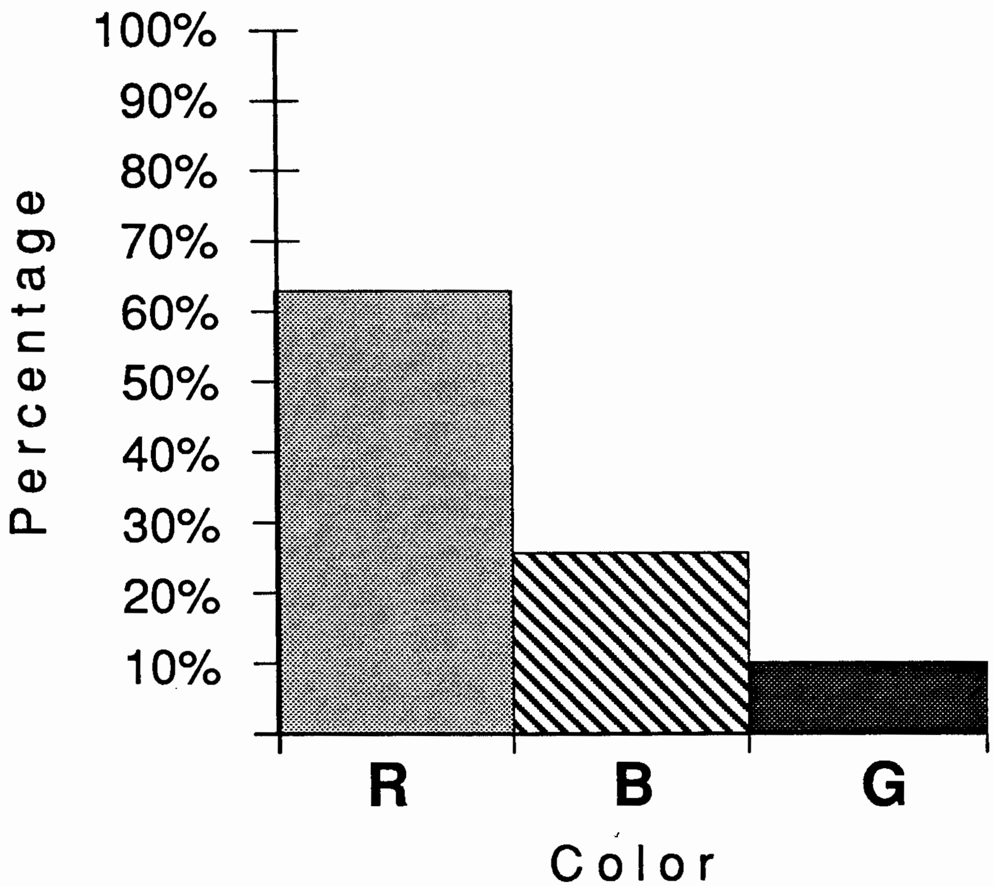


Name ______________________________

Reflections on the Experiment

What suggestions would you make to other classes who were doing this activity? What could they do to improve their experiment? Write your suggestions below in the form of a letter to another class.

Sample Histogram



The Key

	RED	BLUE	GREEN
Ted	7	3	0
Linda	6	2	2
Mike	4	3	3
Patty	7	2	1
Gene	6	2	2

Name ____________________________

The River Crossing*

Directions

This is a game for two players. Each player is given twelve counters representing boats to be placed on the numbers along the bank of a river. The arrangement of the counters (boats) is completely up to the player.

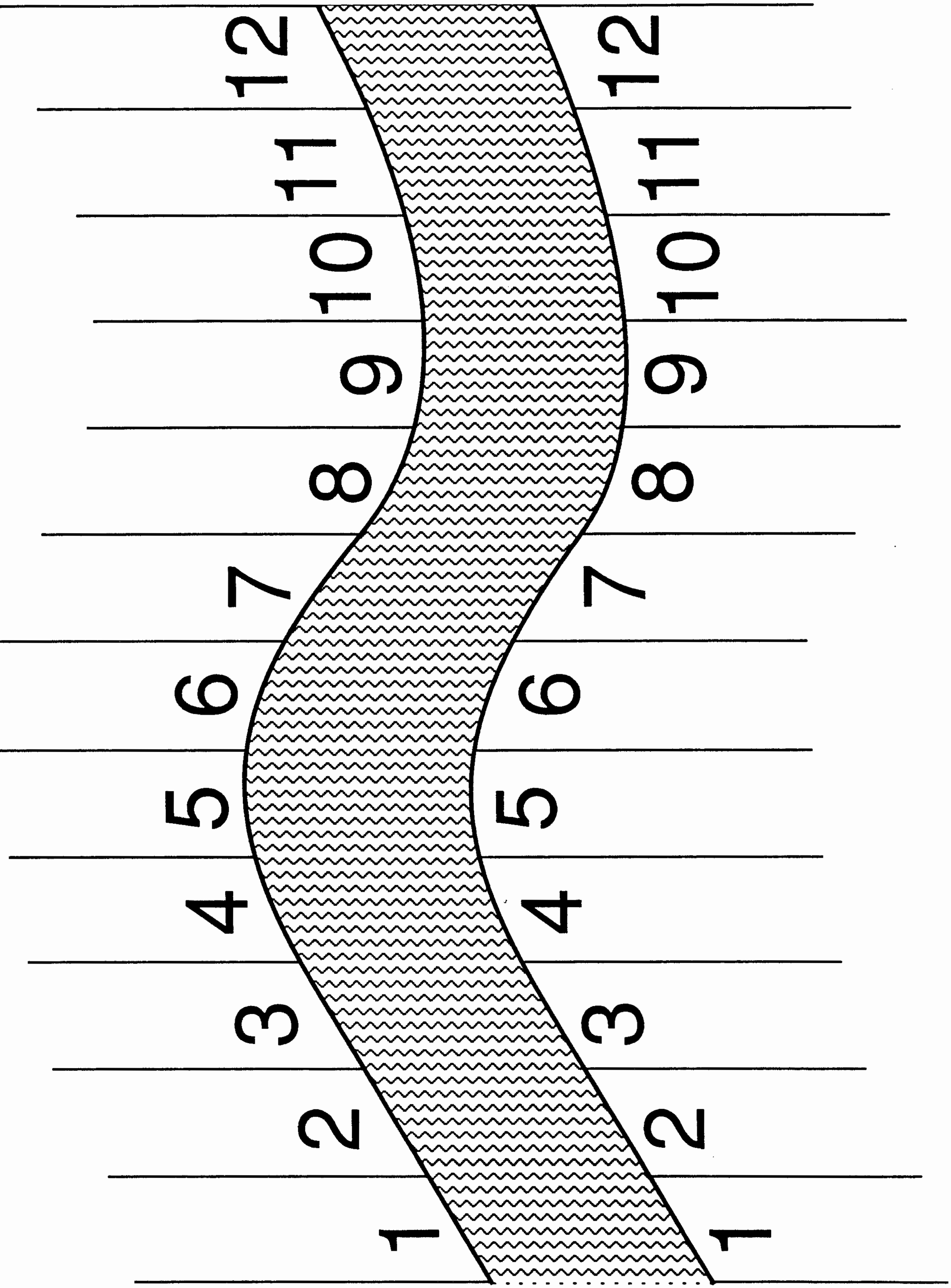
The Players take turns throwing two fair dice. On each roll the sum of the two upturned numbers is determined. If either player has a counter in that position, they may move it across the river to the other side and then remove the counter. Play continues until one player removes all twelve counters from the board.

The Problem

What is the best arrangement for the counters?

* This activity was adapted from a problem in *Mathematics Activities from Poland* by Jerzy Cwirko-Godycki.

The River Crossing Game Board



Name ______________________________

Counter Placements for River Crossing

Game 1

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an **X**.*

Game 2

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an **X**.*

Game 3

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an **X**.*

Game 4

1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an **X**.*

Game 5

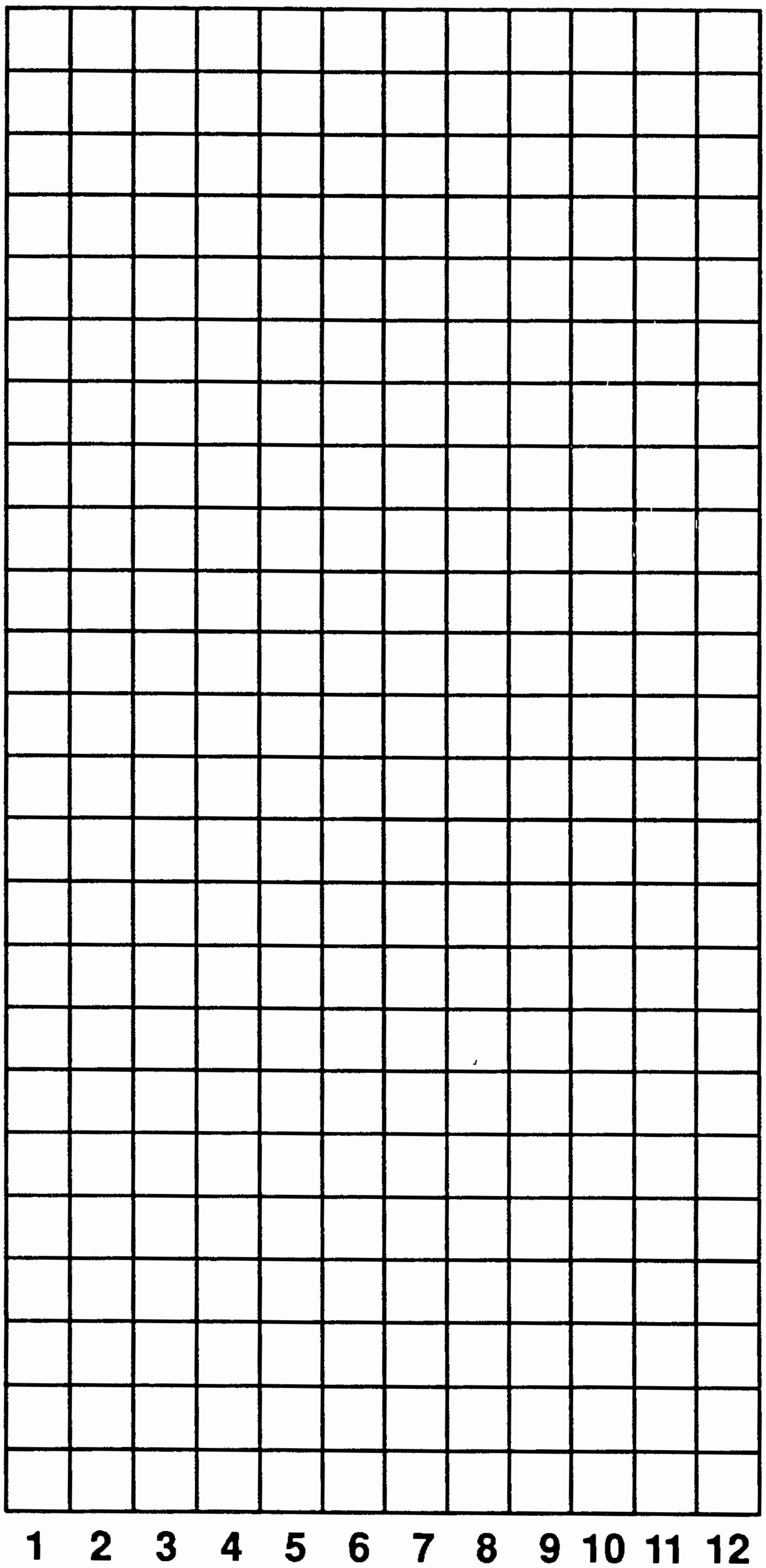
1	2	3	4	5	6	7	8	9	10	11	12

Counter Placement *Indicate counter placement with an **X**.*

Activity Sheet VIII-3-D

Name ______________________________

Line Plot for Sums of Two Dice



Name ______________________________

Possible Sums For Tossing Two Cubical Dice

1st Die	2nd Die	Sum

Name ______________________________

Area Model For Possible Sums From Two Cubical Dice

			2nd die			
+	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
1st die 3						
4						
5						
6						

Theoretical Grid

Theoretical probabilities of outcomes from tossing two cubical dice

Prob(2) =

Prob(3) =

Prob(4) =

Prob(5) =

Prob(6) =

Prob(7) =

Prob(even sum) =

Prob (multiple of 3) =

Prob(8) =

Prob(9) =

Prob(10) =

Prob(11) =

Prob(12) =

Prob(odd sum) =

Prob(number ≥ 5) =

Prob (number < 7) =

Name ____________________________

Reflection on The River Crossing Activity

1. Did you change the placement of your counters after playing a few River Crossing games? Explain why or why not.

2. Did you change the placement of your counters after making the area model of all the outcomes for the sum of two dice? Explain why or why not.

3. Suppose you tossed a pair of dice 100 times. How many times would you expect to get

a sum of 6?	a sum of 1?
a sum of 11?	a sum greater than 7?
an odd sum?	a sum less than 7?

Explain how you determined your answers to these questions.

4. Was there anything that surprised you in The River Crossing Game? Explain.

5. Write a note to a friend on the back of this sheet. Explain The River Crossing Game to your friend Tell your friend what you think is the best placement for the 12 counters. Explain why you think this is the best placement.

Name ____________________________

6. Get your experimental data from the plot of the tosses (Activity Sheet VIII-3-D). Calculate the percentage of time each possible sum occurred on your graph. For example, if you made 52 tosses and got a 2 three times, your percentage of twos would be $3/52 = .057 = 5.7\%$ of the time. Do this for each of the possible sums from 2 to 12. Then compare your experimental percentages to the theoretical percentages of each sum from Activity Sheet VIII-3-F. Fill in the chart below.

SUM	Experimental Percentage	Theoretical Percentage
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Name ______________________________

River Crossing Extension

1. Suppose that you tossed three cubical dice and crossed a boat with each sum of the three dice. What would the river dock look like for a three dice game? Draw a picture.

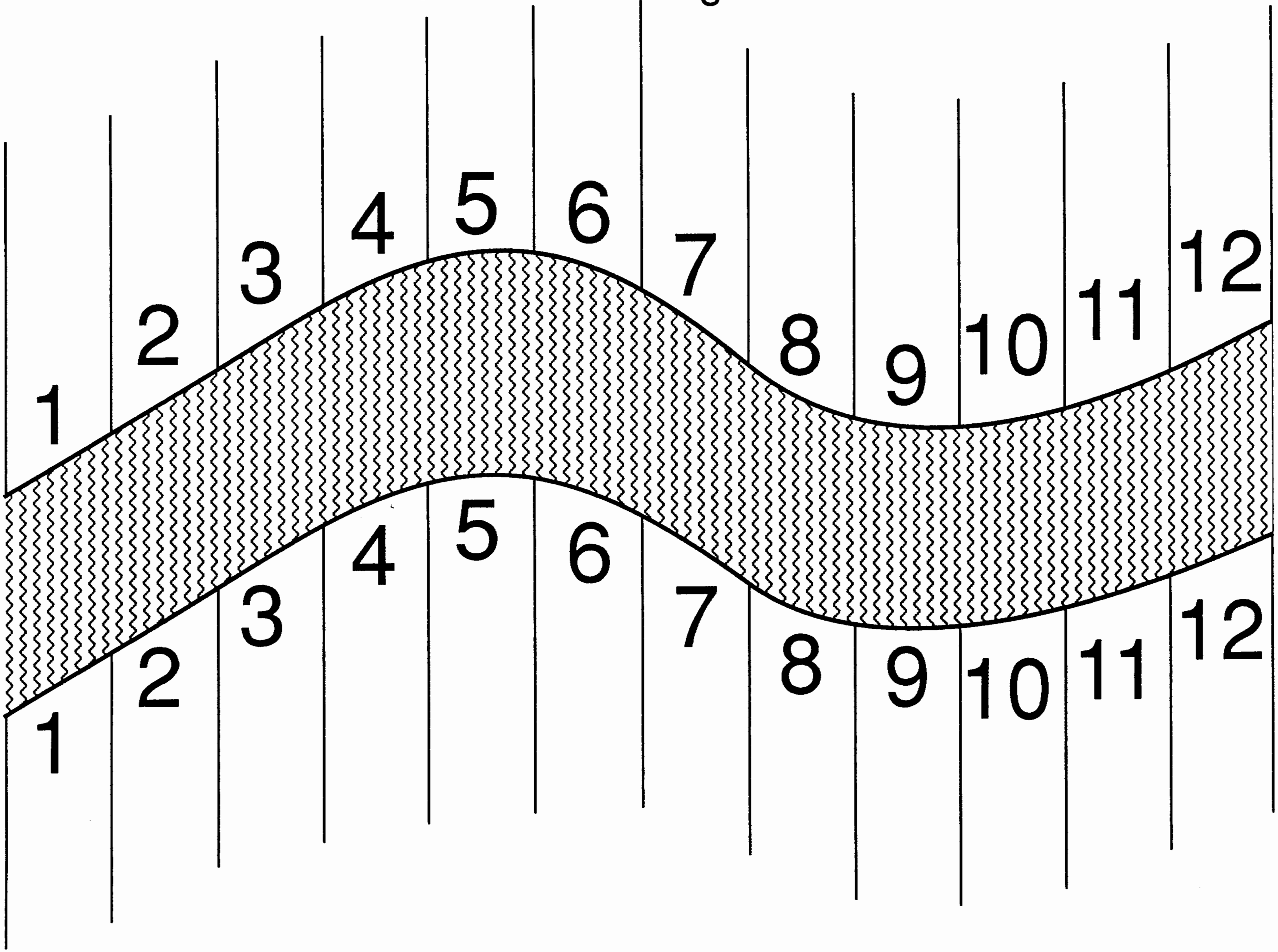
2. If you had 20 counters (boats) to place on this dock, where would you place them? Indicate below.

Counter Placement

3. (Challenge) How many different ways can you get each possible sum from tossing three cubical dice? For example, you can obtain a sum of four in three different ways,

1st Die	2nd Die	3rd Die	Sum
1	1	2	4
1	2	1	4
2	1	1	4

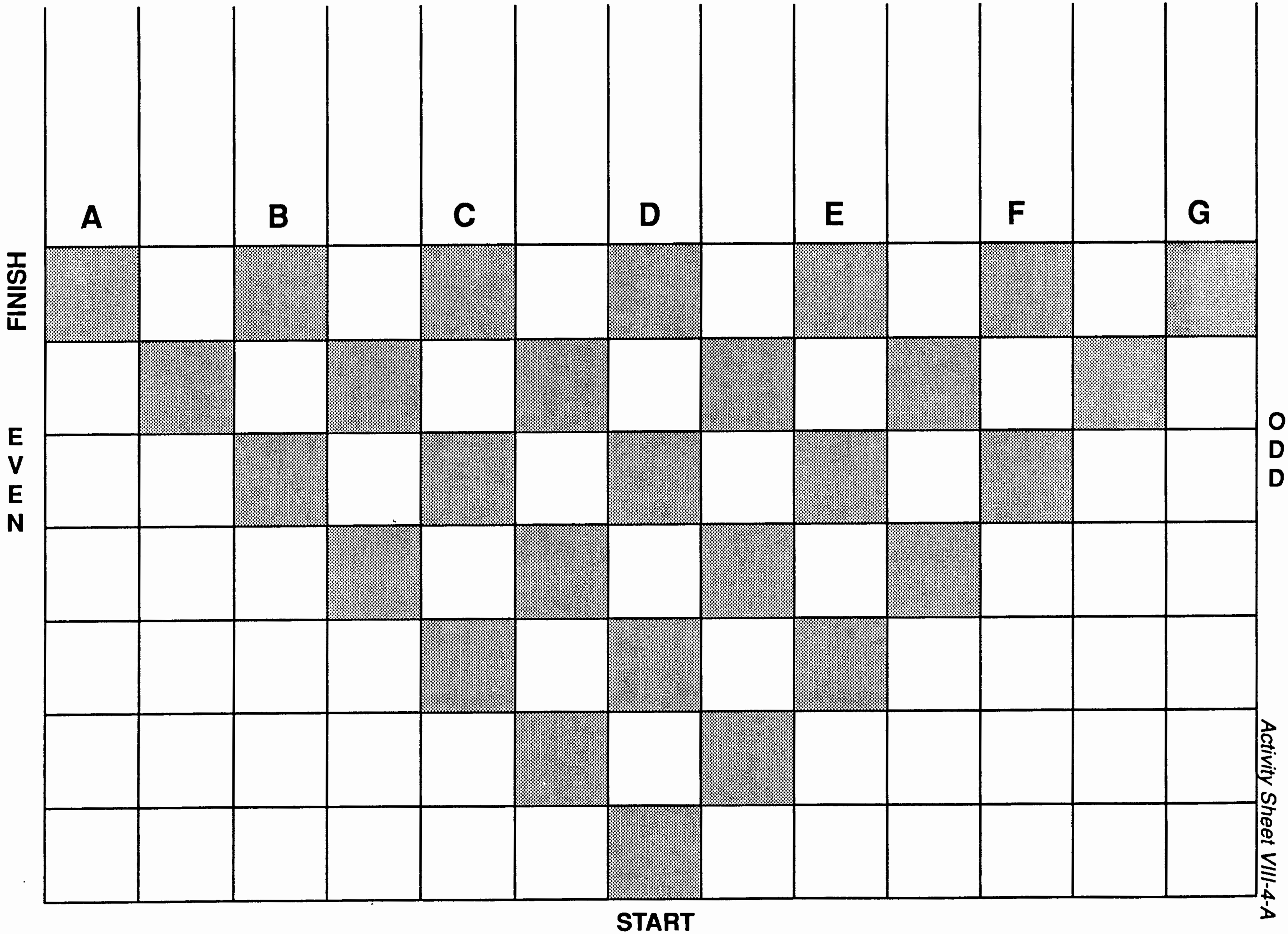
The River Crossing Game Board



Possible Sums For Tossing Two Cubical Dice

1st Die	2nd Die	Sum

CHECKER GAME



Name ______________________________

Checker-A Game

1. Imagine playing the Checker-A Game 100 times. Make some predictions: How many times do you think the marker would end up at square A? at square B? etc. Record your predictions here.

Ending Square:	A	B	C	D	E	F	G
Your Guesses:							

Compare your guesses with those of your groupmates. Were there any general observations about the game that prompted your answers?

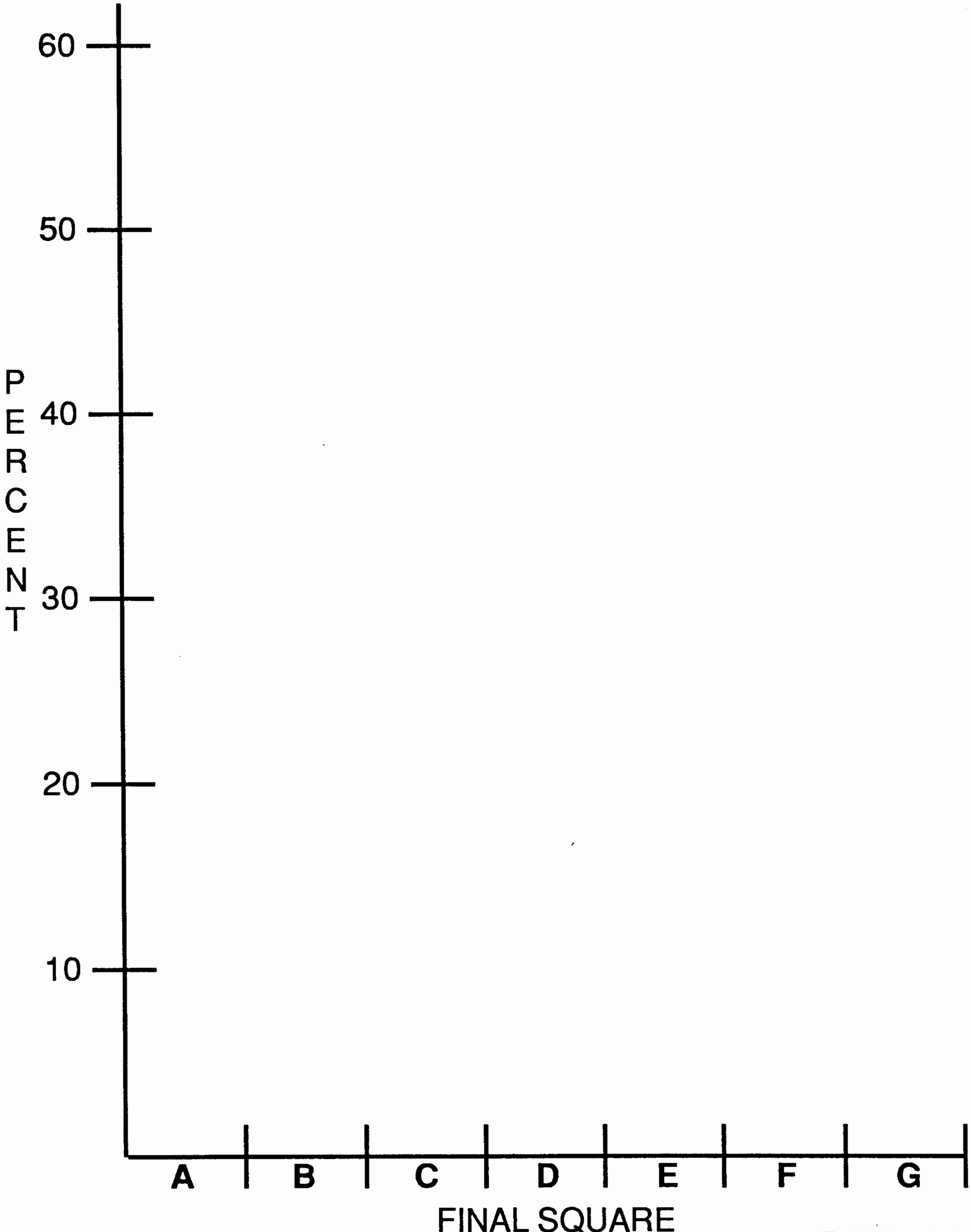
2. Each person in the group now plays the game a number of times. With tally marks (on the game board) keep track of the number of times the marker ends at each of the lettered squares. Put the results obtained by each person into the chart below and complete the chart. Your group will need a total of 100 plays.

	Ending Square							
Player's Name	A	B	C	D	E	F	G	Total Plays
1								
2								
3								
4								
5								
Group Total								100

3. Use your group's results from the chart to calculate the "experimental" probabilities for landing on each of the lettered squares. Write your results below.

P(A) = P(B) = P(C) = P(D) =

P(E) = P(F) = P(G) =


60
50
40
30
20
10
PERCENT
A
B
C
D
E
F
G
FINAL SQUARE

Name ______________________________

Comparison of Experimental and Theoretical Probabilities for the Checker-A Game

Final Square	Experimental Probability	Theoretical Probability
A		
B		
C		
D		
E		
F		
G		

The Checker-A Game

Begin with a marker on the START square. A play of the game consists of tossing a regular die 6 times.

For each toss of an EVEN number, move the marker one square diagonally to the LEFT.

For each toss of an ODD number, move the marker one square diagonally to the RIGHT.

After six tosses, the marker will be on one of the squares marked A, B, C, D, E, F or G.

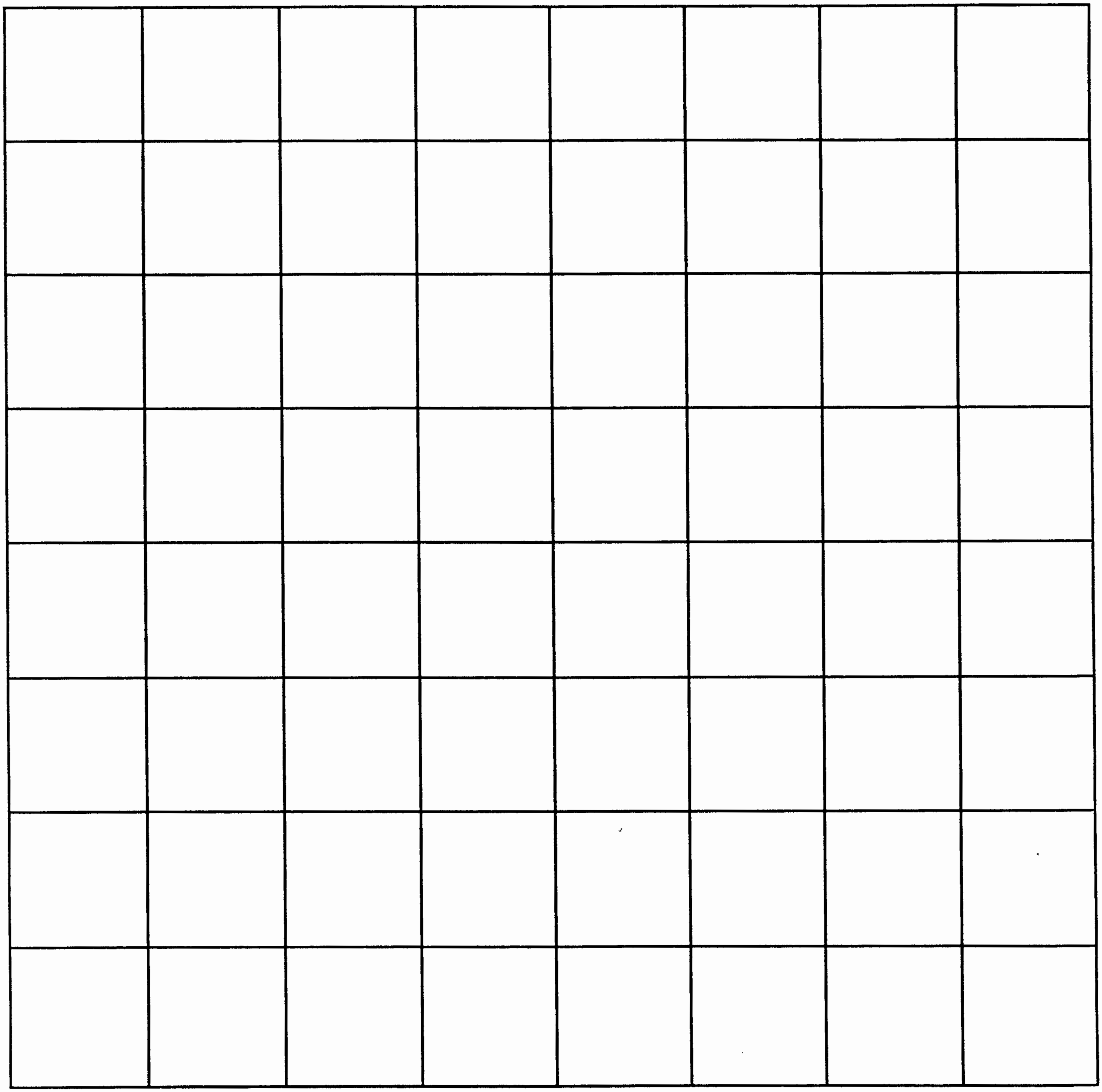
Which square would you choose to be the winner? Why?

E E E E E E	E E E E O E	E E O E E E	E E O E O E	O E E E E E	O E E E O E	O E O E E E	O E O E O E
E E E E E O	E E E E O O	E E O E E O	E E O E O O	O E E E E O	O E E E O O	O E O E E O	O E O E O O
E E E O E E	E E E O O E	E E O O E E	E E O O O E	O E E O E E	O E E O O E	O E O O E E	O E O O O E
E E E O E O	E E E O O O	E E O O E O	E E O O O O	O E E O E O	O E E O O O	O E O O E O	O E O O O O
E O E E E E	E O E E O E	E O O E E E	E O O E O E	O O E E E E	O O E E O E	O O O E E E	O O O E O E
E O E E E O	E O E E O O	E O O E E O	E O O E O O	O O E E E O	O O E E O O	O O O E E O	O O O E O O
E O E O E E	E O E O O E	E O O O E E	E O O O O E	O O E O E E	O O E O O E	O O O O E E	O O O O O E
E O E O E O	E O E O O O	E O O O E O	E O O O O O	O O E O E O	O O E O O O	O O O O E O	O O O O O O

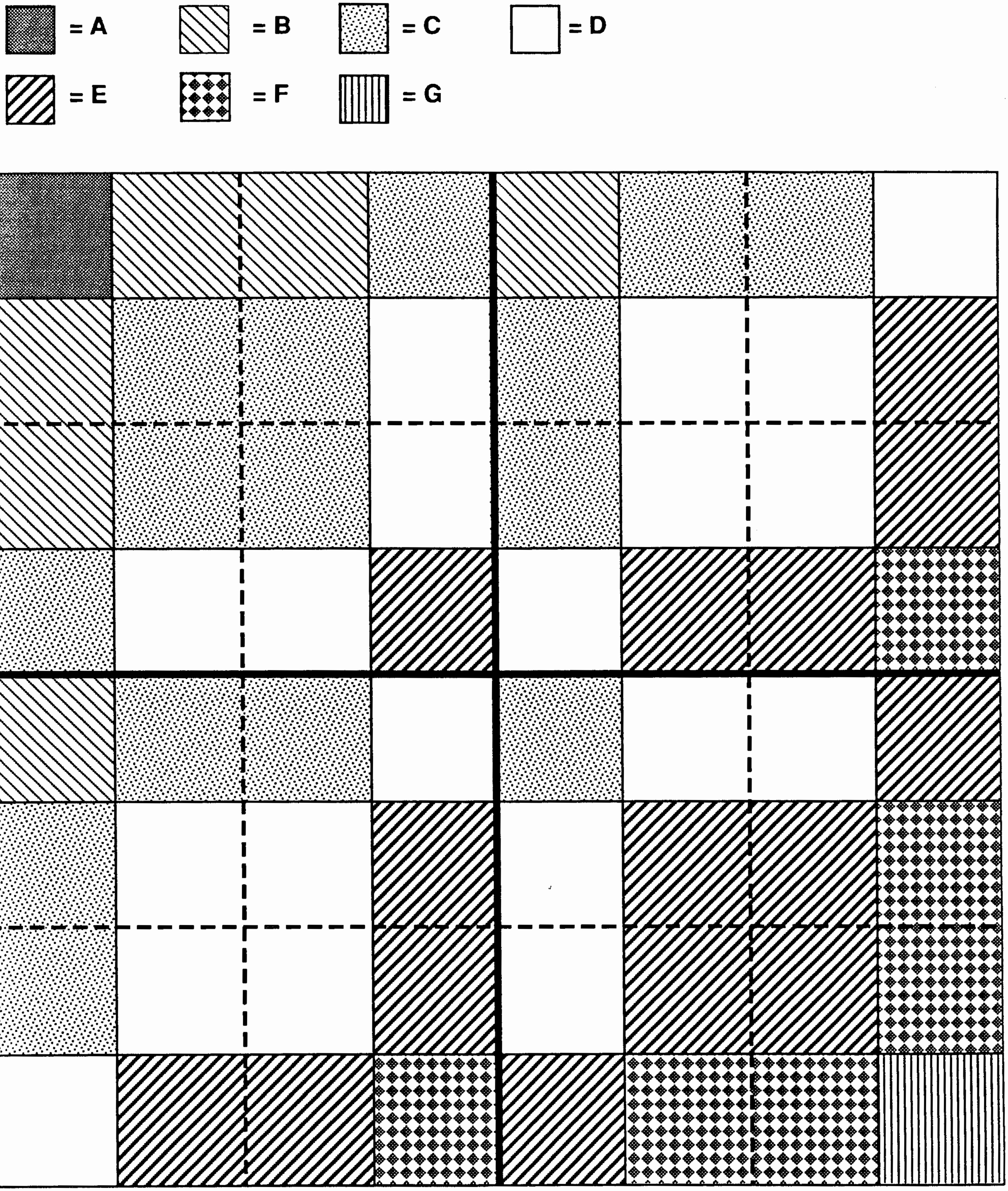
VIII-4 Master for Transparency B

EEEEEE A	B	B	C	B	C	C	D
B	C	C	D	C	D	D	E
B	C	C	D	C	D	D	E
C	D	D	E	D	E	E	F
B	C	C	D	C	D	D	E
C	D	D	E	D	E	E	F
C	D	D	E	D	E	E	F
D	E	E	F	E	F	F	OOOOOO G

VIII-4 Master for Transparency C

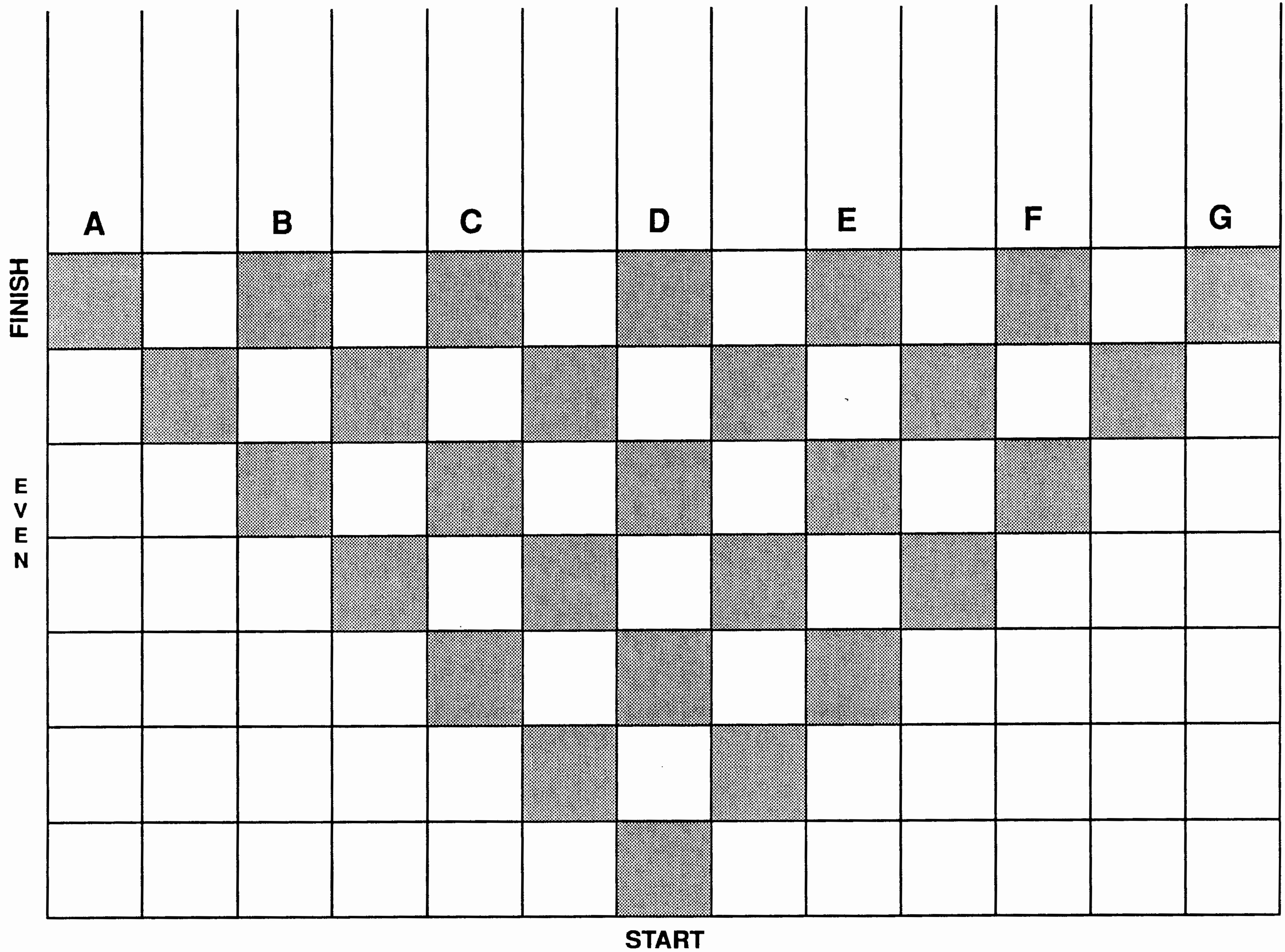


VIII-4 Grid Blank



VIII-4 Master for Transparency D

CHECKER GAME



Name ______________________________

Checker-B Game

1. Imagine playing the Checker-B Game 100 times. Make some predictions: How many times do you think the marker would end up at square A? at square B? etc. Record your predictions here.

Ending Square:	A	B	C	D	E	F	G
Your Guesses:							

Compare your guesses with those of your groupmates. Were there any general observations about the game that prompted your answers?

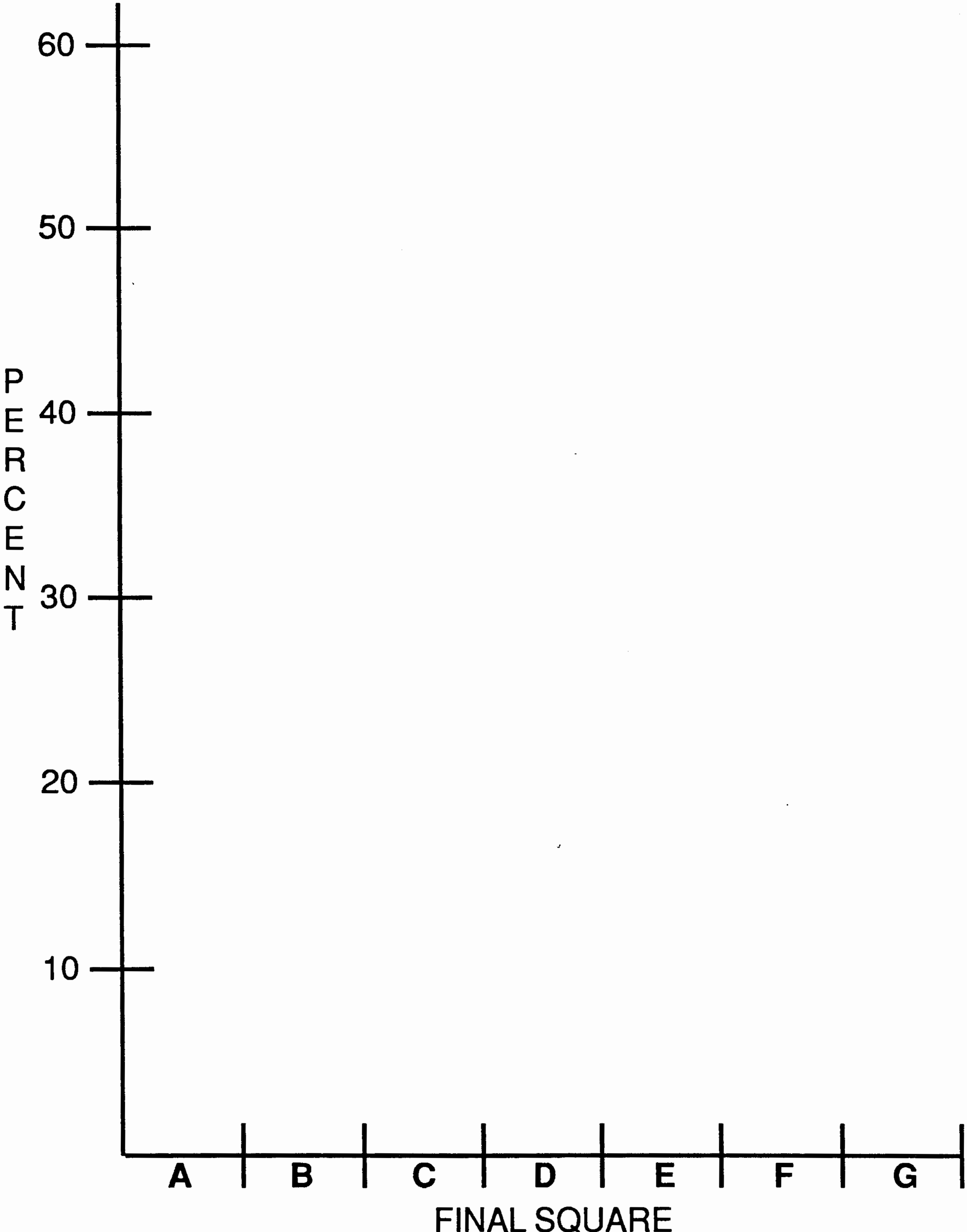
2. Each person in the group now plays the game a number of times. With tally marks keep track of the number of times the marker ends at each of the lettered squares. Put the results obtained by each person into the chart below and complete the chart. Your group will need a total of 100 plays.

	Ending Square							
Player's Name	A	B	C	D	E	F	G	**Total Plays**
1								
2								
3								
4								
5								
Group Total								**100**

3. Use your group's results from the chart to calculate the "experimental" probabilities for landing on each of the lettered squares. Write your results below.

P(A) = P(B) = P(C) = P(D) =

P(E) = P(F) = P(G) =


60
50
40
30
20
10
PERCENT
A
B
C
D
E
F
G
FINAL SQUARE

Name ______________________________

Grid Model for Possible Products From Two Cubical Dice

1. Complete this grid.

2nd die

1st die

X	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

Theoretical Grid

2. Use the grid to find these probabilities of products from tossing two cubical dice

Prob(even product) =

Prob(odd product) =

Prob(12) =

Prob(6) =

Prob(number > 12) =

Prob(number≤ 9) =

Prob(perfect square) =

Prob(multiple of 3) =

Name ______________________________

Comparison of Experimental and Theoretical Probabilities for the Checker-B Game

Final Square	Experimental Probability	Theoretical Probability
A		
B		
C		
D		
E		
F		
G		

Name ____________________________

Reflection on the Checker-A and Checker-B Games

1. What differences would you expect in the results of 1000 plays of the Checker-A and Checker-B Games? Explain.

2. On which square would you bet in the Checker-A Game? Why? On which square would you bet in the Checker-B Game? Why?

3. Although the list of possible paths to a finish square is the same for both the Checker-A and B games, the probability that a path occurs is not the same for both games. Write a note to a friend explaining how you determined the theoretical probabilities of landing on each finish square in the Checker-B game and compare that to the theoretical probability for each square in the Checker-A game.

The Checker-B Game

Begin with a marker on the START square. A play of the game consists of tossing a pair of regular dice 6 times.

After each toss, find the product of the two top faces of the dice.

If the product is an EVEN number, move the marker one square diagonally to the LEFT.

If the product is an ODD number, move the marker one square diagonally to the RIGHT.

After six tosses, the marker will be on one of the squares marked A, B, C, D, E, F or G.

Name ______________________________

1. How many boxes will have to be opened to collect all 5 bike racing stickers? What do you think?

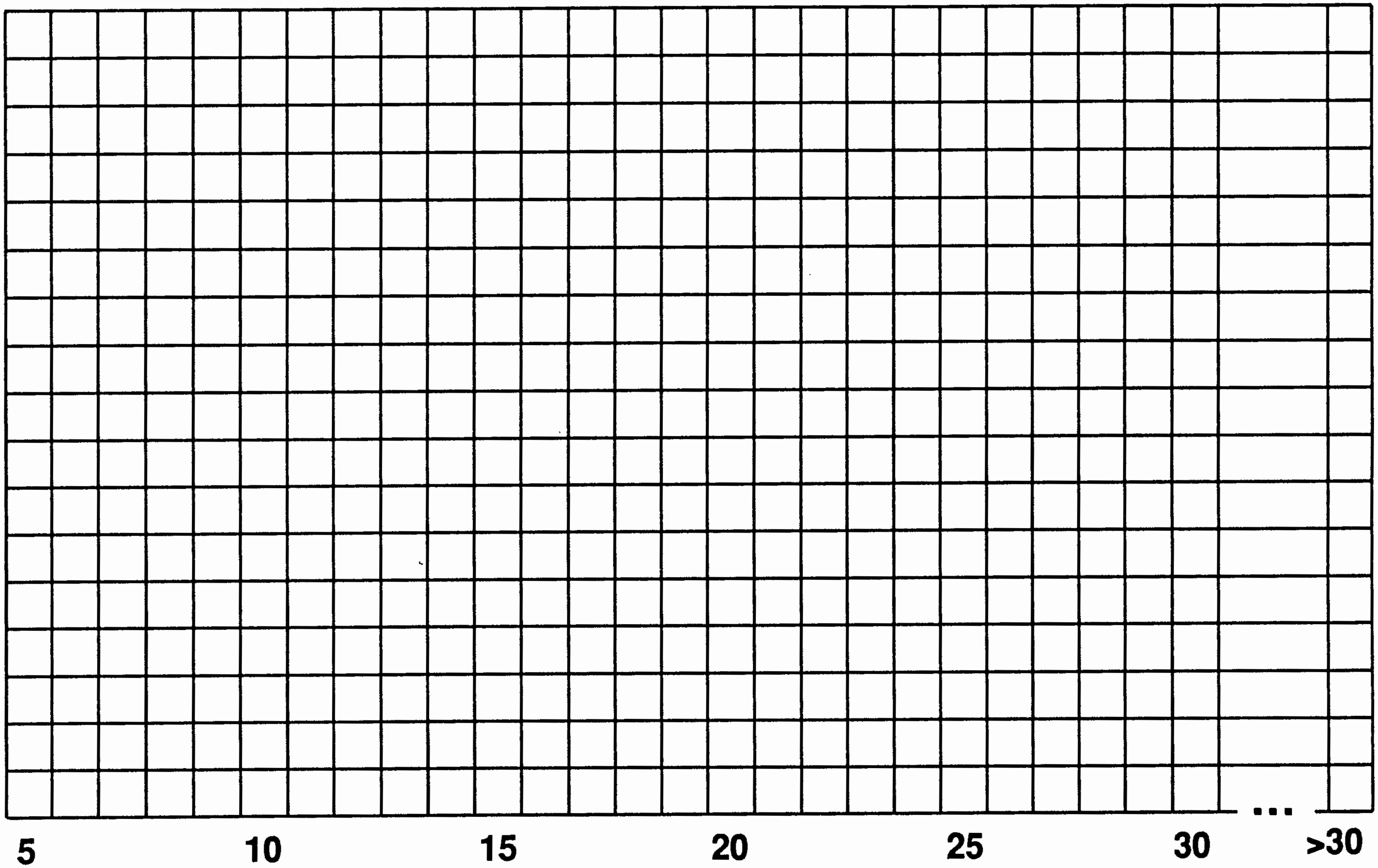
   a) My best guess is ______________ boxes?

   b) The reason I guessed this number of boxes is:

2. Record your data from simulating the cereal box experiment on this chart.

Trial	Tallies	Check off	# Opened
1		1 2 3 4 5	
2		1 2 3 4 5	
3		1 2 3 4 5	
4		1 2 3 4 5	
5		1 2 3 4 5	
6		1 2 3 4 5	
7		1 2 3 4 5	
8		1 2 3 4 5	
9		1 2 3 4 5	
10		1 2 3 4 5	

Cereal Box Line-Plot



The median of this data is ______. The total # of trials is ______.

People will probably need to buy at least ______ boxes.

Name ____________________________

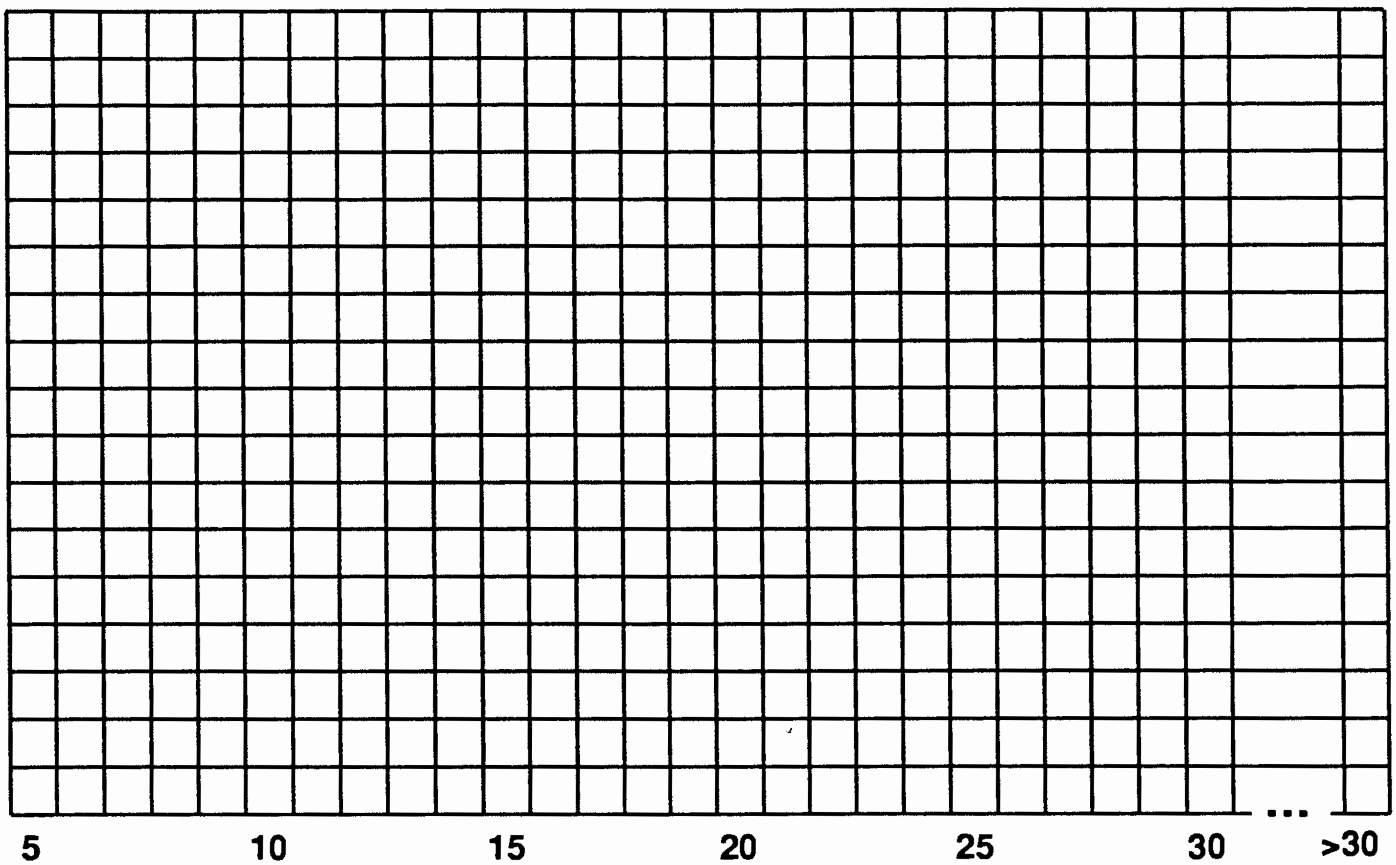
1. Now that you have completed the cereal box activity, how many boxes would you need to buy to collect all 5 stickers? Explain your reasoning.

2. We have used both line-plots and box-plots to visualize the data from the cereal box simulation experiment. What are some advantages of each of these types of plots? What are some disadvantages? Explain.

Name ______________________________

3. Suppose there were 6 stickers in the cereal boxes. How many boxes would you need to buy to collect all 6? ________(your guess).

Devise an experiment to test the number of boxes you would have to buy to collect 6 stickers. Collect some data based on your experiment. Show the results of your data below. Make a 90% box-plot for your data. How did your results compare to your guess above?



Cereal Boxes

General Mills company once included bike racing stickers in their boxes of Cheerios. There were 5 different stickers. Each box contained just 1 of the 5 stickers.

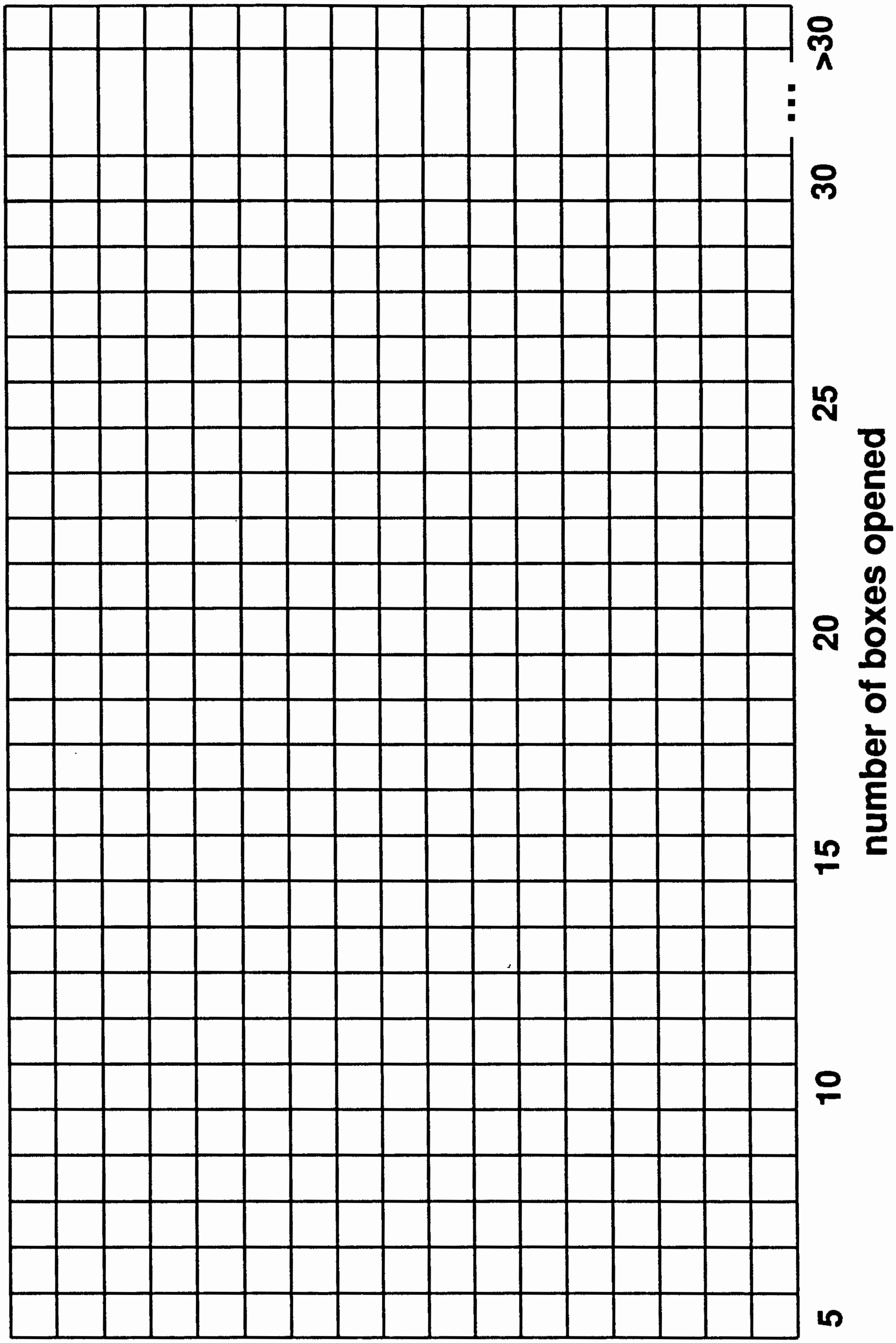
How many boxes of Cheerios would you expect to have to buy in order to collect all 5 stickers?

Make a guess. Write it down.

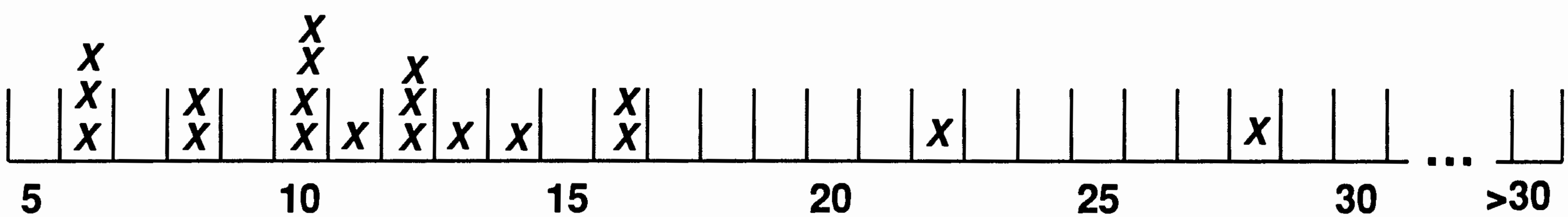
Guesses for the number of cereal boxes needed to collect all the stickers.

Trial	Tallies	Check off	# Opened
1	______	1 2 3 4 5	______
2	______	1 2 3 4 5	______
3	______	1 2 3 4 5	______
4	______	1 2 3 4 5	______
5	______	1 2 3 4 5	______
6	______	1 2 3 4 5	______
7	______	1 2 3 4 5	______
8	______	1 2 3 4 5	______
9	______	1 2 3 4 5	______
10	______	1 2 3 4 5	______
11	______	1 2 3 4 5	______
12	______	1 2 3 4 5	______
13	______	1 2 3 4 5	______
14	______	1 2 3 4 5	______
15	______	1 2 3 4 5	______

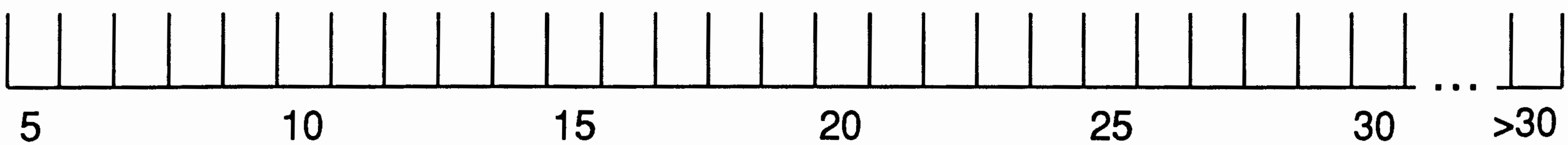
Cereal Box Line-Plot



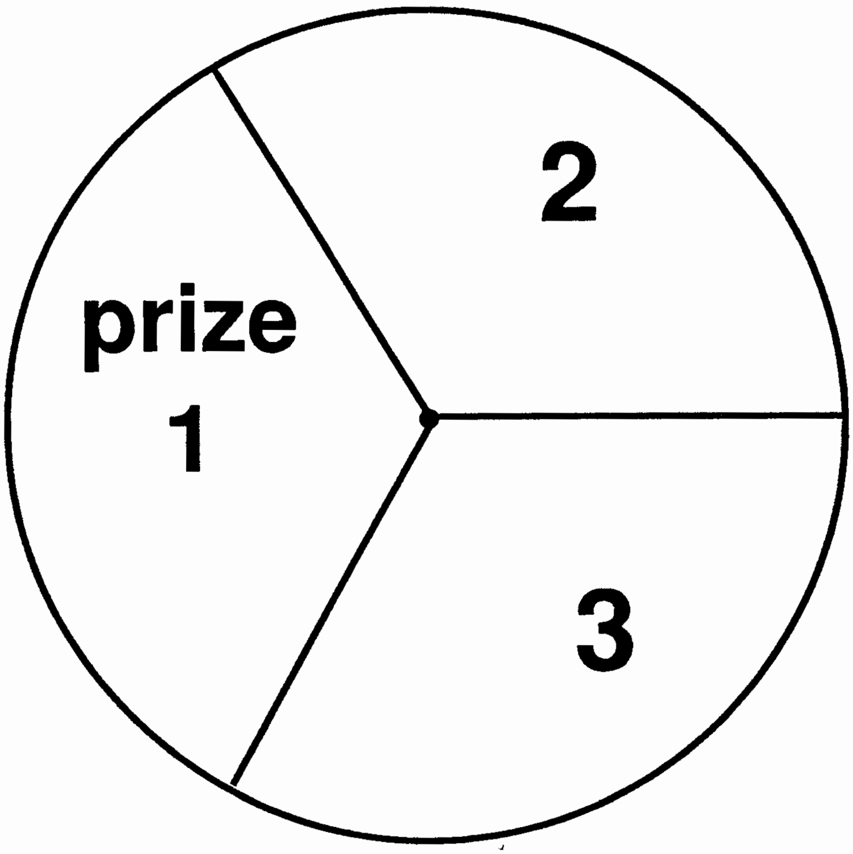
Cereal Box
Sample Line-Plot



Box-Plots of Student Data



Name ______________________________



Name ______________________________

Tallies and Totals for Each Strategy

	WIN	LOSE
STICK		
	Total:	**Total:**
FLIP		
	Total:	**Total:**
SWITCH		
	Total:	**Total:**

Name ______________________________

Write a note to a friend explaining what the chances are of winning under each strategy in Monty's Dilemma. Tell your friend anything that surprised you about this game.

Monty's Dilemma

There is a TV game show in which the contestant is asked to choose one of three doors. Behind one of the doors is a whopping big prize and behind the other two doors are gag prizes.

After the contestant chooses one of the doors, the game show host reveals what is behind one of the other two doors, always showing a gag gift. Then the contestant is presented with the following dilemma:

> ***Would you like to keep the door you chose or switch to the other (still veiled) door?***

Monty's Strategies

Let us pose a mathematical (probabilistic) problem from this dilemma.

Which of these three strategies is most likely to lead the contestant to the winning door?

1) Just stay put and keep the original door you chose, after the door to the gag prize is opened.

(STICK strategy)

2) Choose again by randomly selecting a door from the remaining two closed doors.

(FLIP strategy)

3) Choose again by switching from the door you chose to the other closed door.

(SWITCH strategy)

Results of 100 Trials for Each Strategy

STICK		FLIP		SWITCH	
Won	Lost	Won	Lost	Won	Lost