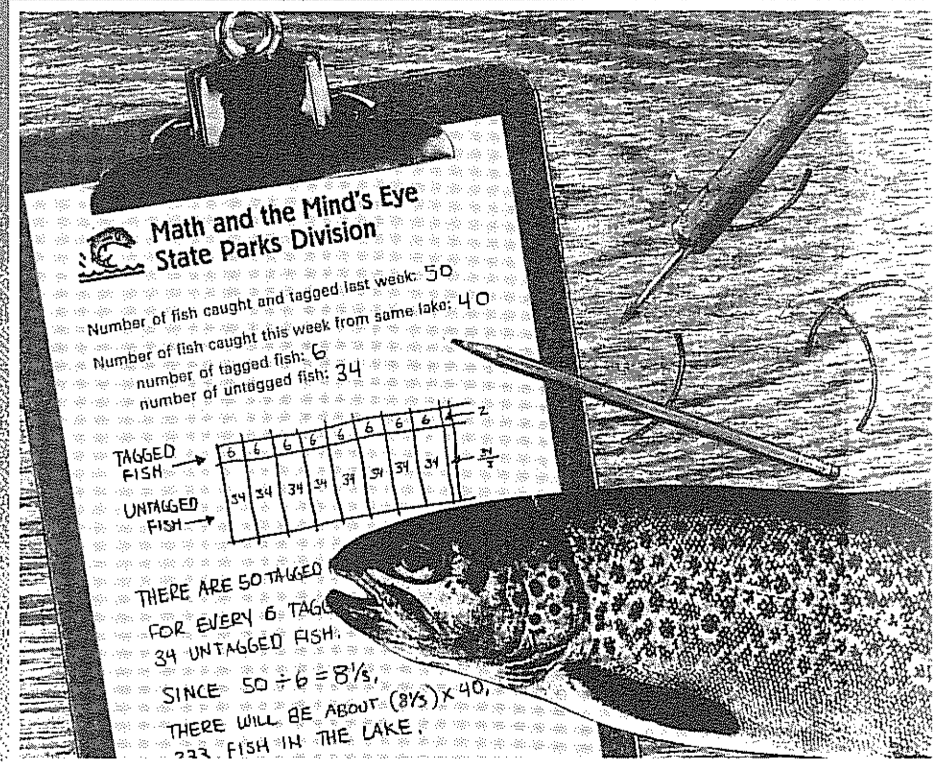

Modeling Percentages & Ratios

Albert Bennett, Eugene Maier
& L. Ted Nelson

Modeling Percentages & Ratios

1 **Introduction to Percentages**
Areas and lengths are used to introduce the meaning of percentage. Given the meaning of 100%, students develop their own procedures for carrying out computations involving percentages.

2 **Fractions, Decimals and Percentages**
A portion of a quantity can be described in terms of a fraction, a decimal, or a percentage. Ways of doing this, and of converting from one way to another, are investigated.

3 **Ratios**
The concept of ratio is introduced as a way comparing the number of black pieces to the number of red pieces in collections of counting pieces.

4 **Percentages and Ratio Problems**
Diagrams and sketches are used to solve story problems involving percentages and ratios.

Math and the Mind's Eye materials are intended for use in grades 4-9. They are written so teachers can adapt them to fit student backgrounds and grade levels. A single activity can be extended over several days or used in part.

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Math and the Mind's Eye

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Introduction to Percentages

O V E R V I E W

Areas and lengths are used to introduce the meaning of percentage. Given the meaning of 100%, students develop their own procedures for carrying out computations involving percentages.

Prerequisite Activity

Unit V, Activity 2, *Geoboard Areas* or Unit V, Activity 3, *Area of Silhouettes*.

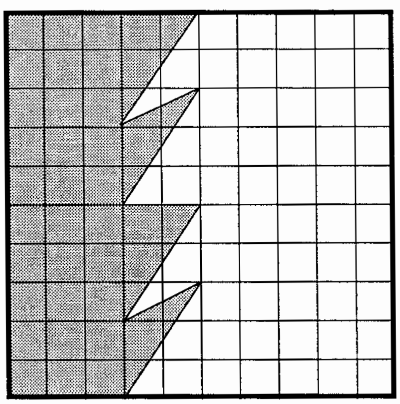
Materials

Copies of *Activity Sheets VII-1-A, VII-1-B, VII-1-C, VII-1-D* and *VII-1-E* for each student. Overhead transparencies as noted.

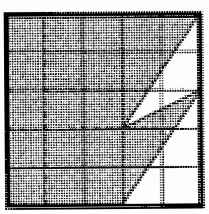
Actions

1. Show the students the pair of squares A and B pictured below. Ask them which square has the larger portion of its area shaded. Discuss the students' responses. Ask for volunteers to explain how they arrived at their conclusion.

B.

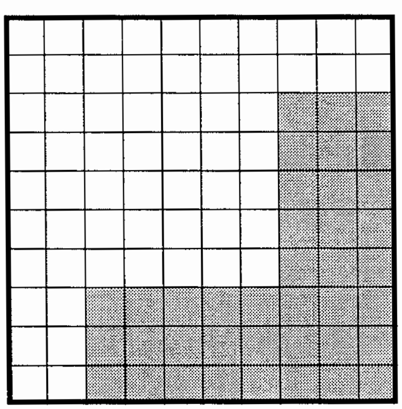


A.

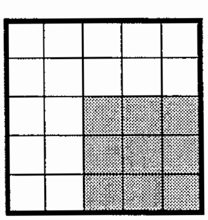


2. Repeat Action 1 for the pair shown below. In concluding the discussion, point out how percentages can be used to compare portions of differently sized quantities.

D.



C.



Comments

1. A master for preparing an overhead transparency for use in Actions 1 and 2 is attached. It is intended that only the top half be shown the students in Action 1.

The students will arrive at their conclusion in a variety of ways. One way is to observe that more than half of region A is shaded and less than half of region B is shaded, hence the greater portion of A is shaded.

Some students may respond "B" since the area shaded in B is twice as large as the area shaded in A. You can agree that the size of the shaded area in B is larger than that in A. Then you may clarify your original question by asking, if the squares were of the same size, e.g., if square A were enlarged until it was the same size as square B, which square would have the larger portion of its area shaded.

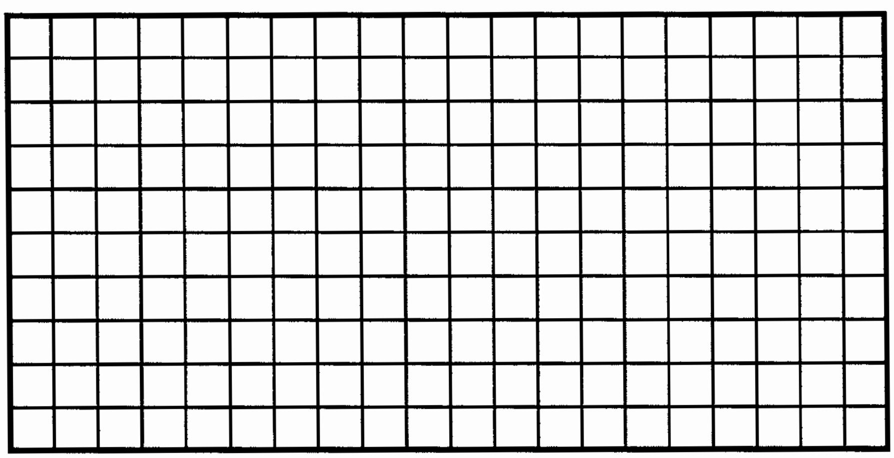
2. The students are likely to have more difficulty in reaching a conclusion than in Action 1. Some may imagine enlarging square C by doubling the length of its sides so that it is the same size as D. In this case the sides of the shaded region will be doubled also, so that its area is 6×6 or 36. Since D contains 39 shaded squares, a larger portion of D is shaded.

Others may base their conclusion on the fact that D has 39 squares out of 100 shaded while C has 9 squares out of 25 shaded, which is equivalent to 36 squares out of 100. This can be illustrated by dividing each square in C into four parts as shown below.

Continued next page.

Actions

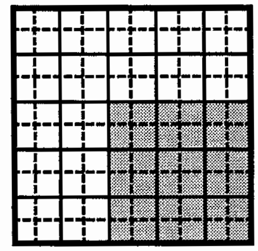
3. Display a 10×20 rectangle on the overhead. Ask for a volunteer to shade in 1% of the area of the rectangle. Have the students determine what percent one square unit is of the rectangle's area. Discuss any questions the students have.



Comments

2. *(Continued.)*

C.

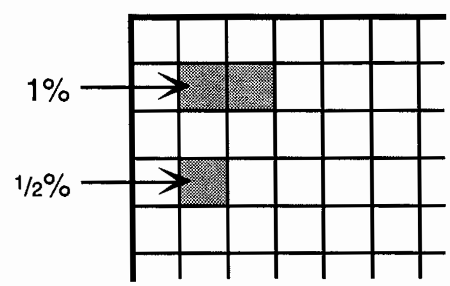


Percentages may be introduced by pointing out that another way of stating "36 parts out of 100 are shaded" is "36 per hundred are shaded" or, using the medieval French word for 'hundred,' "36 per cent are shaded." Since the number of squares gives the area, one can write, using the standard symbol for per cent, "36% of the area of C is shaded" and "39% of the area of D is shaded". Thus percentages are useful in comparing portions of quantities of unlike size.

3. You may have to remind the students that 1% of an area is one part of the area *when the entire area is divided into 100 equal parts.* That is, 1% of an area is one-hundredth of that area. The area of a 10×20 rectangle is 200 square units, so 1% of its area is 2 square units. Thus one square unit is $(^1/_2)$% or .5% of its area.

To summarize:

$$100\% = 200 \text{ square units},$$
$$1\% = 2 \text{ square units},$$
$$(^1/_2)\% = 1 \text{ square unit}.$$



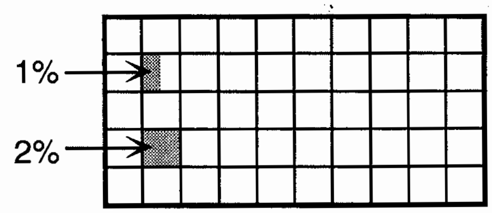
Actions

4. Repeat Action 3 using a 5×10 rectangle.

Comments

4. In this case,

$$100\% = 50 \text{ square units},$$
$$1\% = \tfrac{1}{2} \text{ square unit},$$
$$2\% = 1 \text{ square unit}.$$



5. Distribute a copy of *Activity Sheet VII-1-A* to each student. Ask them to carry out the instructions on their sheet. Discuss the methods the students used to arrive at their answers.

5. Masters for the activity sheets are at the end of the unit.

The students can work in groups of 3 or 4, comparing results and resolving differences.

Sketches for regions G, H and I are shown below. The percentages can be computed in a variety of ways. For example, in region I,

$$75 \text{ square units} = 100\%.$$

Dividing by 25,

$$3 \text{ square units} = 4\%.$$

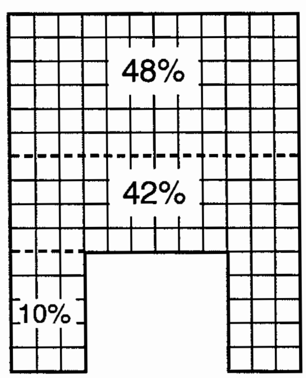
Multiplying by 8,

$$24 \text{ square units} = 32\%.$$

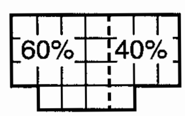
Alternatively,

$$75 \text{ square units} = 100\%,$$
$$1 \text{ square unit} = (^{100}/_{75})\% = {}^{4}/_{3}\%,$$
$$24 \text{ square units} = 24(^{4}/_{3})\% = 32\%.$$

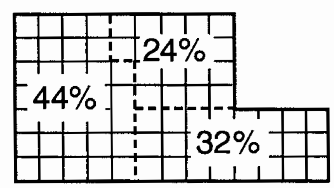
G



H



I



6. Distribute a copy of *Activity Sheet VII-1-B* to each student. Ask them to do part I. Discuss the methods used, then ask the students to do the other two parts. Discuss their results.

6. As you move about the room monitoring the students work, you may wish to invite selected students to show one of the regions they shaded. Providing each of these students with an overhead pen and an overhead transparency of the region you wish presented allows them to prepare ahead of time for showing their work.

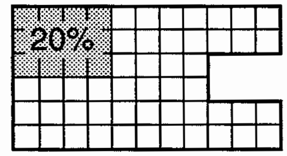
Continued next page.

Actions

Comments

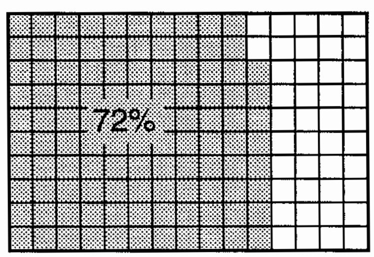
6. *(Continued.)* The students' methods will vary. If a student experiences difficulty, suggest to them that the number of squares to be shaded can be determined from the fundamental fact that 100% is the total area of a region. Thus in C, 100% = 60 square units whence, dividing by 5, 20% = 12 square units.

C.

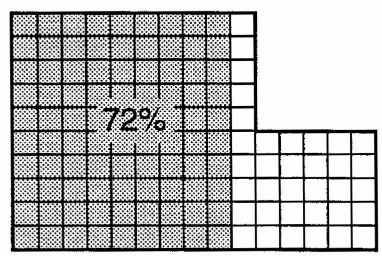


100% = 60 sq. units
20% = 12 sq. units

Finding how many squares to shade in parts II and III is more involved. For example, in the second region in III, 100% = 125 square units and one cannot determine the number of units in 72% by a simple division. There are several ways to proceed. One way is to note that 1% = 1.25 square units and multiply by 72 to get 72% = 90 square units. Another way depends upon recognizing that 72 = .72 × 100. Then, 72% = .72 × 100% = .72 × 125 square units = 90 square units. Calculators are helpful in performing such computations.



1% = 1.5 sq. units
72% = 72 × 1.5 = 108 sq. units

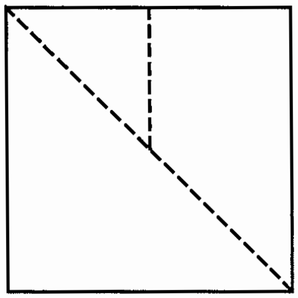


1% = 1.25 sq. units
72% = 72 × 1.25 = 90 sq. units

Parts II and III illustrate that a given percentage of one amount is not generally equal to the same percentage of a second amount.

Actions

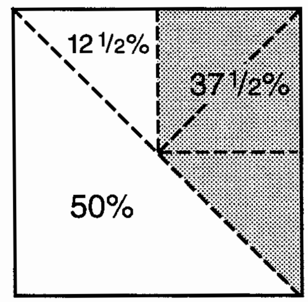
7. (Optional.) Show the students the following square region. Ask them to determine what percent the area of each subregion is of the total area of the square. Discuss their responses.



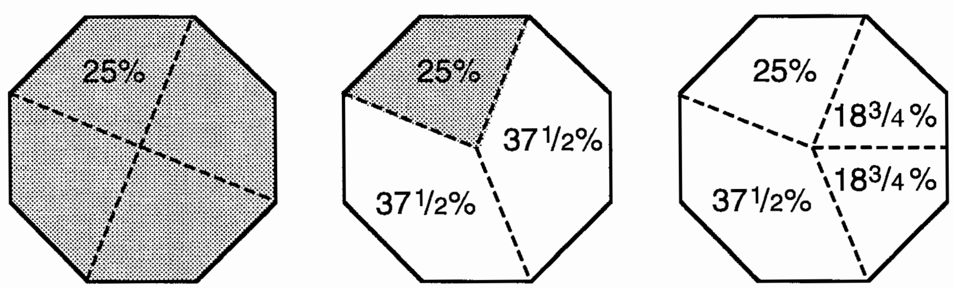
8. (Optional.) Distribute a copy of *Activity Sheet VII-1-C* to each student. Ask them to determine what percent the area of each subregion is of the total area of the region containing it. Discuss the students' methods and any difficulties they encounter.

Comments

7. The portion below the diagonal is 50% of the area. If the students have difficulty determining the remaining percentages, it may be helpful to draw the auxiliary lines shown below to see that the remaining 50% can be divided into four equal parts. One of the original subregions is one of these parts or (12 $\frac{1}{2}$)% of the area. The other original subregion is 3 of these parts or (37 $\frac{1}{2}$)% of the area.



8. Viewing a region, or portion of a region, as divided into equal parts may be helpful. For example, if octagonal region F is divided into 4 equal parts as shown below on the left, each of these parts is 25% of the area. In the middle figure, the unshaded portion is 75% of the total region. If this unshaded portion is divided in two, each part is $\frac{1}{2}$ of 75%, or (37 $\frac{1}{2}$)%, of the total area. Finally, if one of these parts is further divided in two, as shown in the figure on the right, each of the resulting parts will be $\frac{1}{2}$ of (37 $\frac{1}{2}$)%, or (18 $\frac{3}{4}$)%, of the total area.



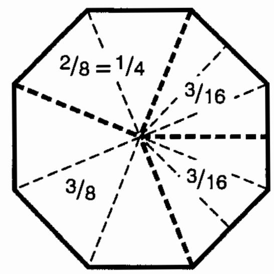
Actions

9. (Optional.) Distribute a second copy of *Activity Sheet VII-1-C* to each student. This time, ask the students to determine what fraction the area of each subregion is of the area of the region containing it. Then ask the students to compare their answers with those obtained in Action 8 to find the percentage equivalents of these fractions. Discuss.

10. Distribute a copy of *Activity Sheet VII-1-D* to each student. Ask the students to do part I of the sheet. Discuss.

Comments

9. This Action leads to finding the percentage equivalents of a number of common fractions. For example, figure F is a regular octagon and readily divides into 8 congruent triangular parts. One of the subregions is comprised of two of these parts, another of three of these parts. If each of the 8 triangular parts is further divided in two, F is divided into 16 congruent parts. The remaining two subregions are each comprised of three of these smaller parts. Hence we have the following:



Comparing this with the results in Comment 8, we see that $3/8$ of the area of a region is the same as $(37\ 1/2)\%$ of the area; $3/16$ of the area is the same as $18\ 3/4\%$. The relation between fractional parts and percentages is further treated in the next activity, *Fractions, Decimals and Percentages.*

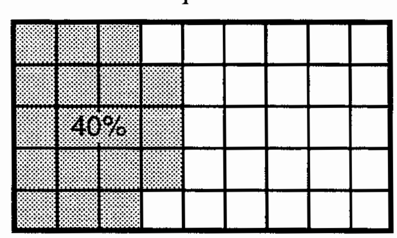
10. In a statement of the form, "X is Y% of Z," if two of the quantities X, Y and Z are known, the third can be determined. In Action 5, X and Z are given and the students find Y. In Action 6, Y and Z are given and they find X. In this Action, X and Y are given and the students are asked to find Z.

To discover the variety of methods used, you can ask for volunteers to show one of their completed rectangles and explain how they determined its size

One can proceed in a variety of ways. For example, in figure D,

40% = 18 square units.

D.



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Actions

11. Ask the students to do part II of *Activity Sheet VII-1-D*. Discuss.

Comments

10. *(Continued.)*

Hence, dividing by 40 (with the use of a calculator if one wishes),

$$1\% = 0.45 \text{ square units,}$$

multiplying by 100,

$$100\% = 45 \text{ square units.}$$

Thus, the completed rectangle is 5×9.

Alternatively,

$$40\% = 18 \text{ square units,}$$
$$20\% = 9 \text{ square units,}$$

and hence, multiplying by 5,

$$100\% = 45 \text{ square units.}$$

Or,

$$40\% = 18 \text{ square units}$$
$$400\% = 180 \text{ square units,}$$

and dividing by 4,

$$100\% = 45 \text{ square units.}$$

11. Part II is similar to part I except that measurements are in units of length rather than units of area. The lengths of the completed segments can be determined by methods similar to those used in part I.

In part E,

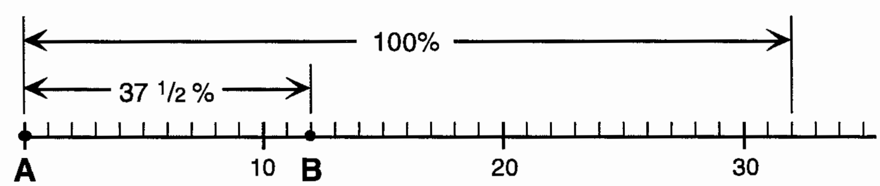
$$(37\ ^1/_2)\% = 12 \text{ units.}$$

Using a calculator to divide by 37.5,

$$1\% = .32 \text{ units,}$$
$$100\% = 32 \text{ units.}$$

Thus the completed segment is 32 units long.

E.



1% = .32 units, 100% = 32 units

Alternately,

$$(37\ ^1/_2)\% = 12 \text{ units.}$$

Then multiplying by 2,

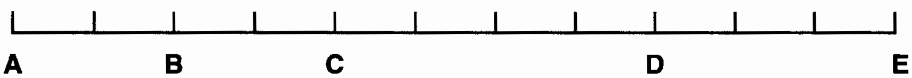
$$75\% = 24 \text{ units.}$$

Hence,

$$25\% = 8 \text{ units,}$$
$$100\% = 32 \text{ units.}$$

Actions

12. (Optional.) Show the students the following line segment. Ask them what percent the length AB is of length AC. Then ask them what percent length AD is of length AC. Repeat for length AE. Discuss any questions the students have.



13. (Optional.) Distribute a copy of *Activity Sheet VII-1-E* to each student. Ask the students to complete part 1. Discuss any difficulties the students have in completing this part. Then ask the students to complete the rest of the Activity Sheet. Discuss the methods they use to arrive at their conclusions.

Comments

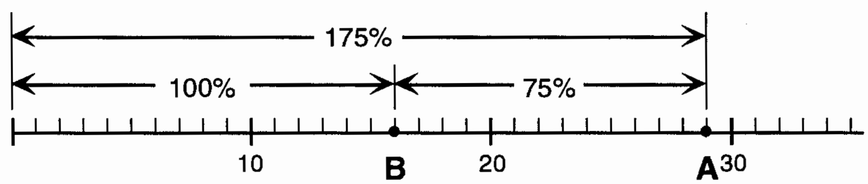
12. Actions 12 and 13 deal with percentages that are greater than 100%.

If each subinterval is 1 unit of length, AC is 4 units in length. Hence the length of each unit is 25% of the length of AC. Thus lengths AB, AD, and AE are, respectively, 50%, 200% and 275% of the length of AB.

13. In part 1a, the length of SB = 12 units = 100% and the length of SA = 24 units = 200% of the length of SB.

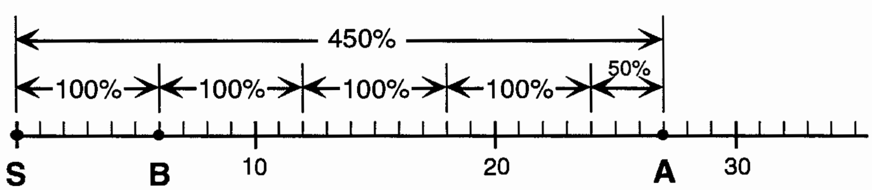
In part 1b., the length of SA is the length of SB plus 12 units. Since the length of SB is 16 units and 12 is 3/4 of 16, 12 units is 75% of the length of SB. Hence the length of SA is 100% + 75%, or 175% of the length of SB.

1b.



In 1c, 100% = 6 units. Since the length of SA = 27 units = 4(6) + 3 units, the length of SA = 4(100%) + 50%, or 450%, of the length of SB.

1c.



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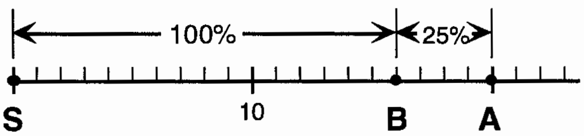
Actions

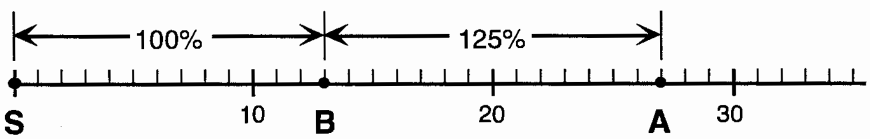
Comments

13. *(Continued.)* Parts 5 and 6 use common language that may seem ambiguous to the students. In both cases it is intended that the length of segment SB represents 100%. So in part 5, the length of SA is 125% of the length of SB. Thus SA is 20 units long. In part 6, the length of SA is 225% of the length of SB or 27 units long.

5.



6.



In part 8, the length of SA is 30 units and is given to be 120% of the length of SB. Hence,

$$120\% = 30 \text{ units.}$$

Dividing by 6,

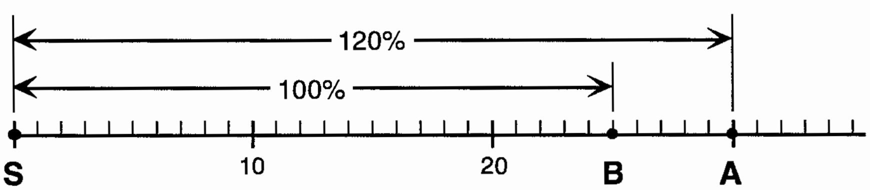
$$20\% = 5 \text{ units.}$$

Multiplying by 5,

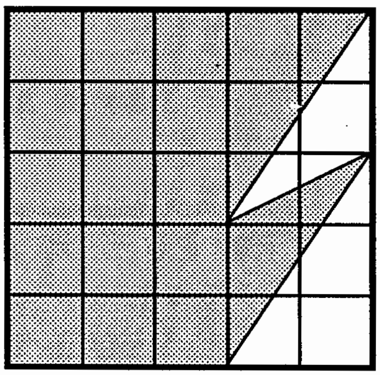
$$100\% = 25 \text{ units.}$$

Hence , the length of SB is 25 units.

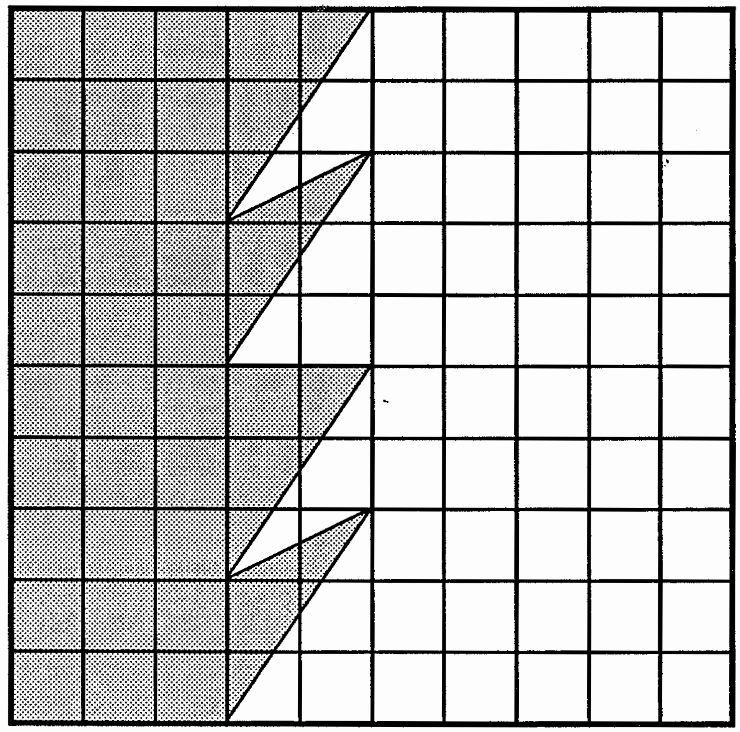
8.



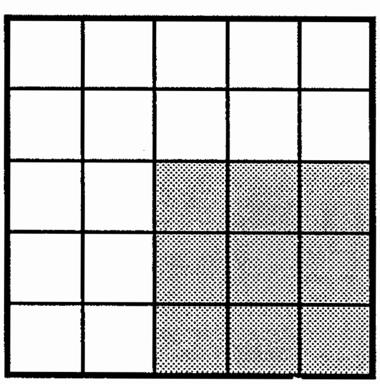
120% = 30, 20% = 5, 100% = 25



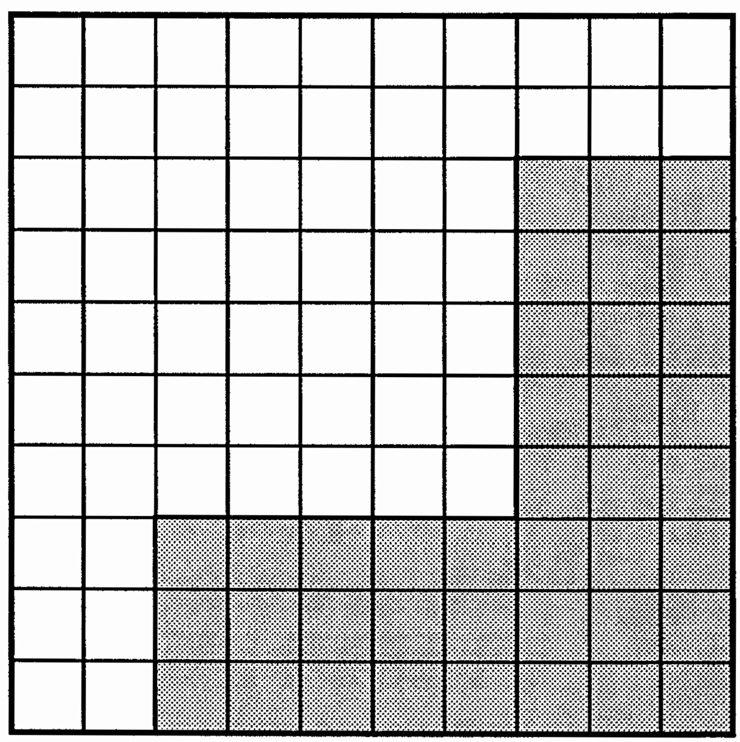
A



B



C

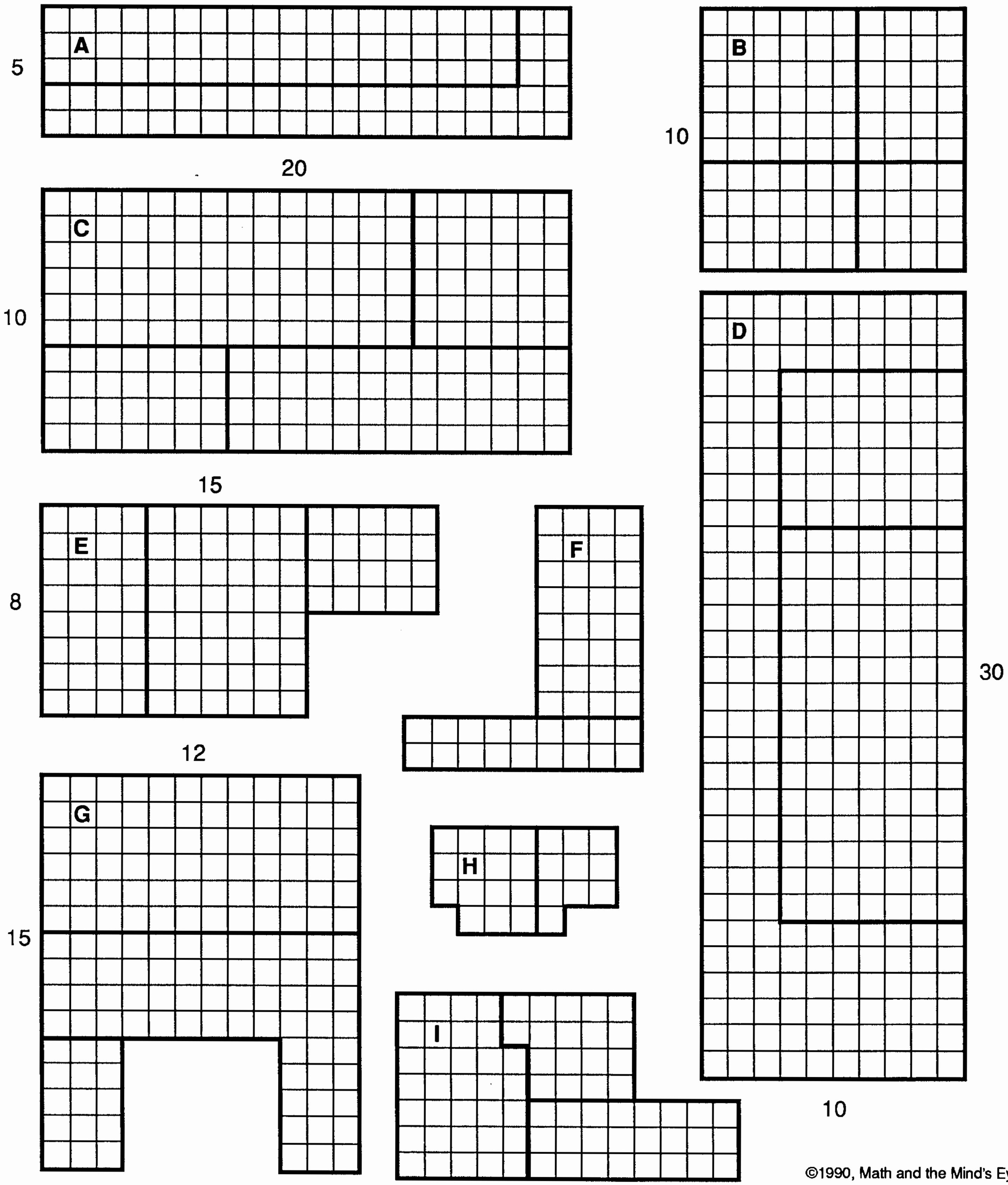


D

Name ____________________________

Activity Sheet VII-1-A

Find what percent the area of each subregion is of the entire region.

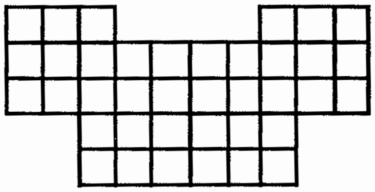


Activity Sheet VII-1-B

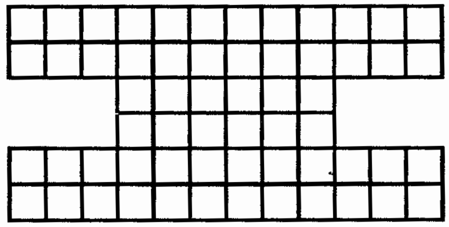
Name ______________________________

I. For each of the following regions, shade in the given percentage of its area.

A. 50%



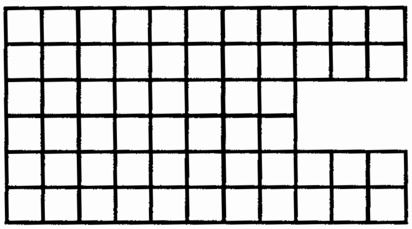
B. 75%



12

C. 20%

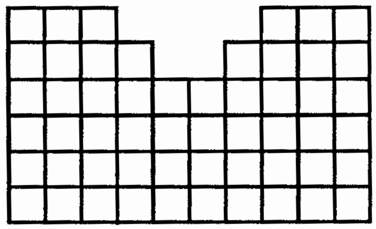
6



11

D. 33⅓%

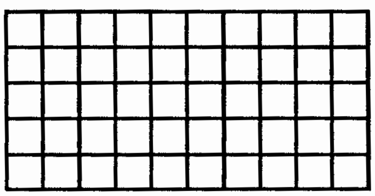
6



10

II. Shade in 30% of the area of each of the following regions.

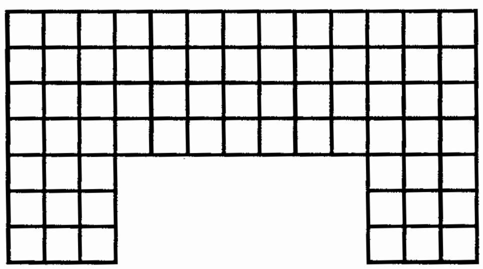
5

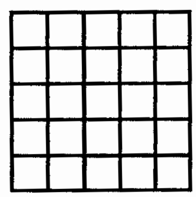


10

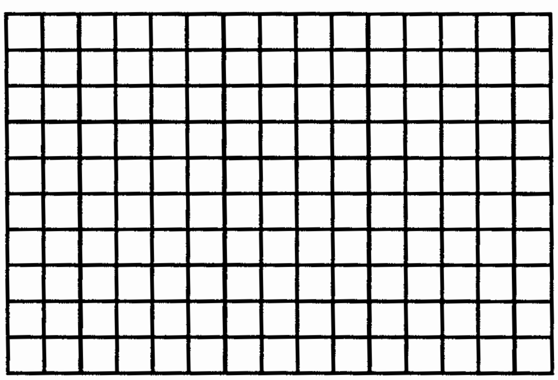
13

7



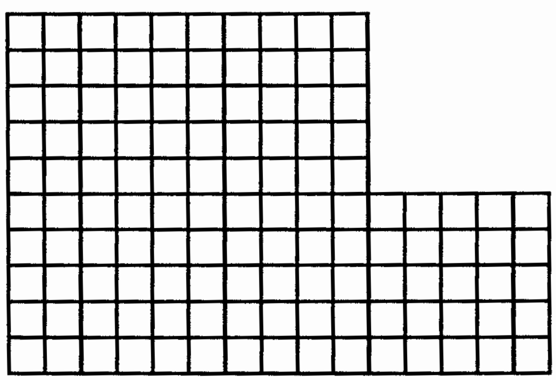


III. Shade in 72% of the area of the following regions.



15

10

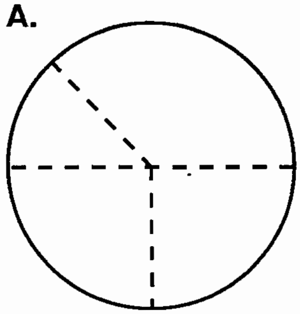


15

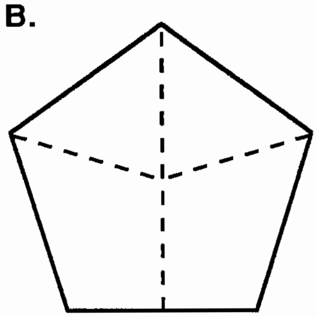
Name ______________________________

What percent is the area of each subregion of the area of the entire region?

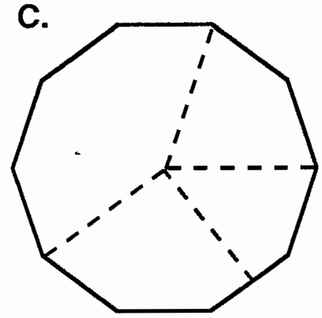
A.



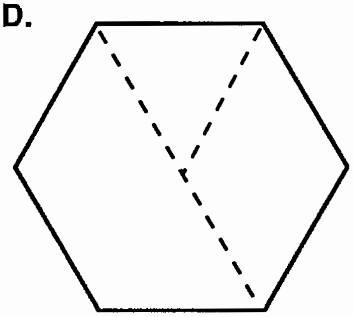
B.



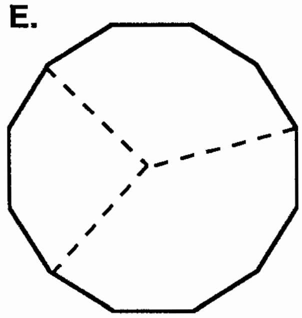
C.



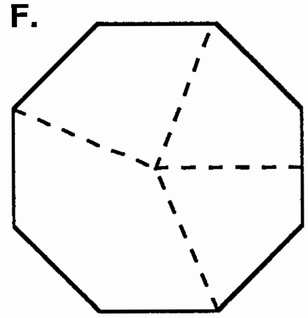
D.



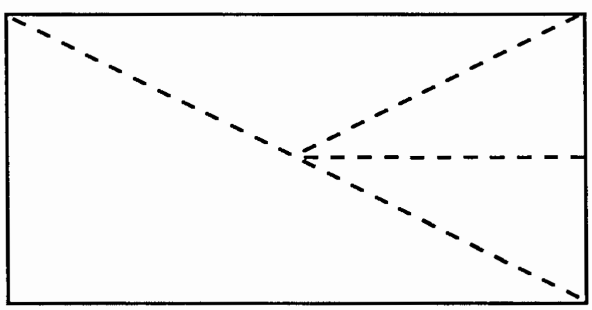
E.



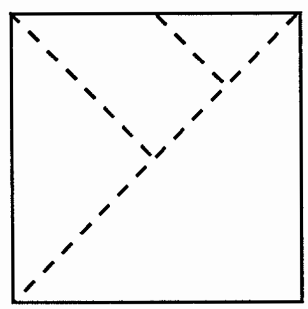
F.



G.

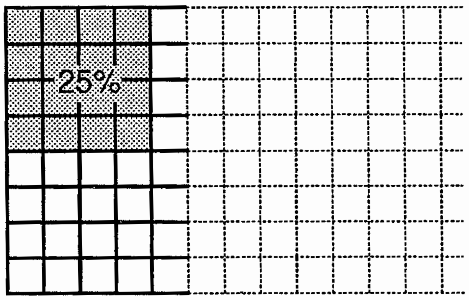


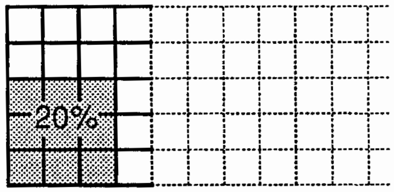
H.

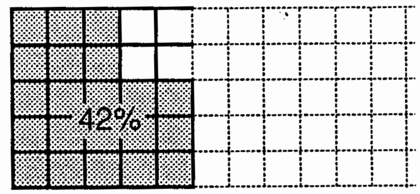


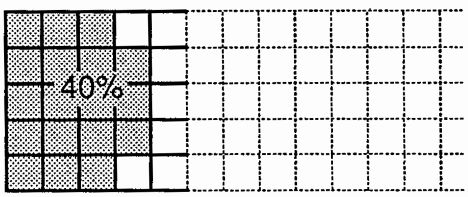
Name ____________________________

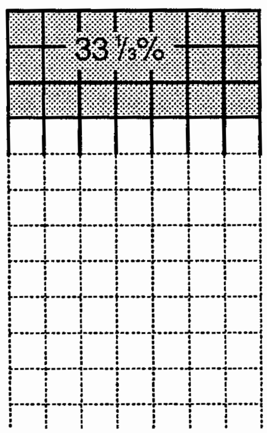
I. Complete the rectangle so the shaded region is the given percentage of its area.

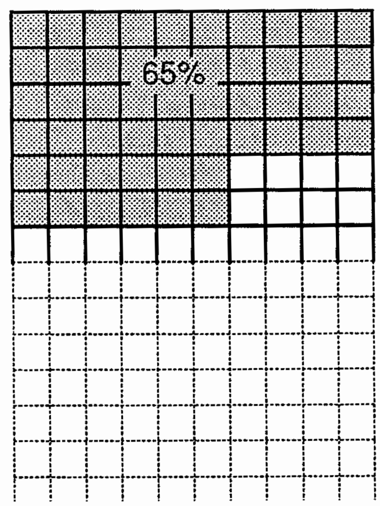
A.


B.


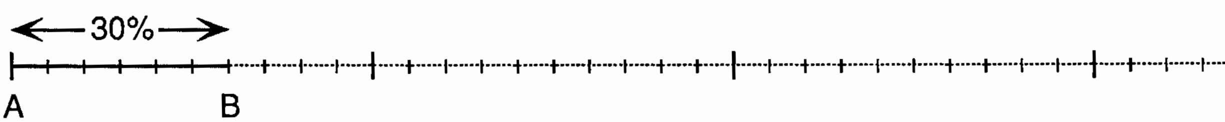
C.


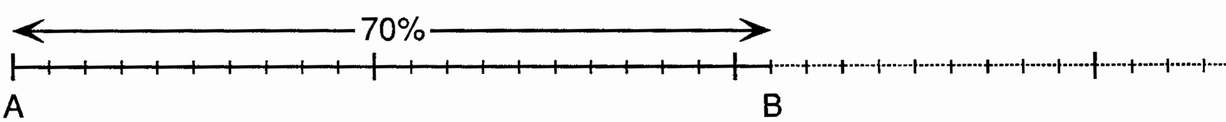
D.


E.


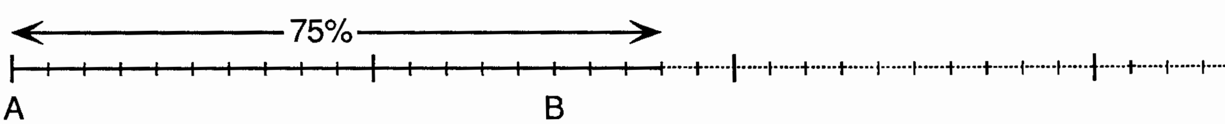
F.


II. In each of the following, AB is part of a line segment. Complete the line segment if AB is the given percentage of its length.

A.


B.


C.

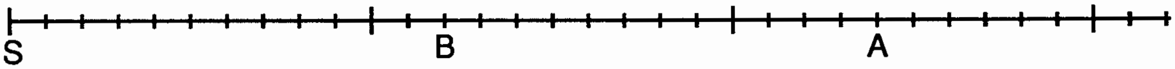

D.


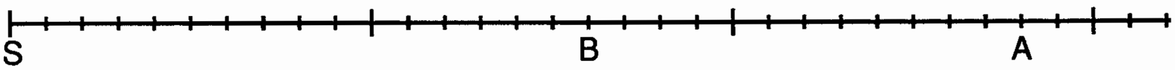
E.

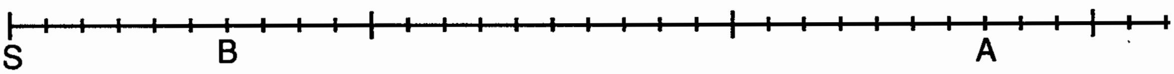

Name ______________________________

Activity Sheet VII-1-E

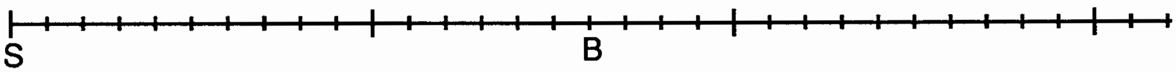
1. For each of the following find what percent length SA is of length SB.

a. 

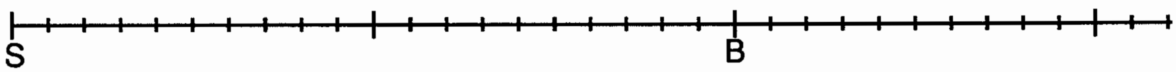
b. 

c. 

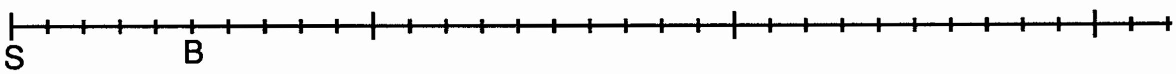
2. Find point A if the length of SA is 150% of the length of SB.



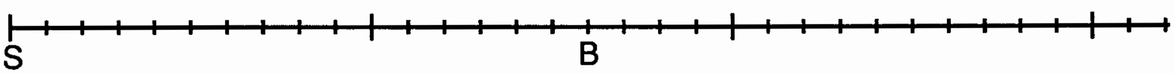
3. Find point A if the length of SA is 135% of the length of SB.



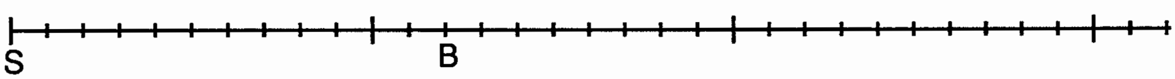
4. Find point A if the length of SA is 460% of the length of SB.



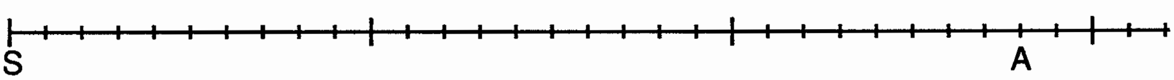
5. Find point A if SA is 25% longer than SB.



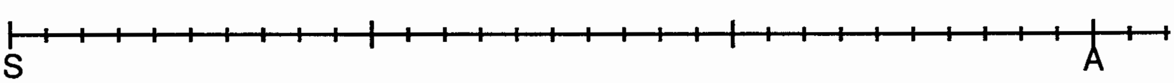
6. Find point A if the length of SA is the length of SB increased by 125%.



7. Find point B if the length of SA is 200% of the length of SB.



8. Find point B if the length of SA is 120% of the length of SB.



Unit VII • Activity 2

Fractions, Decimals & Percents

O V E R V I E W

A portion of a quantity can be described in terms of a fraction, a decimal, or a percentage. Ways of doing this, and of converting from one way to another, are investigated.

Prerequisite Activity

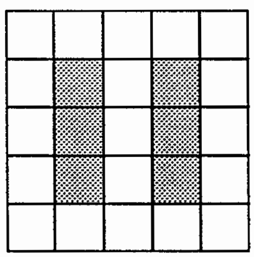
Unit IV, Activity 6, *Introduction to Decimals;* Unit VII, Activity 1, *Introduction to Percentage.*

Materials

Activity sheets as indicated for each student.

Actions

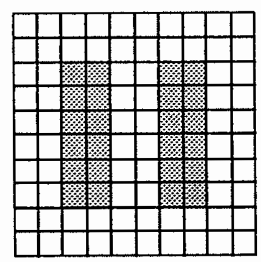
1. Show the students the figure below. Ask them: (1) what fraction of the area of the region is shaded, (2) what the decimal equivalent of this fraction is and (3) what percentage of the area is shaded. Allow time between questions to discuss the students' answers. Point out the different ways the amount of shading can be described.



Comments

1. A master for preparing an overhead transparency for use in Actions 1 and 2 is attached. It is intended that only the top half be shown the students in Action 1.

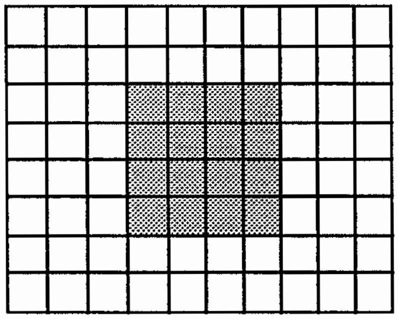
Since 6 squares out of 25 are shaded, 6/25 of the area is shaded. The fraction 6/25 is equivalent to 24/100 which, written in decimal form, is .24. This equivalence can be shown by subdividing each square into 4 parts (see figure below) and observing that now 24 parts out of 100 are shaded. This also means that 24% of the area is shaded.



The amount of area shaded can be expressed as an ordinary fraction ("6/25 of the area is shaded"), as a decimal fraction (".24 of the area is shaded") or as a percentage ("24% of the area is shaded").

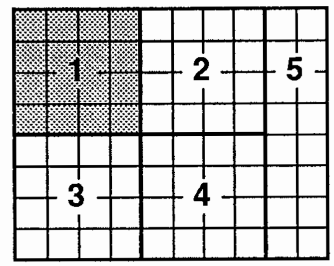
Actions

2. Repeat Action 1 for the following figure.



Comments

2. Since 16 out of 80 squares are shaded, 16/80 of the area of the region is shaded. Some of the students may recognize that 16/80 is equivalent to 2/10 or 1/5. One can also see that 1/5 of the area is shaded by observing that the entire region can be divided into 5 parts each with the same area as the shaded portion:



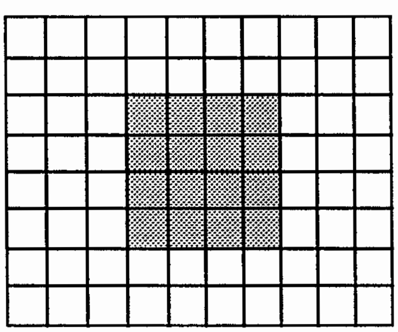
Since 16/80 is equivalent to 2/10, the equivalent decimal fraction is .2. Some students may find the decimal equivalent by dividing 16 by 80 on the calculator. This method is discussed later. (See Action 5).

Shading 1 out of 5 squares is equivalent to shading 20 out of 100, hence 20% of the region is shaded. There are other ways of determining this percentage. For example, 80 square units = 100%. Hence, 8 square units = 10% and thus, 16 square units = 20%.

Actions

3. Distribute a copy of *Activity Sheet VII-2-A* to each student. For each region, ask the students to write three statements about the amount of area shaded: one statement using ordinary fractions, one using decimal fractions and one using percentages. Examples of such statements are shown below for region A.

A.



16/80 or 1/5 of the area is shaded.
.2 of the area is shaded.
20% of the area is shaded.

Comments

3. Masters of the activity sheets are attached.

Region A of the activity sheet is the region discussed in Action 2. It is used here to provide examples of the requested statements.

For regions A-G, the fraction of the area shaded can be represented by a fraction whose denominator is 100, from which both the decimal fraction and percentage can be determined.

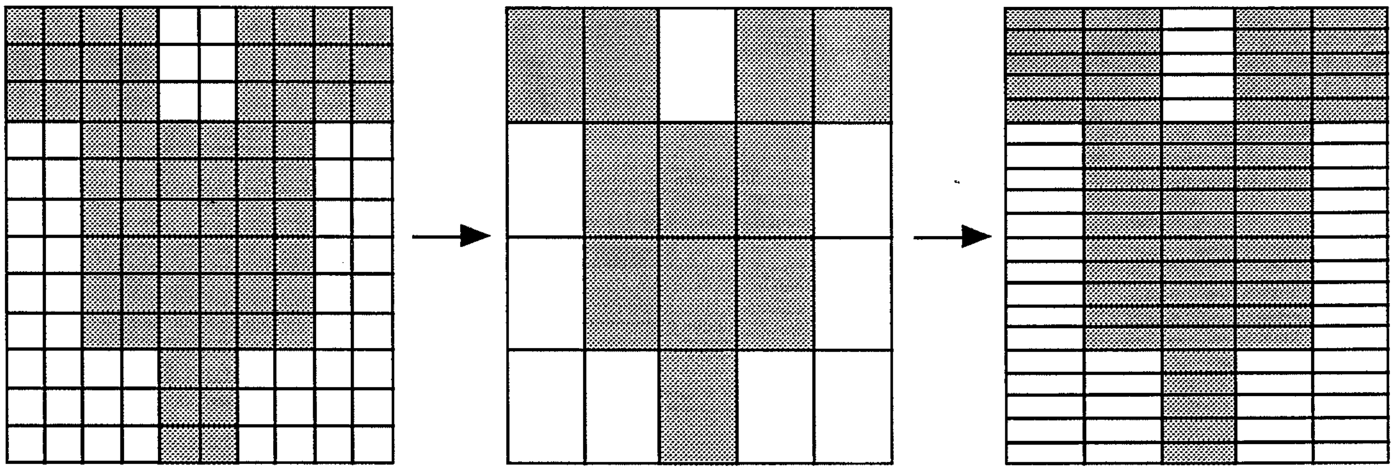
Continued next page.

Actions

Comments

3. (*Continued.*) For example, 66/120 of the area of region F is shaded. This fraction reduces to 11/20 which is equivalent to 55/100. Thus the decimal equivalent is .55. Also, shading 66 out of 120 parts is equivalent to shading 11 out of 20 which, in turn, is equivalent to shading 55 out of 100.

F.



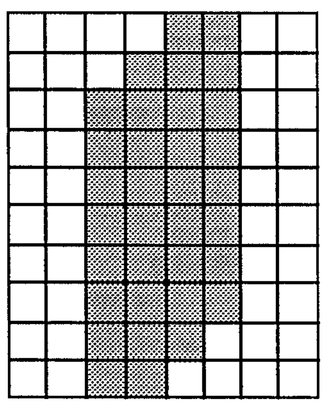
66 parts out of 120 are shaded.

11 parts out of 20 are shaded.

55 parts out of 100 are shaded.

Thus if the region were divided into 100 equal parts, 55 of them would be shaded. Hence 55% of the area is shaded.

H.



17/40 of the area is shaded.
.425 of the area is shaded.
42.5% of the area is shaded.

In region H, 34/80 or 17/40 of the area is shaded. This can not be written as a fraction with 100 as denominator. However, since $40 \times 25 = 1000$, 17/40 is equivalent to $(17 \times 25)/1000 = 425/1000$. This fraction, in decimal form, is .425. Since shading 425 squares out of 1000 is equivalent to shading 42.5 out of 100, 42.5% of the area is shaded. There are others ways to determine this percentage.

For example:

80 square units = 100%;

dividing by 80 (a calculator is recommended),

1 square unit = 1.25%;

multiplying by 34,

34 square units = 42.5%.

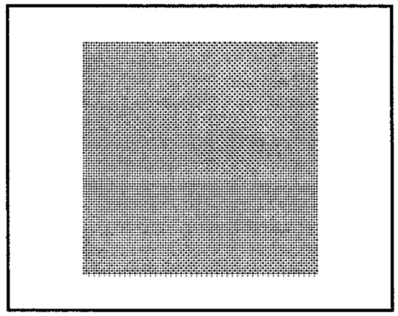
In region I, 30/80 or 3/8 of the region is shaded. Since $125 \times 8 = 1000$, 3/8 is equivalent to 375/1000 which, written as a decimal, is .375.

The students can compare their results with one another, discussing any discrepancies.

Actions

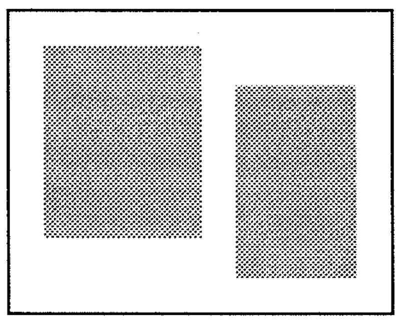
4. Repeat Action 3 with *Activity Sheet VII-2-B* . When the students have completed the sheets, discuss their methods and observations, concluding with a discussion of procedures for converting from decimal fractions to percentages.

D.



.45 of the area is shaded.
$\frac{9}{20}$ of the area is shaded.
45% of the area is shaded.

H.



.4375 of the area is shaded.
$\frac{7}{16}$ of the area is shaded.
43.75% of the area is shaded.

Comments

4. Point out to the students that, for each figure on the sheet, one of the statements has been provided and they are to provide the other two.

One way to find the fractional part of the area of region D that is shaded is to convert .45 into its fractional form:

$$.45 = \frac{45}{100} = \frac{9}{20}.$$

The percentage of the area of region D that is shaded can be determined by noting:

.45 of the area of the entire region D
= .45 of 100% of the area of D
= (.45 × 100)% of the area of D
= 45% of the area of D.

Notice that it is not necessary to know the numerical value of either the area of region D or the area of the shaded portion.

A similar procedure can be used for the other regions in which the portion of shaded area is expressed as a decimal. For example, for region H:

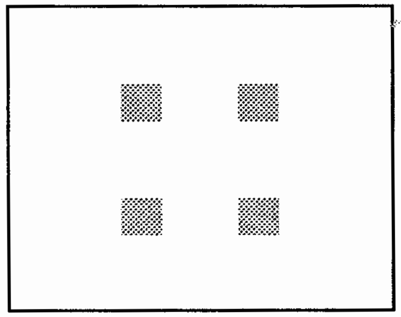
.4375 of the area of the entire region H
= .4375 of 100% of the area of H
= (.4375 × 100)% of the area of H
= 43.75% of the area of H.

Since $.75 = \frac{75}{100} = \frac{3}{4}$, this can also be written as $(43\ \frac{3}{4})\%$.

Expressed as a fraction, .4375 is $\frac{4375}{10000}$ which reduces to $\frac{7}{16}$.

Continued on next page.

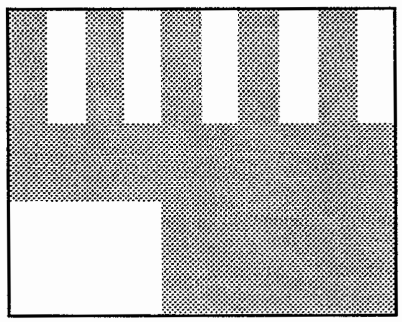
E.



5% of the area is shaded.
$5/100$ of the area is shaded.
.05 of the area is shaded.

4. (*Continued.*) If the percentage is given, the students can use the meaning of per cent to find the other forms. For example, in region E, 5% of the area is shaded. Thus if the region were divided into 100 equal parts, 5 of them would be shaded, i.e., 5 out of 100 parts, or $5/100$ of the area is shaded. This can be reduced to $1/20$. Expressed in decimal form, $5/100$ is .05.

I.



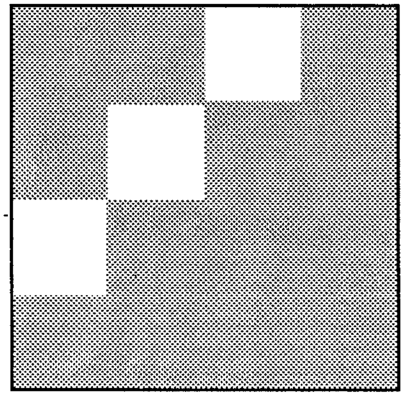
66 ¼% of the area is shaded.
.6625 of the area is shaded.
$53/80$ of the area is shaded.

In region I, 66 ¼% of the area is shaded. Thus, if the region were divided into 100 equal parts, 66 ¼, or 66.25, of them would be shaded. If each of these 100 parts were further divided into 100 equal parts, resulting in 10000 equal parts, 66.25 × 100 or 6625 of them would be shaded. Hence $6625/10000$ of the area is shaded. Written as a decimal, this is .6625; left in fractional form, it can be reduced to $53/80$.

The above examples illustrate that a decimal can be converted to a percentage by multiplying it by 100. Conversely, then, a percentage can be converted to a decimal by dividing it by 100. For example, .525 of an area is the same as 52.5% of that area, whereas 18.75% of an area is .1875 of that area.

Actions

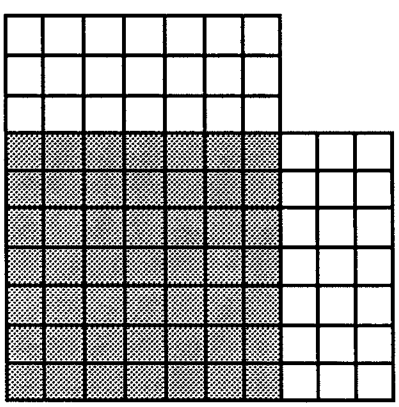
5. Show the students the following region which has 13/16 of its area shaded. Ask the students for their ideas on how the percentage of area that is shaded can be determined. Discuss how fractional parts can be converted to percentages.



13/16 of the area is shaded.

6. Provide each student with a copy of *Activity Sheet VII-2-C*. Ask them to rank the regions from least percentage of area shaded to greatest percentage of area shaded.

B.



Comments

5. A master to prepare a transparency of the region is attached.

The students may have various suggestions. One way to convert a statement about a fractional part of an area to a statement about a percentage of the area is to use a procedure similar to that described in Comment 4:

area of entire region = 100% of area,

13/16 of area = 13/16 of 100% of area,

= (13/16 × 100)% of area,

= (1300 ÷ 16)% of area,

= 81.25% of area.

The above computation could have been performed by first computing 13 ÷ 16 and then multiplying by 100. In view of the discussion in Comment 4 concerning the relationship between decimal fractions and percentages, finding 13 ÷ 16 and omitting the multiplication by 100 gives the decimal fraction, .8125, of the amount of area shaded.

With the aid of a calculator, statements involving fractional parts can be quickly converted to statements involving decimals or percentages, e.g., using a calculator, 17 ÷ 32 = .53125. Hence, 17/32 of the area of a region is .53125, or 53.125%, of the area. It is customary in many settings in which percentages are reported, such as the sports page in newspapers, to report decimals to the nearest thousandth and percentages to the nearest tenth. Thus in the above case, the decimal would be reported as .531 and the percentage as 53.1%.

6. It is anticipated that students will have calculators available for this activity.

For each region, the portion of area shaded can readily be expressed in terms of a fraction. For example, in region B, 49 out of 91 squares are shaded. Hence, 49/91 of the area is shaded. Now 49 ÷ 91 = .5384... . Thus, using the rounding convention mentioned in Comment 5, .538 or 53.8% of the area of region B is shaded.

Note that it is easy to express the amount of the areas that are shaded as fractions of the total area. However, it is difficult to compare the relative size of these fractions. Converting to percentages, or decimals, facilitates this comparison.

Continued next page.

Actions

7. (Optional.) Ask the students to bring to class examples of the use of percentages they find in newspapers and other popular media. Have the students share their findings with the class.

Comments

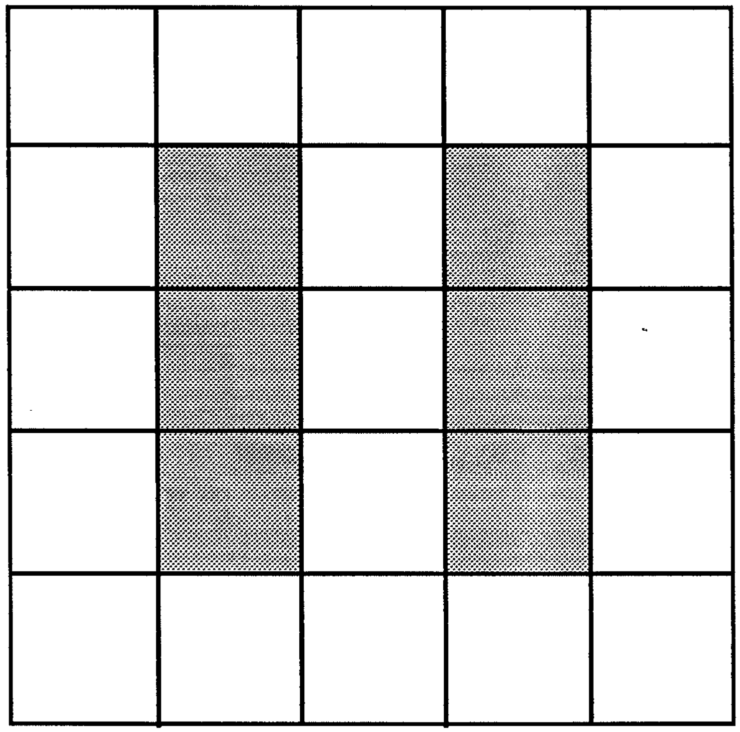
6. (*Continued.*) The regions, ranked from the least to the greatest percentage of area shaded, with percentages given to the nearest tenth, are:

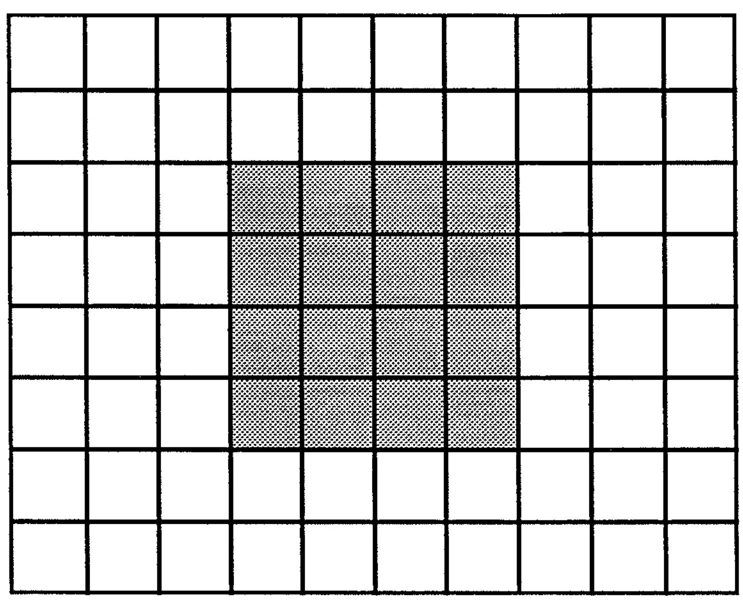
G	52.6%
B	53.8%
E	54.3%
F	55.6%
D	56.2%
A	57.1%
H	57.3%
C	57.7%

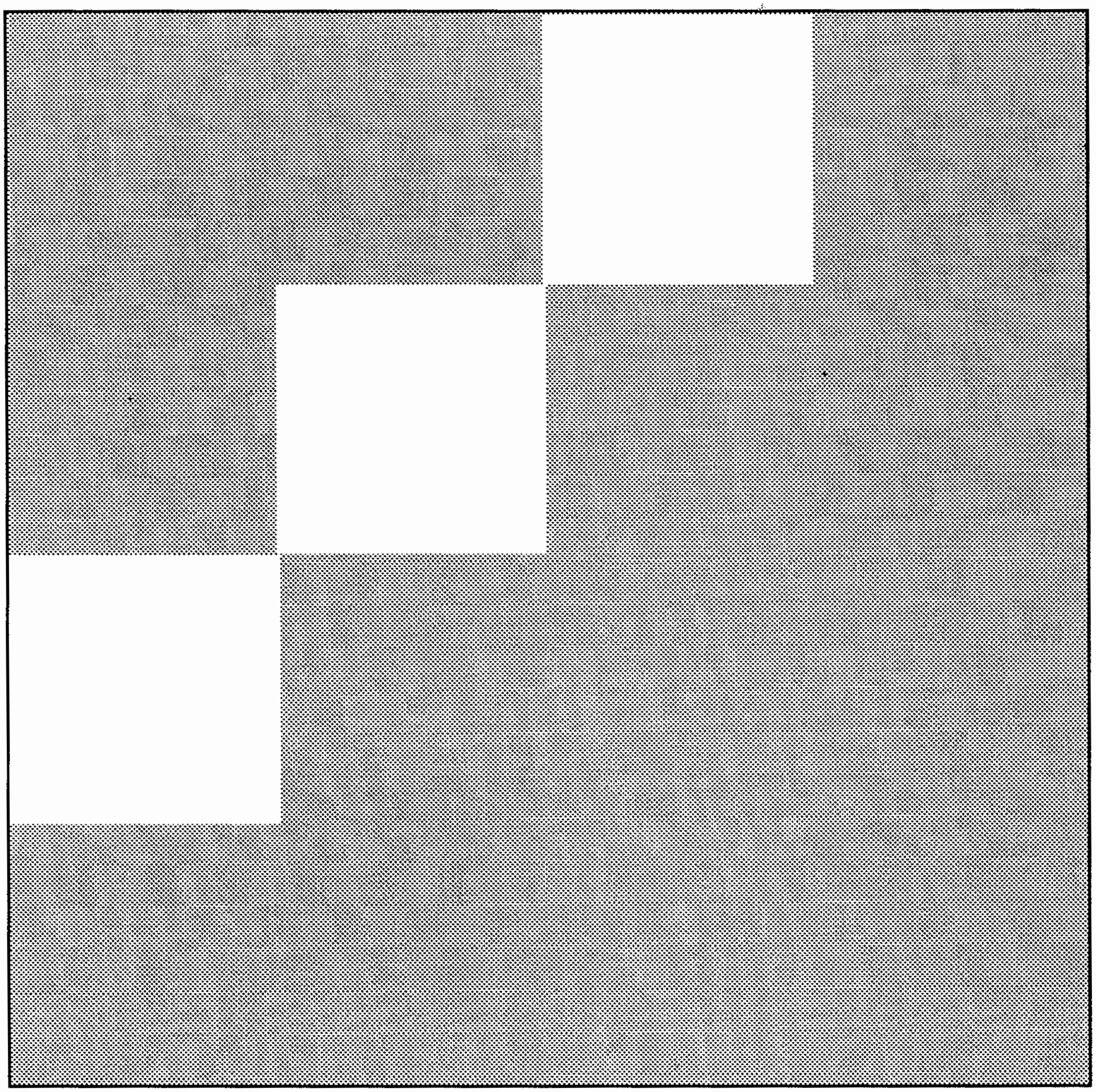
7. The language used in the media may differ from that of the mathematics class. For example, in the following listing of early season standings in the Pacific Division of the National Basketball Association, the column headed "Pct" is not actually the percentage of games won, but the decimal fraction of games won. Thus, on the date of the listing, the Portland Trailblazers had won .727 of the games they had played. The actual percentage of games they had won was 72.7%. Referring to a decimal fraction as a percentage is not uncommon in sports statistics.

	W	L	Pct.
L.A. Lakers	8	1	.889
Portland	**8**	**3**	**.727**
Seattle	6	5	.545
Phoenix	4	4	.500
Sacremento	3	6	.333
L.A. Clippers	2	5	.286
Golden State	2	7	.222

Sometimes a decimal fraction is referred to as an "average". For example, a baseball player who gets 36 hits in 129 times at bat is said to have a "batting average" of .279 (the decimal equivalent of $36/129$). This means that the player has hit safely in .279 of the times he or she has been at bat, or in 27.9% of the times at bat. Mathematically, the "batting average" is more akin to a percentage than an average. This statistic can be thought of as a mathematical average in the following sense: If a player is given a score of 1 if he or she hits safely and a score of 0 otherwise, the average of these scores will be the same as the batting average described above. For example, in the above situation, 36 times the player would have received a score of 1 and 93 times a score of 0. The average of 36 1's and 93 0's is .279.







$^{13}/_{16}$ of the area is shaded.

Name ______________________________

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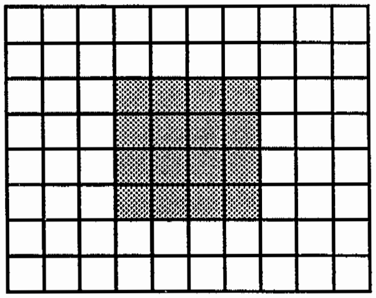

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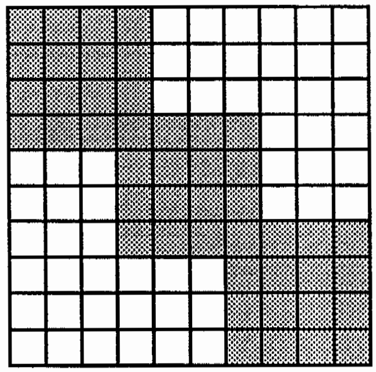

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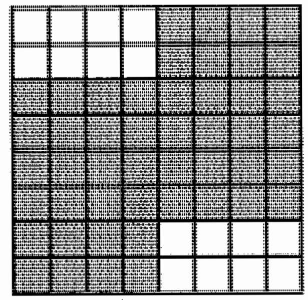

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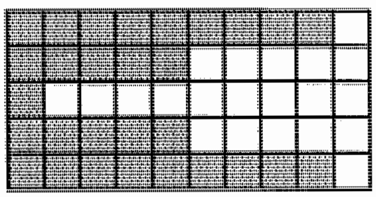

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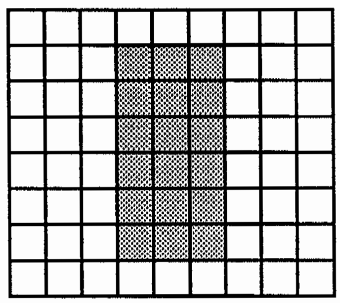

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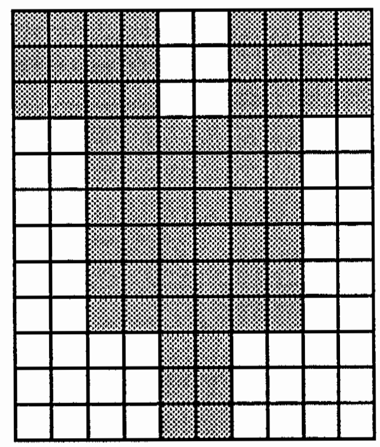

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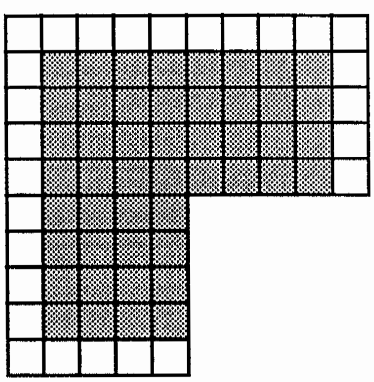

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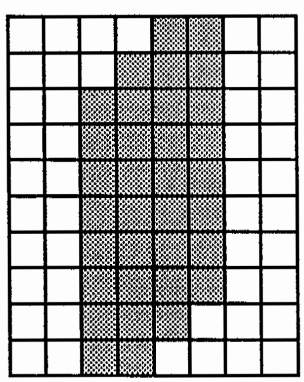
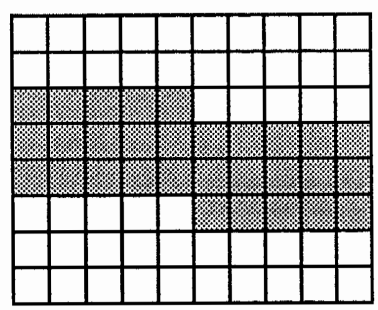


I.



Name ______________________________

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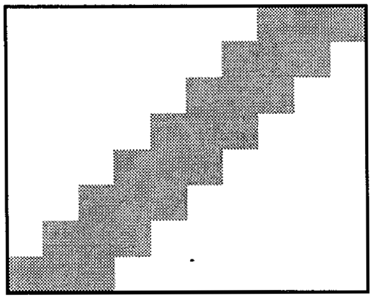
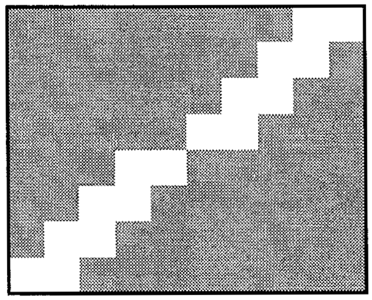


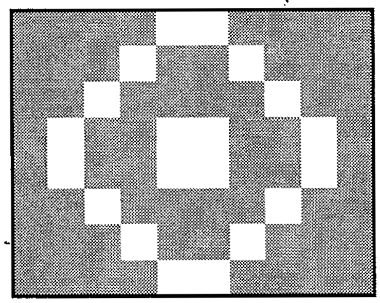
.3 of the area is shaded.

B.



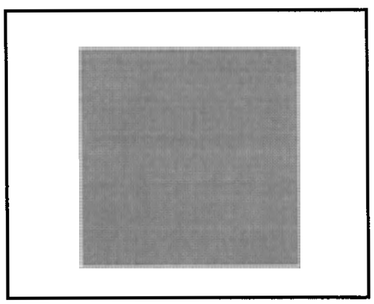
80% of the area is shaded.

C.



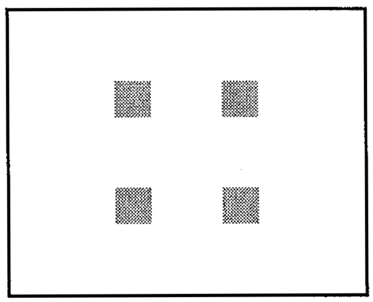
75% of the area is shaded.

D.



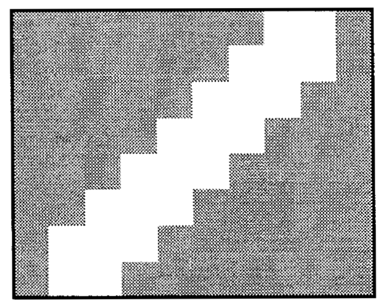
.45 of the area is shaded.

E.



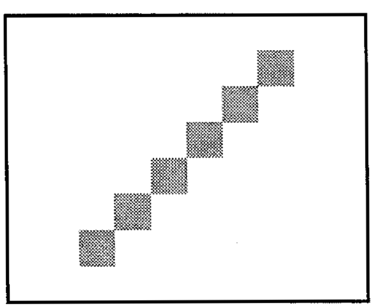
5% of the area is shaded.

F.



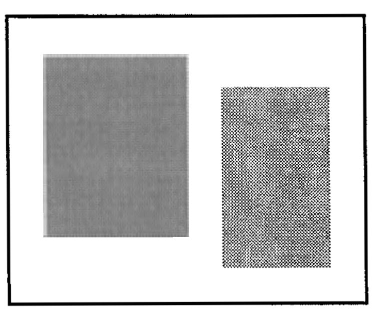
.725 of the area is shaded.

G.



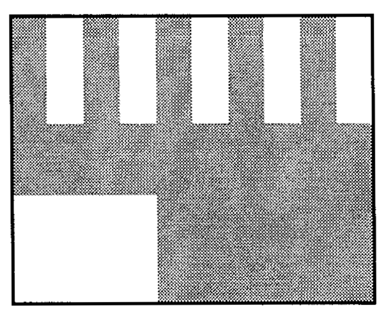
.075 of the area is shaded.

H.



.4375 of the area is shaded.

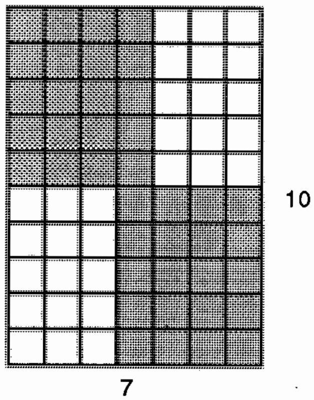
I.



66 ¼% of the area is shaded.

Name ______________________________

A.



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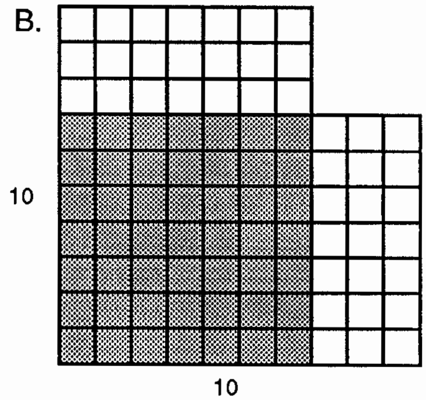

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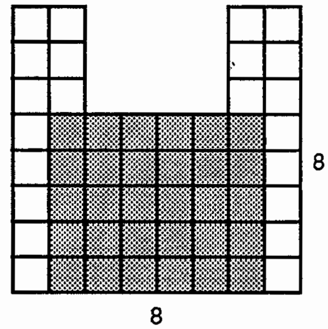
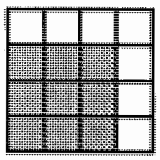


D.



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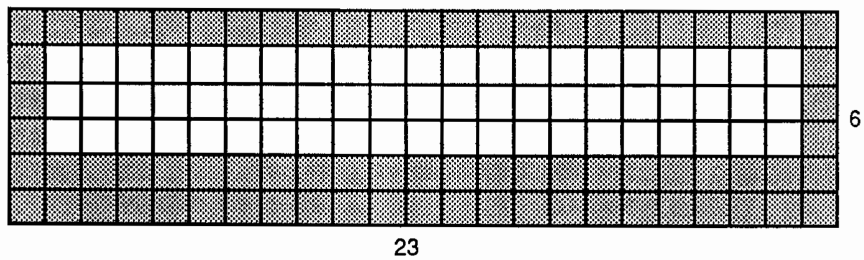

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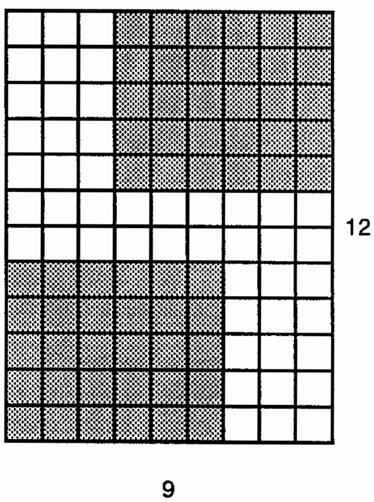

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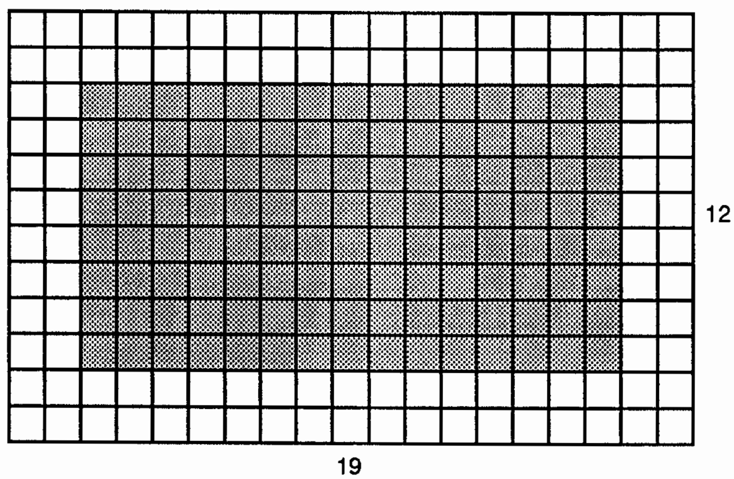

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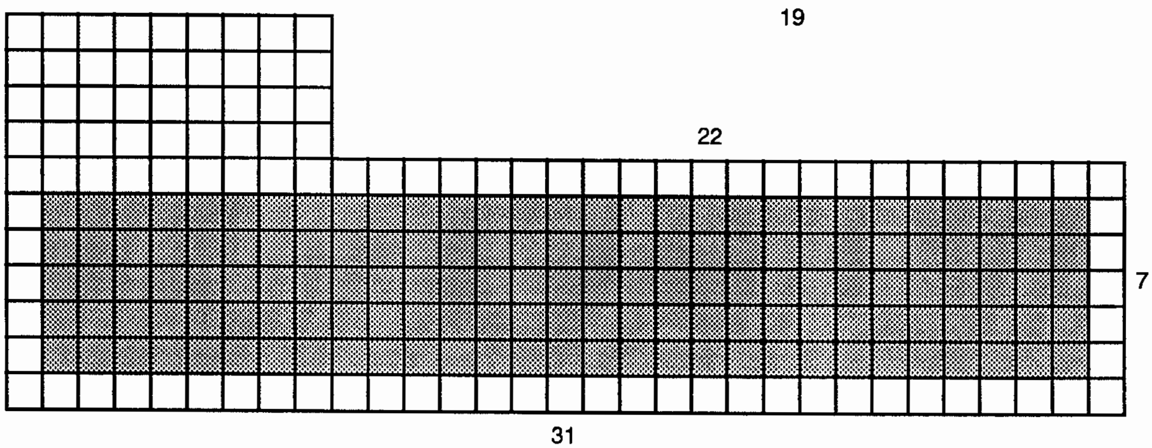


Ratios

O V E R V I E W

The concept of ratio is introduced as a way of comparing the number of black pieces to the number of red pieces in collections of counting pieces.

Prerequisite Activity

None.

Materials

Red and black counting pieces or other markers of two different colors.

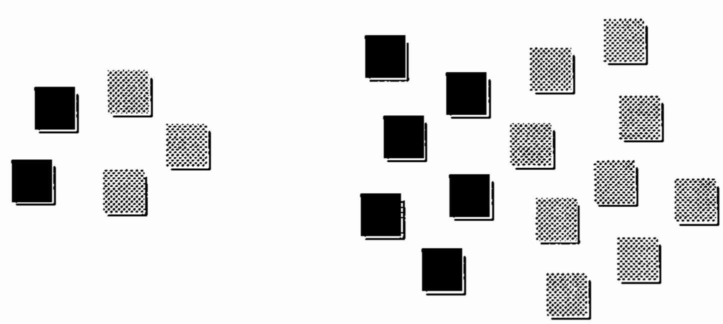
Actions

1. Distribute counting pieces to each student. Ask each student to form a collection containing 2 black and 3 red pieces. Then ask them to form another collection which has a different number of pieces but the same ratio of black to red pieces.

Comments

1. Each student, or group of students, should have at least thirty counting pieces. In place of counting pieces, tile of two different colors may be used or any two sets of objects that can be distinguished from each other, provided the wording in Actions 1–6 is changed accordingly.

Some students may question what it means for a collection to have a ratio of 2 black to 3 red pieces. One response is to tell the students it means for every 2 black pieces in the collection there are exactly 3 red pieces. To put it another way, a collection has a ratio of 2 black to 3 red pieces if the pieces in the collection can be placed in groups each containing 2 black and 3 red pieces. Note that each group will contain 5 pieces.



Collection A **Collection B**

These two collections have the same ratio of black to red pieces.

Actions

2. Ask the students to describe other collections in which the ratio of black to red pieces is 2 to 3. Record this information. Discuss with the students their observations about the data collected.

Comments

2. Here is a listing of some possible collections:

Collection	*Black*	*Red*	*Total*
A	4	6	10
B	20	30	50
C	12	18	30
D	6	9	15
...			

Some observations: The number of black pieces is a multiple of 3 and the number of red pieces is the same multiple of 2. The total number of pieces is a multiple of 5. If the number of black pieces is divided by the number of red pieces, the quotient is always ⅔.

One interesting observation is that, for any two collections, the product of the number of black in the first and the number of red in the second is the same as the product of the number of red in the first and the number of black in the second. The reason is that the number of blacks in a collection is a multiple of 2 and the number of reds is the same multiple of 3. For example, the above table could be written as follows:

Collection	*Black*	*Red*	*Total*
A	2×2	2×3	10
B	10×2	10×3	50
C	6×2	6×3	30
D	3×2	3×3	15
...			

Thus the product of the number of blacks in B and the number of reds in C is $10 \times 2 \times 6 \times 3$ which, interchanging the 10 and 6, can be written as $6 \times 2 \times 10 \times 3$, which is the product of the number of reds in B and the number of blacks in C.

Actions

3. Ask each student to form a collection of 15 black pieces and 9 red pieces, then ask them to describe other collections which have the same ratio of black to red pieces. Discuss ways of describing and recording this ratio.

4. Construct a table with the following headings on the chalkboard or overhead. Make the entries shown for the collection of counting pieces described in line 1 of the table. Ask the students to determine the missing information. If a ratio is to be recorded, ask for the smallest possible whole numbers. Discuss the methods the students use.

	Ratio of Black to Red Pieces	No. of Black Pieces	No. of Red Pieces	Total No. of Pieces
1.	2 to 3			20
2.	to	10		30
3.	3 to 4		12	
4.	to	6	14	

Repeat for lines 2, 3 and 4.

Comments

3. A collection of 15 black pieces and 9 red pieces contains 5 black pieces for every 3 red pieces. Hence, the ratio of black to red pieces in the collection is the same as the ratio of black to red pieces in a collection of 5 black and 3 red pieces. Examples of other collections which have the same ratio of black to red pieces are 10 black and 6 red, 20 black and 12 red, 25 black and 15 red, etc. Thus, we say the following ratios are equal: 15 to 9, 5 to 3, 10 to 6, 20 to 12, 25 to 15, etc.

The notation 15:9 is used to indicate a ratio of 15 to 9. Since a ratio of 15 to 9 is the same as a ratio of 5 to 3, one can write 15:9 = 5:3. A statement of equality of ratios is sometimes called a *proportion*. In writing a proportion, the symbol :: is sometimes used instead of an equal sign, e.g. 15:9::5:3. It is customary to read this statement as "15 is to 9 as 5 is to 3."

4. A master for making an overhead transparency of the table is attached. If it is used, it is suggested that you cover the entries and uncover them one line at a time as they are discussed.

The students will use a variety of methods in arriving at their answers. Here are some possibilities:

For the collection described in line 1, each group of 5 pieces will contain 2 black and 3 red. Since 20 pieces makes 4 groups of 5, there are 8 black and 12 red pieces in the collection.

The collection in line 2 will have 20 red pieces. Hence the ratio of black to red is 1 to 2.

In line 3, for every 4 red pieces there will be 3 black. Since there are 3×4 red pieces, there will be 3×3 or 9 black pieces.

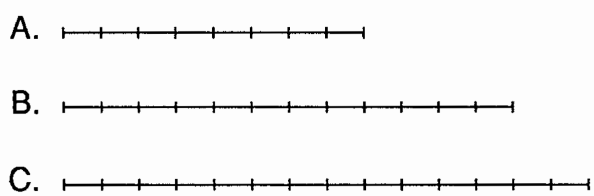
In line 4, the only common divisor of 6 and 14 is 2. The black pieces can be divided into 2 groups of 3 and the red into 2 groups of 7. Hence for every 3 black pieces there are 7 red pieces.

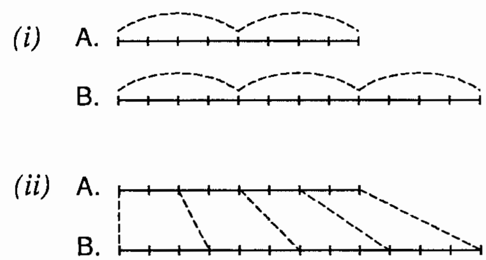
Actions

5. (Optional.) Enter the information from line 5 below in the chart and repeat Action 4 for these entries. Do the same for lines 6, 7, and 8.

	Ratio of Black to Red Pieces	No. of Black Pieces	No. of Red Pieces	Total No. of Pieces
	...	...	...	...
5.	to	50	250	
6.	to		280	700
7.	5 to 7			240
8.	5 to 2			350

6. Distribute ½ cm grid paper to each student. Ask the students to draw line segments A, B and C so that their lengths are 8, 12 and 14 units, respectively, as shown below. Then ask the students to draw line segment D so that the ratio of the length of C to the length of D is the same as the ratio of the length of A to the length of B. Discuss the methods the students use.





Comments

5. The difference in these entries and those in Action 4 is that the size of the numbers prevents the students from actually forming the collections. You and/or the students may wish to add other lines. Two entries in any line are sufficient to determine the remaining entries in the line. If entries are made at random, one may encounter fractional pieces.

In line 5, there is 1 black piece for every 5 red pieces so the ratio is 1 to 5. In line 6, there will be 420 black pieces. Since 420 is 140 groups of 3 and 280 is 140 groups of 2, there will be 3 black pieces for every 2 red pieces. Note 140 is the greatest common divisor of 420 and 280.

In line 7, 5 of every 12 pieces are black, so in 240 pieces, which is 20 groups of 12, there will be 20×5, or 100, black pieces. In line 8, 5 of every 7 pieces are black, resulting in 5×50, or 250, black pieces altogether.

6. A master for ½ cm grid paper is attached.

Some students may note that segment B is 1 ½ times as long as segment A and hence segment D should be 1 ½ times as long as segment C. Thus, segment D is $1\frac{1}{2} \times 14$, or 21, units long.

Other students may recognize that for every 2 units in A there are 3 units in B and thus for every 2 units in C there should be 3 units in D. Since there are 7 groups of 2 units in C, there should be 7 groups of 3 units, or 21 units, in D.

Still others may see that ½ of the length of A is ⅓ of the length of B. Hence, ½ of the length of C should be ⅓ of D. Since ½ of the length of C is 7, the length of D is 3×7, or 21.

The ratio of the length of A to the length of B is 2:3. This can be seen visually by observing either *(i)* that A consists of 2 groups and B of 3 groups, all of the same length, or *(ii)* for every 2 units in A there are 3 units in B.

Actions

7. Repeat Action 6 for other lengths.

8. (Optional.) Distribute copies of *Activity Sheet VII-3* to the students. Ask the students to complete the sheet. Discuss.

Comments

7. Here are some possibilities for lengths of segments A, B, and C, respectively:
   a. 6, 10 and 12,
   b. 30, 24 and 25,
   c. 8, 14 and 12.

The following choices for A, B and C involve fractional lengths:
   d. 2.5, 10 and 4,
   e. 12, 6, and 15,
   f. 4, 13, and 6.

8. If the students know the Pythagorean Theorem, they can use it to find that the hypotenuse of triangle C is 15 and that of D is 5. Thus the perimeter of C is 36 and the perimeter of D is 12, so that the requested ratio is 36:12 or 3:1.

Alternatively, this ratio can be determined by noting that each side of C is 3 times as long as the corresponding side of D, as shown in figure 1.

By computing the areas, one finds the ratio of the area of C to the area of D is 54:6 or 9:1. This conclusion can also be reached by observing that C is composed of 9 copies of D, as shown in figure 2.

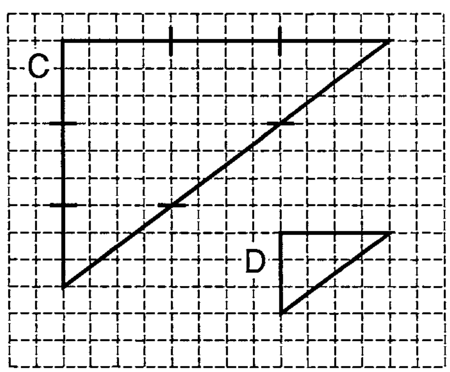


Figure 1

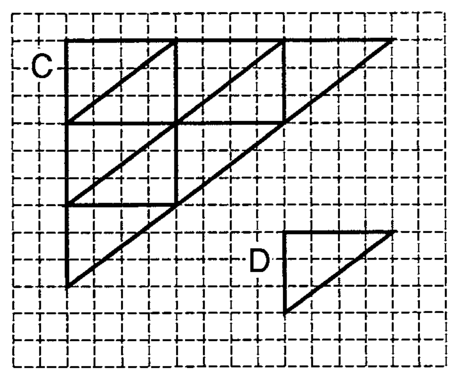


Figure 2

Continued next page.

Actions

Comments

8. (*Continued.*) One can see that the ratio of the area of F to the area of G is 1:4 by observing that every square unit in F corresponds to 4 square units in G, as shown in figure 3. Figure 4 is another way of showing this ratio. In this figure, F is pictured as the union of a rectangle and a triangle and G is shown to be composed of 4 copies of each. One can also determine the ratio by computing the areas of F and G.

Figure 5 shows that the ratio of the area of J to the area of K is 9:4.

This activity sheet illustrates the following: If the ratio of the perimeters of two similar regions is a:b, then the ratio of their areas is a^2:b^2.

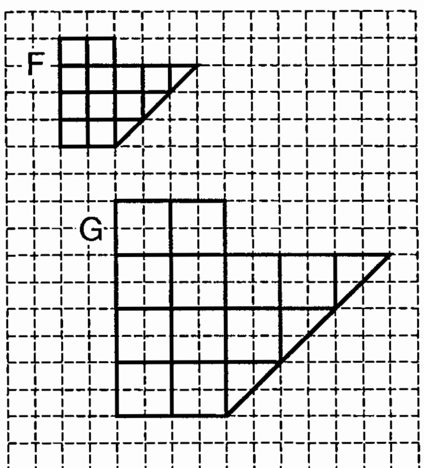


Figure 3

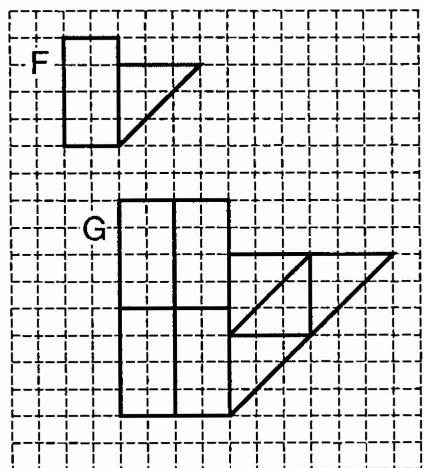


Figure 4

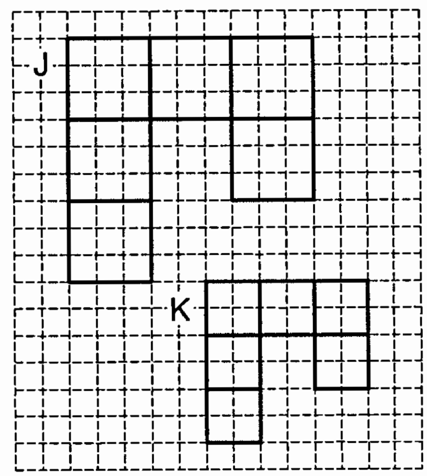
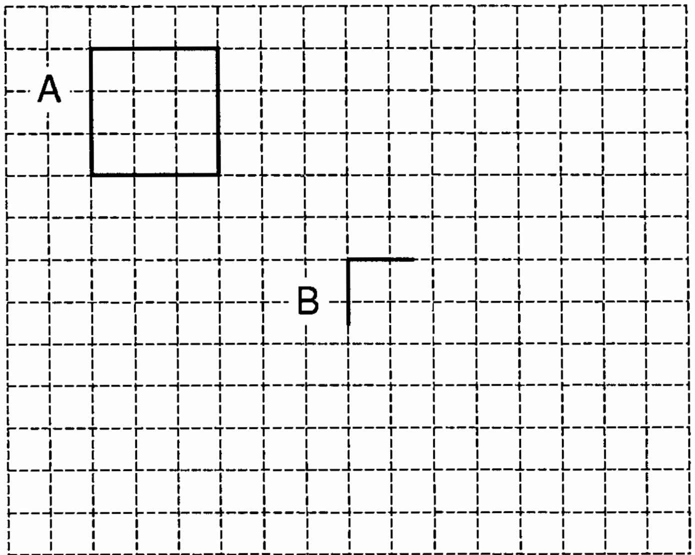


Figure 5

	Ratio of Black to Red Pieces	Number of Black Pieces	Number of Red Pieces	Total Number of Pieces
1.	2 to 3			20
2.	to	10		30
3.	3 to 4		12	
4.	to	6	14	
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				

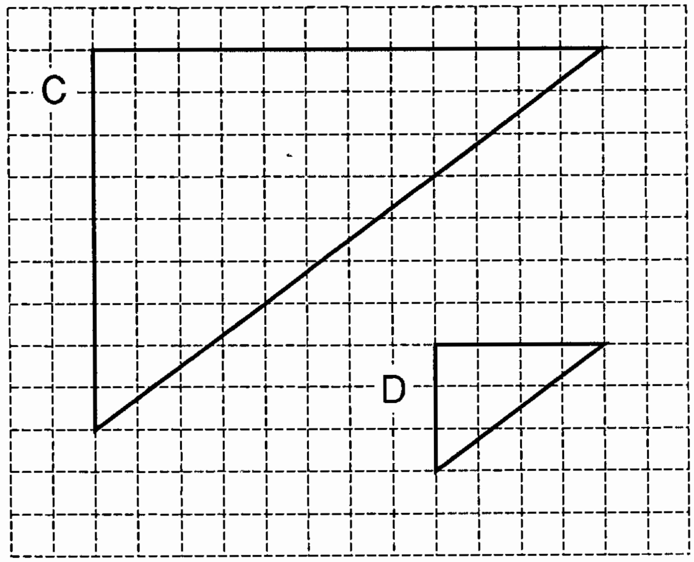
Name ______________________________

1. Complete square B so the ratio of the length of its side to the side of square A is 2 to 1. What is the ratio of the area of square B to the area of square A?



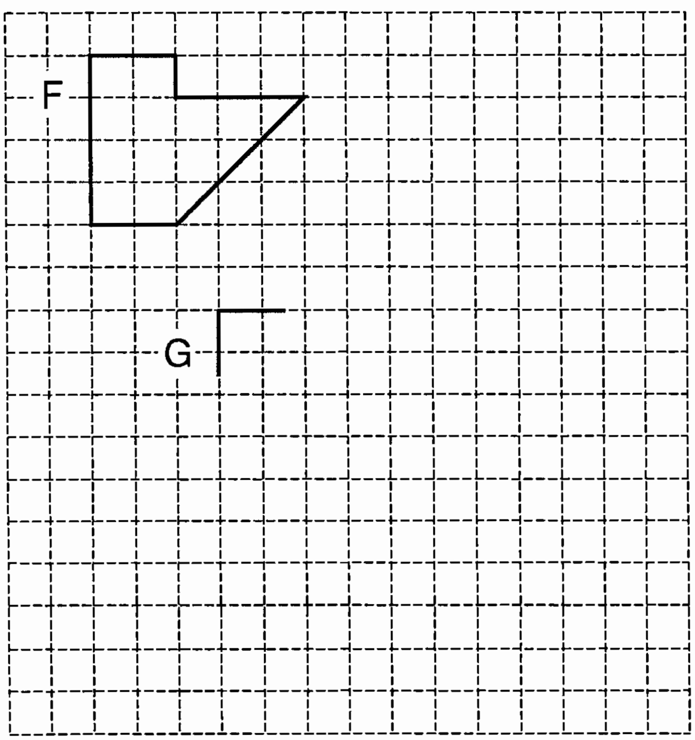
2. What is the ratio of the perimeter of triangle C to the perimeter of triangle D?

What is the ratio of the area of triangle C to the area of triangle D?



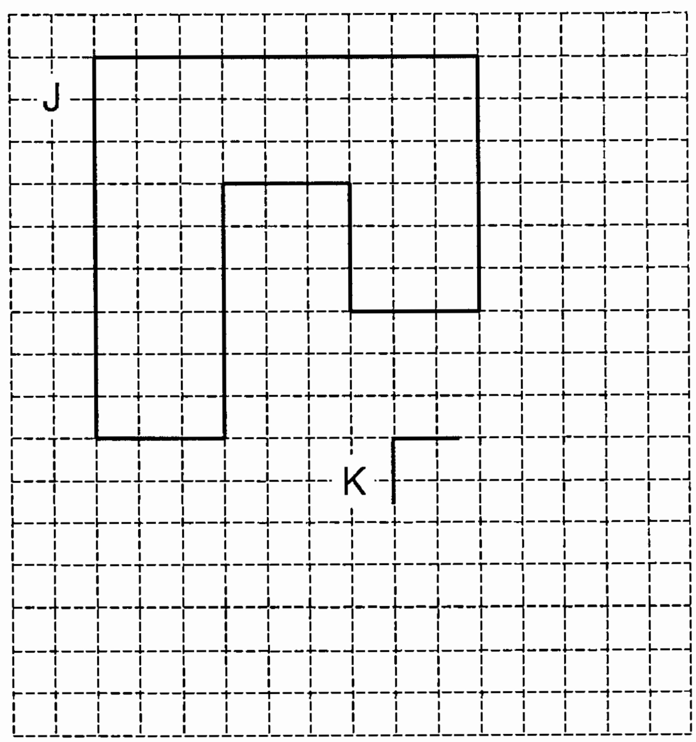
3. Complete figure G so it has the same shape as figure F and the ratio of the perimeter of F to the perimeter of G is 1 to 2.

What is the ratio of the area of F to the area of G?



4. Complete figure K so it has the same shape as figure J and the ratio of the perimeter of J to the perimeter of K is 3 to 2.

What is the ratio of the area of J to the area of K?



Percentage & Ratio Problems

O V E R V I E W

Diagrams and sketches are used to solve story problems involving percentages and ratios.

Prerequisite Activity

Unit 1, Activity 4, *Diagrams and Sketches*; Unit VII, Activity 1, *Introduction to Percentage*, Unit VII, Activity 2, *Fractions, Decimals and Percentages*, Unit VII, Activity 3, *Ratios.*

Materials

Copies of *Percentage and Ratio Problems* and *More Percentage and Ratio Problems.* For optional use, copies of *Still More Percentage and Ratio Problems.*

Actions

1. Write the following problem, which is problem ONE from *Percentage and Ratio Problems,* on the overhead or chalkboard. Ask the students to draw a sketch or diagram that enables them to solve the problem. Discuss.

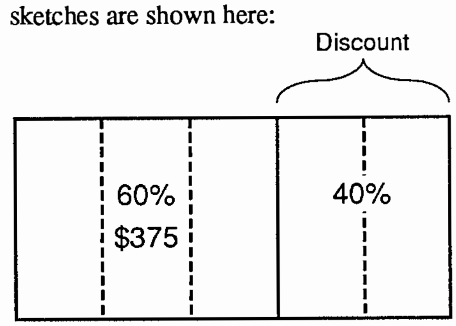
ONE

At a clearance sale, the price of a VCR was discounted 40%. The discounted price was \$375. What was the original price?

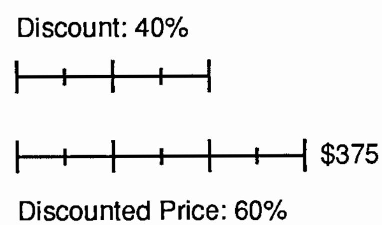
Comments

1. A master of *Percentage and Ratio Problems* is included with this activity.

You may want to ask for volunteers to show their sketches and how they used them to arrive at solutions. Two possible sketches are shown here:



"Each of the 5 regions represents \$375 ÷ 3, or \$125. Hence, the original price was \$125 × 5, or \$625."



"The discount is two-thirds of the discounted price of \$375. Thus the discount is \$250. So the original price is \$375 + \$250, or \$625."

Actions

2. Repeat Action 1 for problem TWO from *Percentage and Ratio Problems.*

TWO

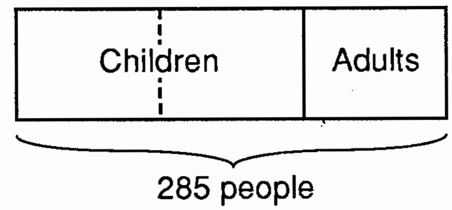
The ratio of children to adults at the circus is 2 to 1. If the total attendance is 285, how many children are present?

3. Ask the students to use diagrams and sketches to help them solve other word problems from *Percentage and Ratio Problems* and *More Percentage and Ratio Problems.*

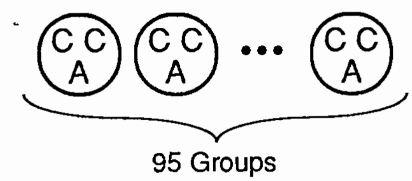
4. (Optional) Ask the students to solve *Still More Percentage and Ratio Problems* with the use of diagrams and sketches.

Comments

2. Shown below are two possible sketches.



"Each section represents 285 ÷ 3, or 95, people. Hence there are 95 adults and 190 children."



"If the people are divided into groups of three, each containing two children and one adult, there will be 285 ÷ 3, or 95, groups. So there are 95 adults and twice that many, or 190, children."

3. Other problems can be assigned from *Percentage and Ratio Problems* or from *More Percentage and Ratio Problems*, a master of which is also included with this activity.

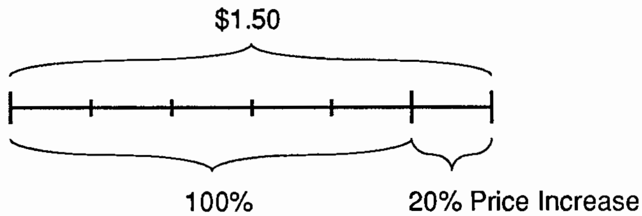
You may wish to assign a problem or two a day over a several day period. Volunteers can be asked to present their solutions to the class.

Sample sketches for THREE through TEN of *Percentage and Ratio Problems* and for *More Percentage and Ratio Problems* are found on pages 3 through 8 of this activity.

4. A master for *Still More Percentage and Ratio Problems* is attached. These problems are intended for those students who wish to work on more challenging problems. Pages 9 through 13 contain sample sketches for these problems.

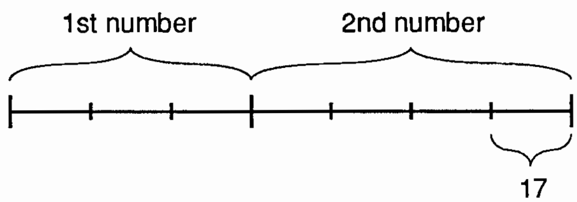
Percentage & Ratio Problems
Sample Sketches

THREE



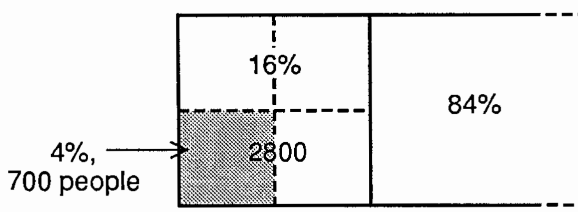
Price increase = $\$1.50 \div 6 = 25¢$.
So before the increase, the bar cost $1.25.

FOUR



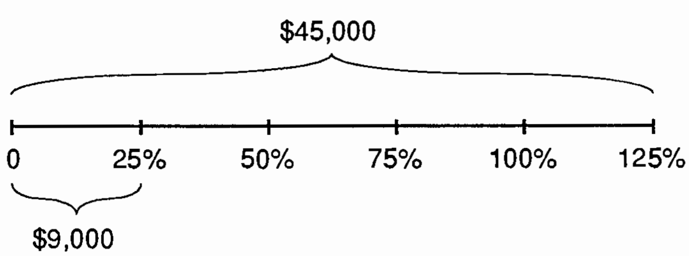
First number = $3 \times 17 = 51$.
Second number = $4 \times 17 = 68$.

FIVE



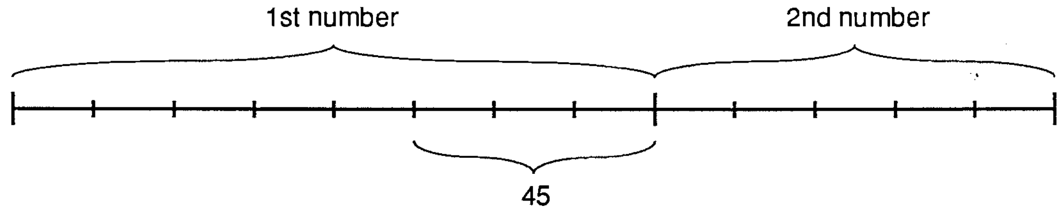
4% of the population is 700.
Hence, $100\% = 25 \times 700 = 17{,}500$.

SIX



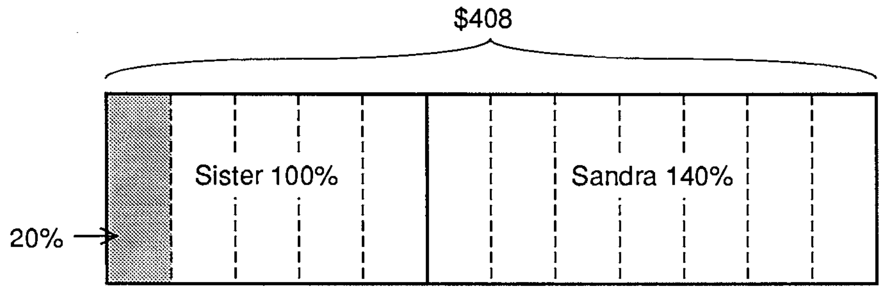
25% of goal = $\$45{,}000 \div 5 = \$9{,}000$.
Goal = $4 \times \$9000 = \$36{,}000$.

SEVEN



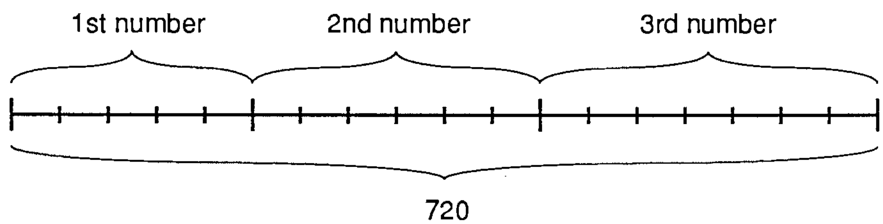
Each interval = 45 ÷ 3 = 15.
So, first number = 8 × 15 = 120 and second number = 5 × 15 = 75.

EIGHT



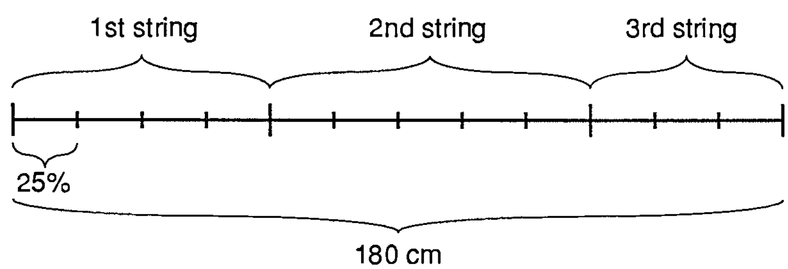
Each section = $408 ÷ 12 = $34.
Thus, the sister's sales = 5 × $34 = $170 and
Sandra's sales = 7 × $34 = $238.

NINE



Each interval = 720 ÷ 18 = 40. Thus, the first number = 5 × 40 = 200,
the second number = 6 × 40 = 240, the third number = 7 × 40 = 280.

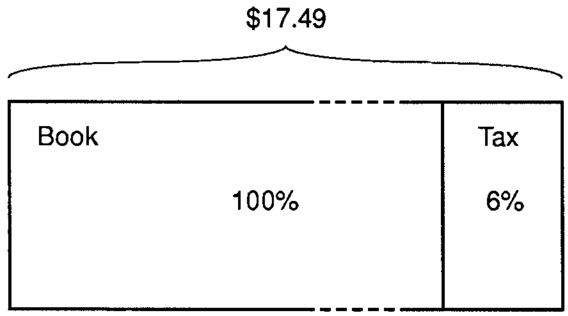
TEN



Each section = 180 cm ÷ 12 = 15 cm.
So, length of first string = 4 × 15 cm = 60 cm,
length of second string = 5 × 15 cm = 75 cm,
length of third string = 3 × 15 cm = 45 cm.

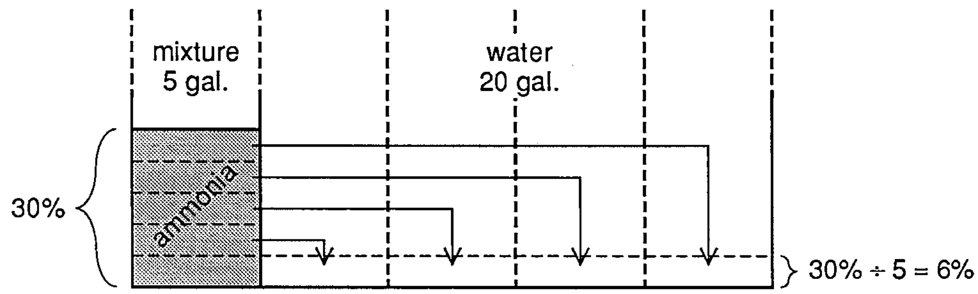
More Percentage & Ratio Problems
Sample Sketches

ONE



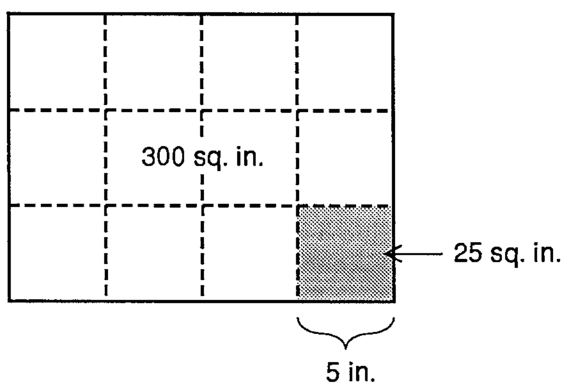
106% of book price = \$17.49.
So, 1% of book price = \$17.49 ÷ 106 = 16 ½¢.
Thus, book price = 100 × 16 ½¢ = \$16.50.

TWO



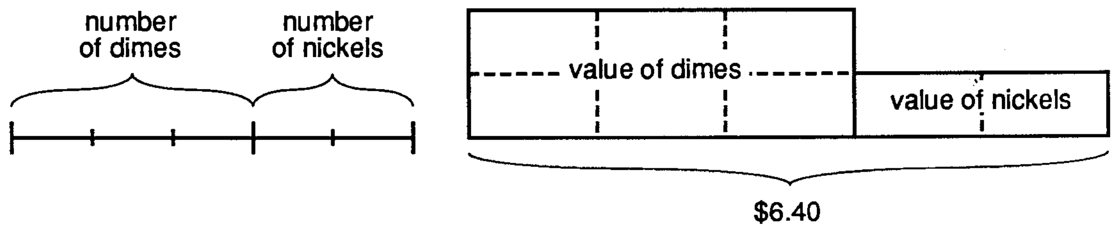
Spreading the ammonia evenly throughout the mixture with the water added gives a level of ammonia of 6%.

THREE



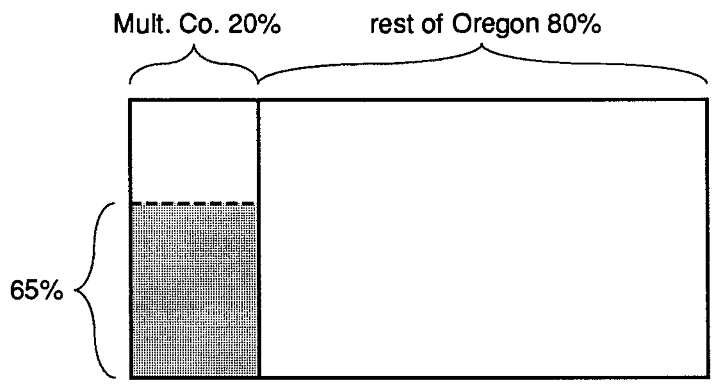
Area of each square = 300 sq. in. ÷ 12 = 25 sq. in.
Thus, side of square = 5 in.
Hence, length = 4 × 5 in. = 20 in. and width = 3 × 5 in. = 15 in.

FOUR



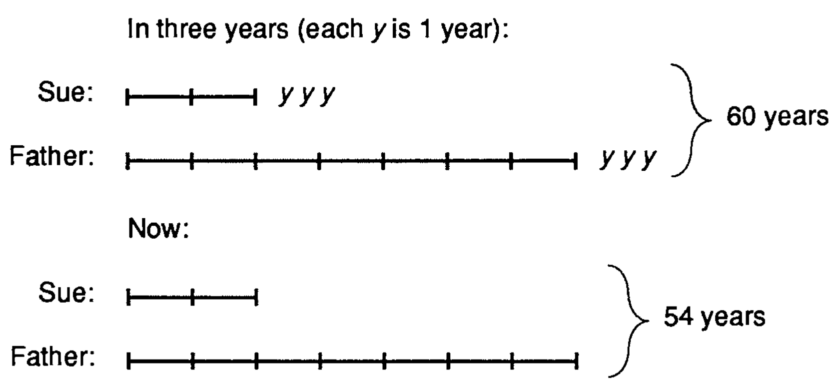
The value of each section = $6.40 ÷ 8 = 80¢.
Thus, value of dimes = 6 × 80¢ = $4.80 and
value of nickels = 2 × 80¢ = $1.60.
Hence, there are 48 dimes and 32 nickels.

FIVE



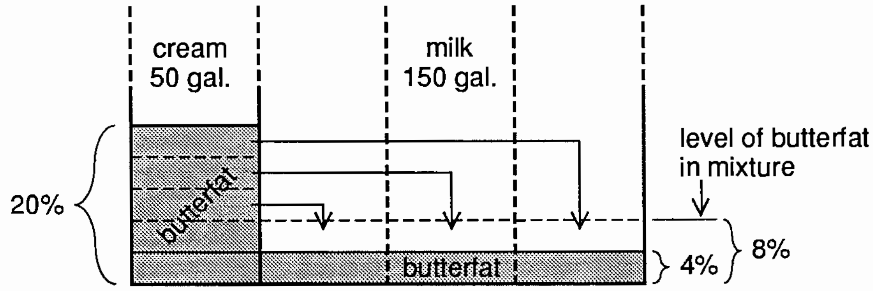
The shaded area = 65% of 20% = .65 × 20% = 13%.
Hence, the population of Portland within Multnomah County is 13% of the population of Oregon.

SIX



Each interval = 54 years ÷ 9 = 6 years.
Thus, Sue's age = 2 × 6 years = 12 years and
her father's age = 7 × 6 years = 42 years.

SEVEN



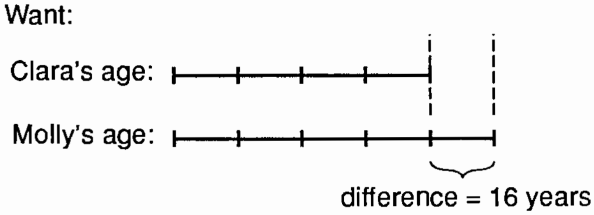
Averaging the level of butterfat in the cream with that in the milk gives a butterfat level of 8%.

EIGHT

Now:

Clara's age = 31 years, Molly's age = 47 years, difference = 16 years.

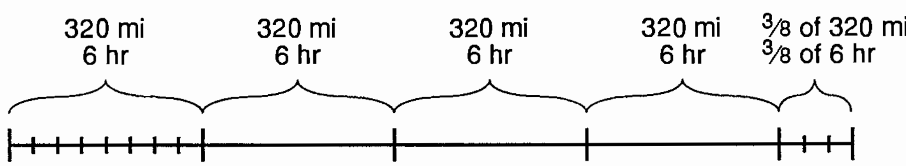
Want:



Each interval = 16 years. When the ratio of ages is 4 to 5, then Clara's age = 4 × 16 years = 64 years and Molly's age = 5 × 16 years = 80 years.
Since Clara is now 31, this will happen in 33 years.

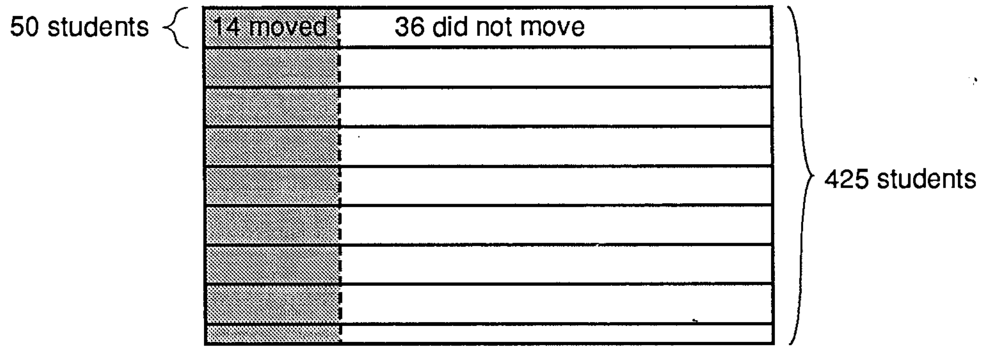
NINE

1400 mi ÷ 320 mi = 4 ⅜



$$\begin{aligned}\text{Driving time} &= (4 \times 6)\text{ hr} + (3/8 \times 6)\text{ hr}\\ &= (24 + 9/4)\text{ hr}\\ &= 26\ 1/4\text{ hr.}\end{aligned}$$

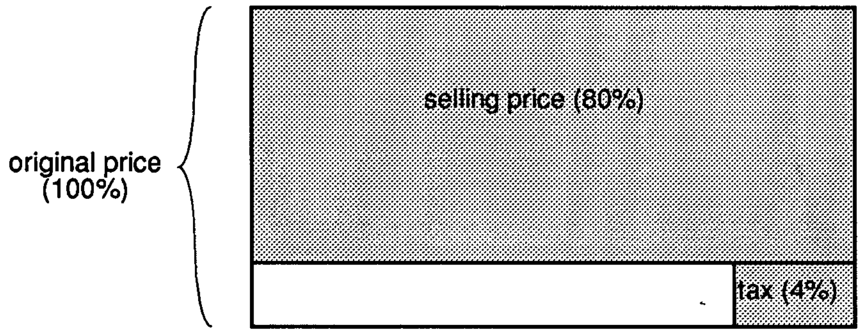
TEN



$425 \div 50 = 8\frac{1}{2}$.
Number of students who moved $= 8\frac{1}{2} \times 14 = 119$.

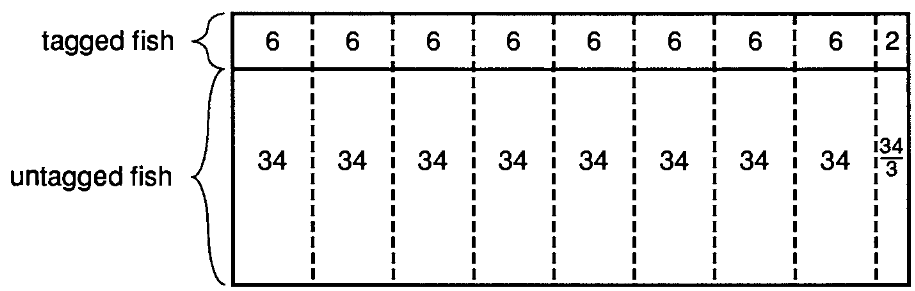
Still More Percentage & Ratio Problems
Sample Sketches

ONE



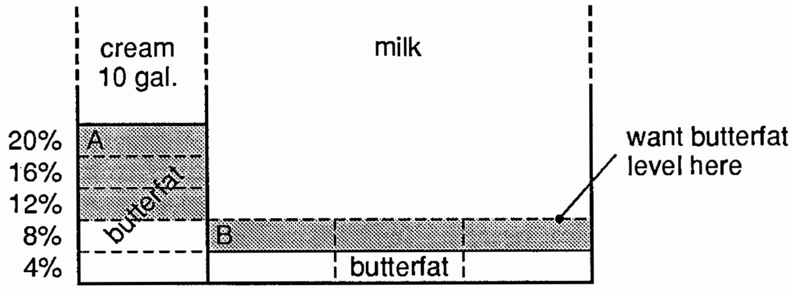
Tax paid = 5% of selling price = 4% of original price.
$33.39 = selling price plus tax = 84% of original price.
Thus 1% of original price = $33.39 ÷ 84 = 39.75¢.
Hence, original price = 100 × 39.75¢ = $39.75.

TWO



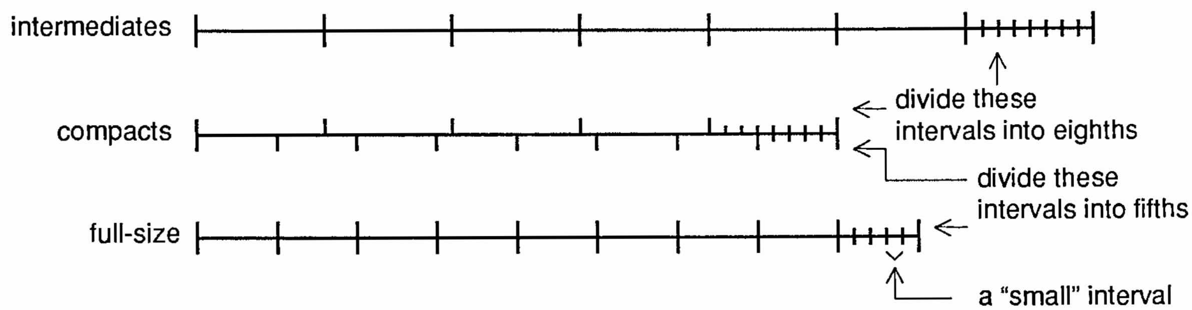
There are 50 tagged fish in the lake.
For every 6 tagged fish there are 34 untagged fish.
Since $50 \div 6 = 8\ 1/3$,
there will be about $(8\ 1/3) \times 40$, or 333, fish in the lake.

THREE



Butterfat in region A must fit into region B.
Hence, B should be 3 times as wide as A. Thus,
amount of milk = 3 × (amount of cream) = 30 gallons.

FOUR

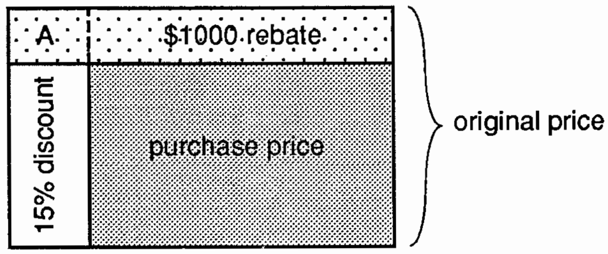


There will be $7 \times 8 = 56$ "small" intervals in "intermediates", $5 \times 8 = 40$ "small" intervals in "compacts" and $9 \times 5 = 45$ "small" intervals in "full-size", for a total of 141 "small" intervals. These 141 "small" intervals represent 2820 cars; hence, each represents 20 cars. Thus,

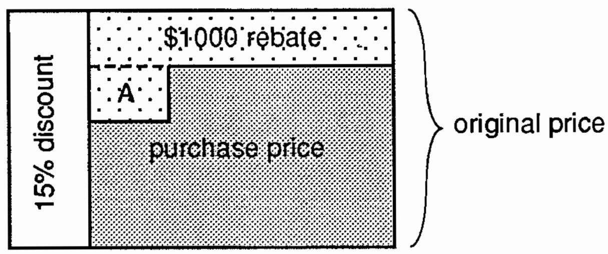
number of intermediate cars = $20 \times 56 = 1120$,
number of compact cars = $20 \times 40 = 800$,
number of full-size cars = $20 \times 45 = 900$.

FIVE

(a) Rebate, then discount:

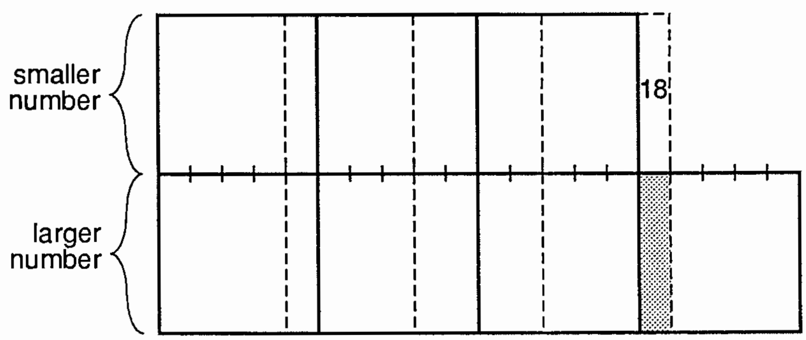


(b) Discount, then rebate:



The discount, then rebate, price is less by amount A which is 15% of \$1000, or \$150.

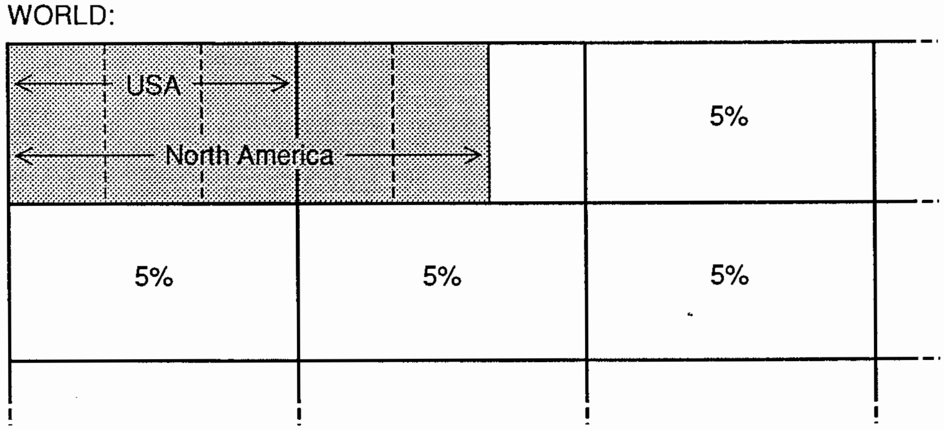
SIX



Dotted lines show the 4 to 5 ratio when smaller number is increased.
The shaded region is one "section".

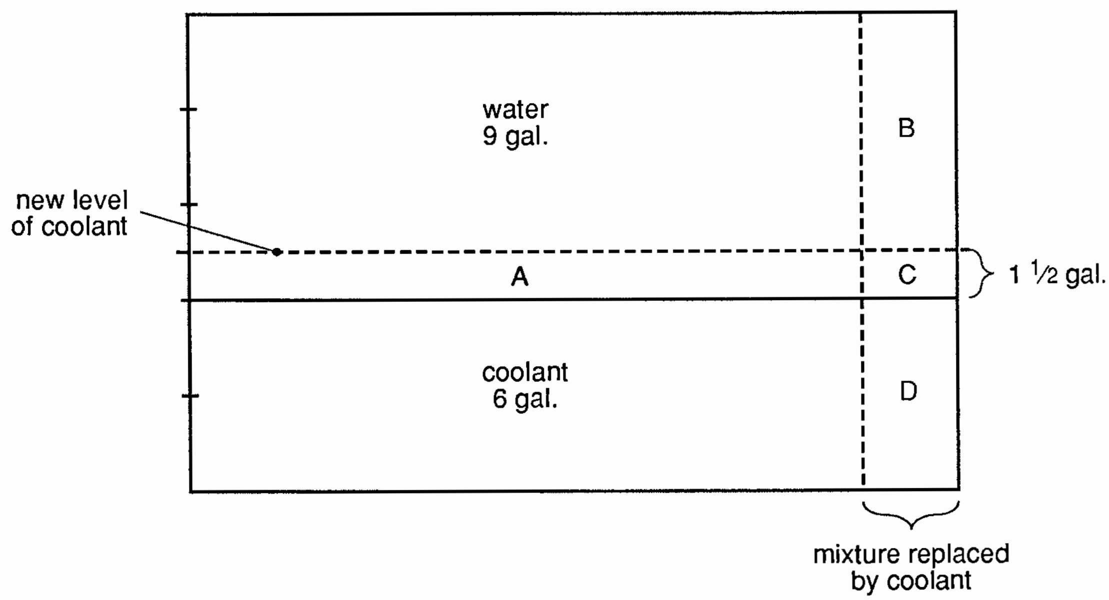
The smaller number is equivalent to 15 "sections", the larger number to 20 "sections". Since each section is 18, the smaller number = 15 × 18 = 270 and the larger number = 20 × 18 = 360.

SEVEN



North America population = 5% + (2⁄3 of 5%) = (8 1⁄3)%.

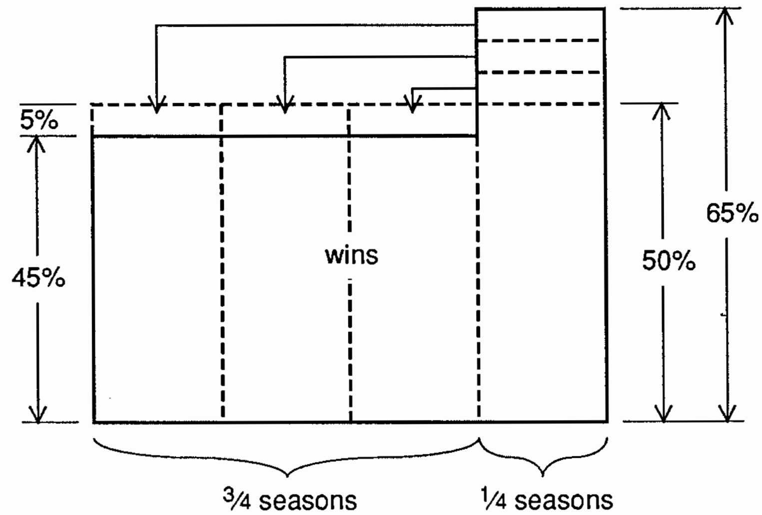
EIGHT



The amount of coolant in B and C must fill A and C.
Since A and C combined is 1 1⁄2 gallons, so is B and C.
D is two-thirds of B and C, so D is 1 gallon.
Hence, the amount of mixture replaced by coolant is 2 1⁄2 gallons.

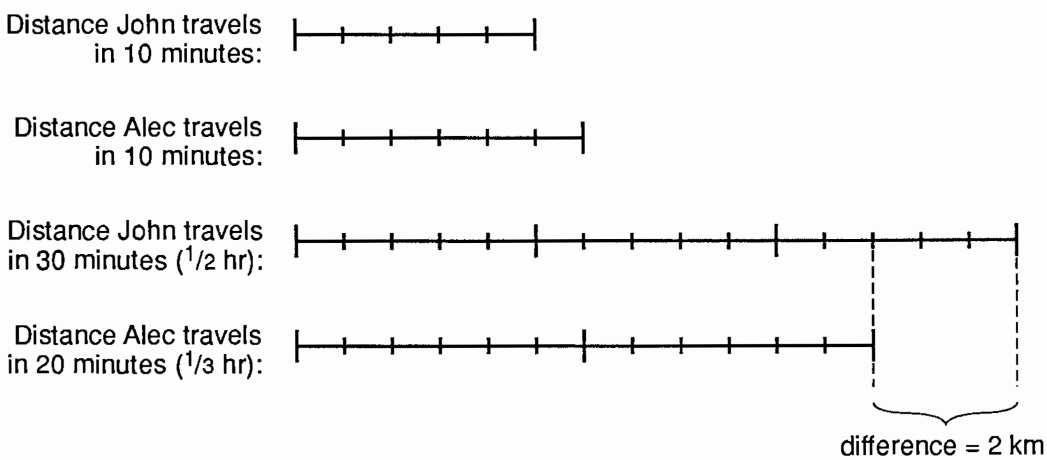
Sample Sketches • Still More Percentage & Ratio Problems
continued

NINE



For the fourth column to raise the average of the first three columns to 50%, the fourth column must reach 65%. Hence, the team must win 65% of their remaining games to have a break-even season.

TEN



Each interval represents $\frac{2}{3}$ km.
So,

John's distance = $15 \times \frac{2}{3}$ km = 10 km,

Alec's distance = $12 \times \frac{2}{3}$ km = 8 km

Thus, John travels 10 km in $\frac{1}{2}$ hour, or 20 km/hr, and Alec travels 8 km in $\frac{1}{3}$ hour, or 24 km/hr.

ONE
At a clearance sale, the price of a VCR was discounted 40%. The discounted price was $375. What was the original price?

TWO
The ratio of children to adults at the circus is 2 to 1. If the total attendance is 285, how many children are present?

THREE
The price of a large chocolate bar was increased by 20%. It now costs $1.50. What did it cost before the increase?

FOUR
Two numbers are in the ratio 3 to 4. One number is 17 more than the other. What are the numbers?

FIVE
In a certain town, 16% of the population lives in apartment buildings. If 2800 people live in apartments, what is the town's population?

SIX
A telethon for a local charity raised $45,000. This was 125% of the goal. What was the goal?

SEVEN
The ratio of two numbers is 8 to 5. Their difference is 45. What are the numbers?

EIGHT
Sandra sold 40% more Campfire Girls cookies than her sister. Together they sold $408 worth. How much did each girl sell?

NINE
Three numbers are in the ratio 5 to 6 to 7 and their total is 720. Find the numbers.

TEN
A length of string that is 180 cm long is cut into 3 pieces. The second piece is 25% longer than the first and the third piece is 25% shorter than the first. How long is each piece?

ONE
Jane paid $17.49 for a book, including tax. If the sales tax is 6%, what was the price without tax?

TWO
If 5 gallons of a mixture with a 30% concentration of ammonia is added to 20 gallons of water, what is the concentration of ammonia?

THREE
The ratio of the length of a rectangle to its width is 4 to 3. Its area is 300 square inches. What are its length and width?

FOUR
In a collection of dimes and nickels worth $6.40, the ratio of the number of dimes to the number of nickels is 3 to 2. Find the number of each type of coin.

FIVE
The population of Multnomah County is 20% of the population of Oregon. Also, 65% of the population of Multnomah County is within the Portland city limits. What percent of the Oregon population is in Multnomah County within the Portland city limits?

SIX
The ratio of Sue's age to her father's age is 2 to 7. In three years their ages will total 60. How old is each now?

SEVEN
If 50 gallons of cream with 20% butterfat is mixed with 150 gallons of milk with 4% butterfat, what percent butterfat is the mixture?

EIGHT
Clara is 31 years old. Her sister Molly is 47. In how many years, will their ages be in the ratio of 4 to 5?

NINE
Mrs. Howe drove 320 miles in 6 hours. If she continues at that rate, how many hours of driving will it take her to complete a trip of 1400 miles?

TEN
Of 50 students in a school who took part in a survey, 14 said they had moved within the last year. If this is representative of the school as a whole and there are 425 students in the school, how many of them moved within the last year?

ONE
During its January clearance sale, a gift shop offered all its merchandise at a 20% discount. Arnold bought an item for $33.39, which included a 5% sales tax. What was the original price of the item, without tax?

TWO
Last week, a state park employee caught 50 fish, tagged them and threw them back. Today he caught 40 fish from the same lake and 6 were tagged. About how many fish are there in the lake?

THREE
How much milk with 4% butterfat should be added to 10 gallons of cream with 20% butterfat to obtain a mixture that is 8% butterfat?

FOUR
There are 2820 cars in Smithtown. The ratio of intermediates to compacts is 7 to 5 and the ratio of compacts to full-size cars is 8 to 9. How many full-size cars are there?

FIVE
A car is sold with a $1000 rebate and a 15% discount. Which gives the better price for the buyer: (a) rebate and then discount the price after rebate, (b) discount the price and then rebate?

SIX
The ratio of two numbers is 3 to 4. If the smaller number is increased by 18, the ratio becomes 4 to 5. What are the numbers?

SEVEN
The population of the United States is 60% of the population of the North America and 5% of the population of the World. What percent is the population of North America of the population of the World?

EIGHT
A 15-gallon radiator contains 40% coolant and 60% water. How much of this mixture should be replaced by coolant to increase the percentage of coolant to 50%.

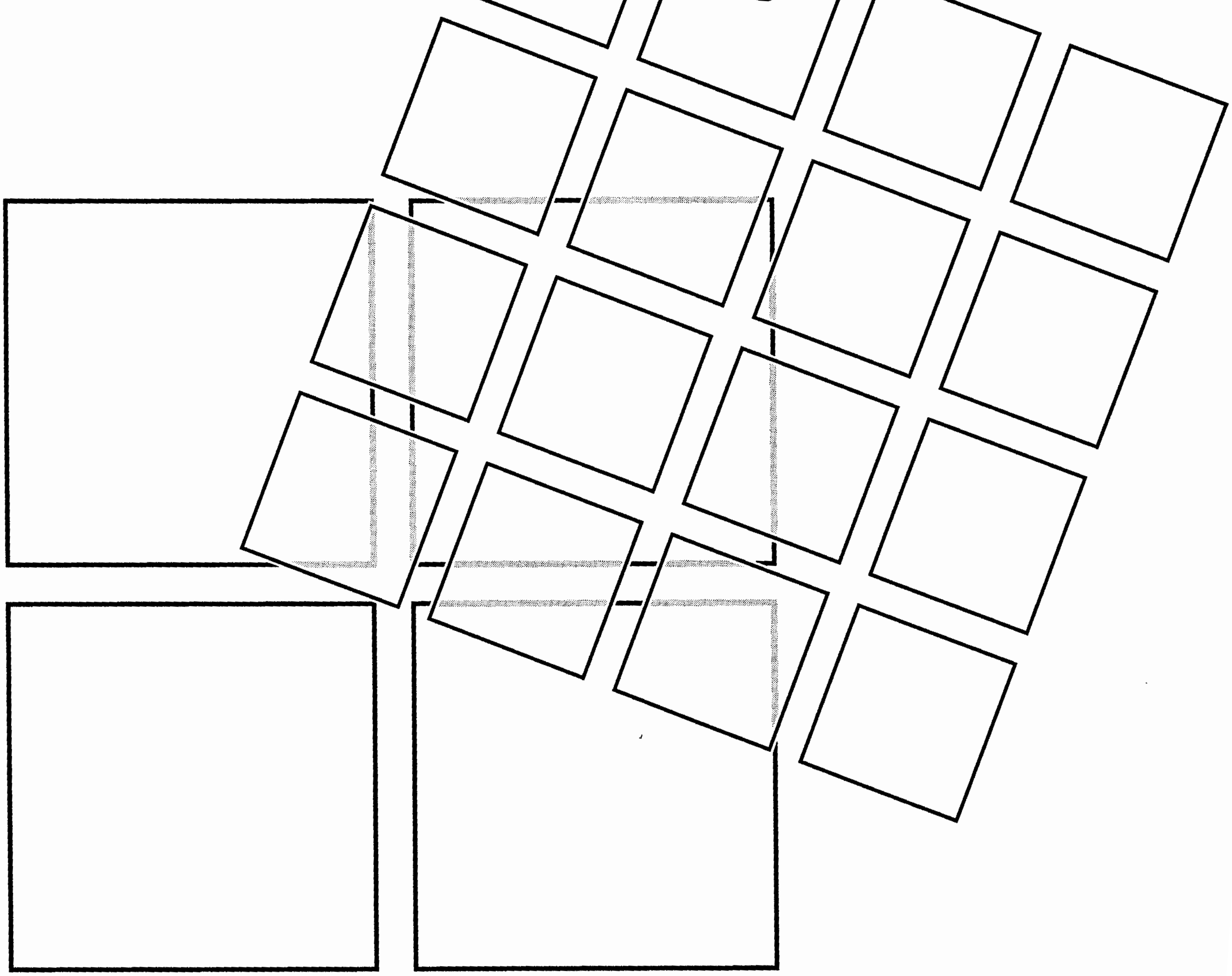
NINE
A baseball team has played 3/4 of its season and won 45% of its games. What percent of its remaining games must it win to have a break-even season?

TEN
The ratio of Alex's cycling speed to John's is 6:5. John leaves school 3:00 p.m. and Alex leaves at 3:10 p.m. By 3:30, Alex is only 2 km behind John. How fast is each boy going?

Unit VII / Math and the Mind's Eye

Transparencies

for Modeling Percentages and Ratios



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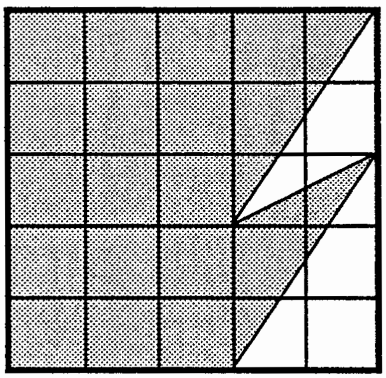
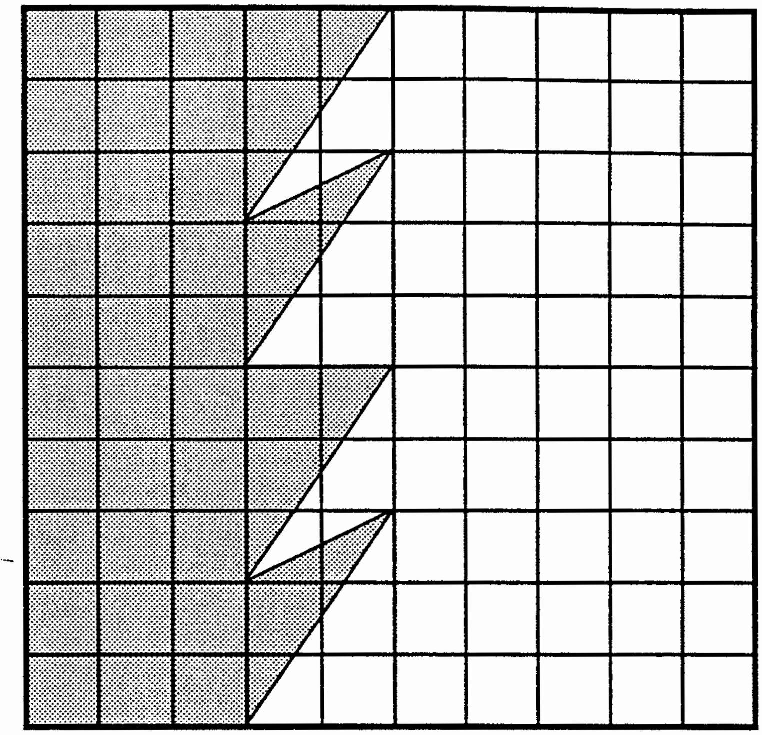
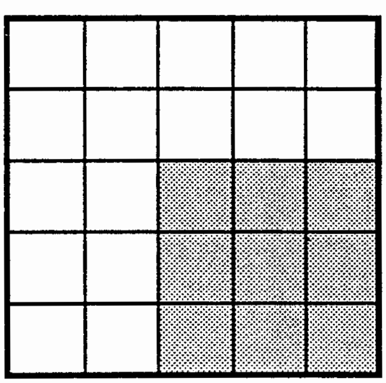
Catalog #MET7



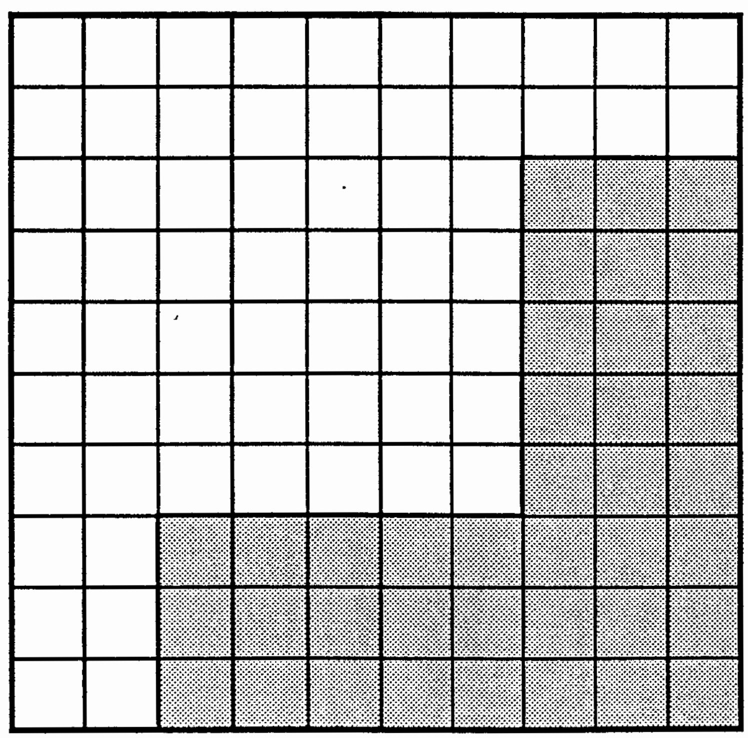
A



B

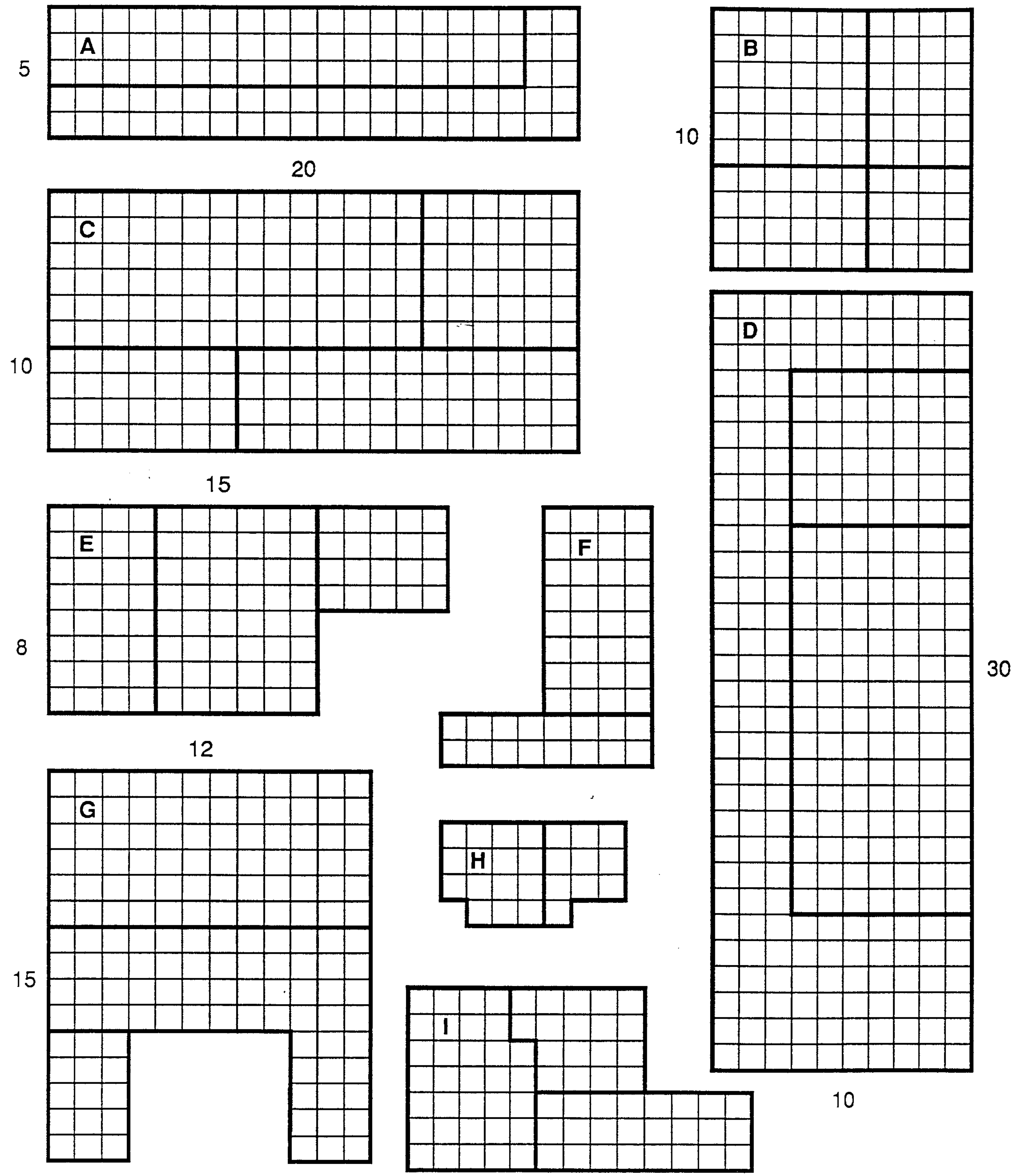


A



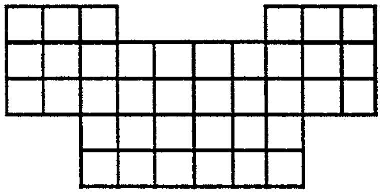
B

Find what percent the area of each subregion is of the entire region.

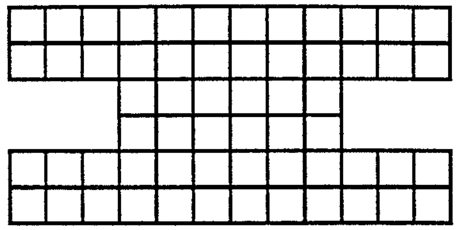


I. For each of the following regions, shade in the given percentage of its area.

A. 50%



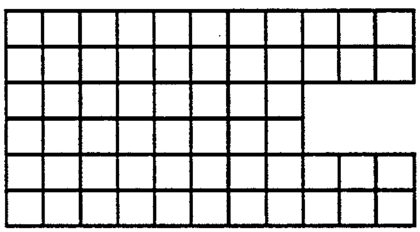
B. 75%



12

C. 20%

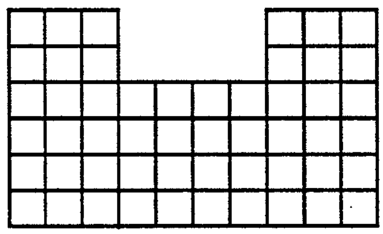
6



11

D. 33 ⅓%

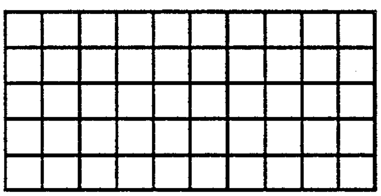
6



10

II. Shade in 30% of the area of each of the following regions.

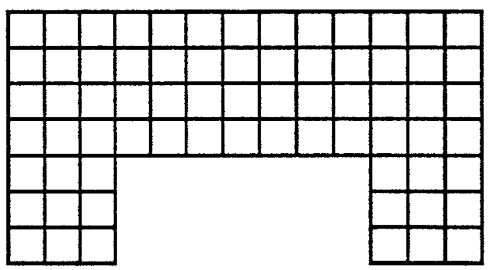
5

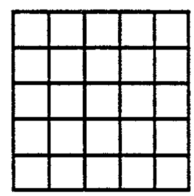


10

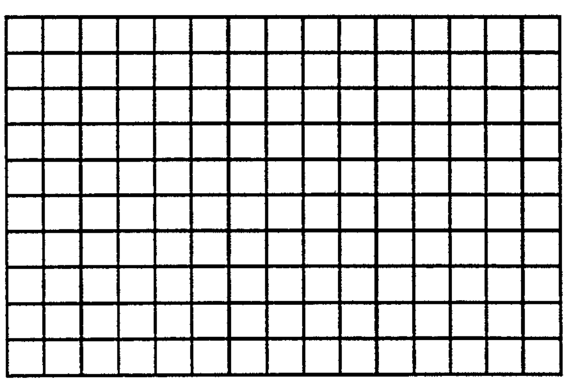
13

7



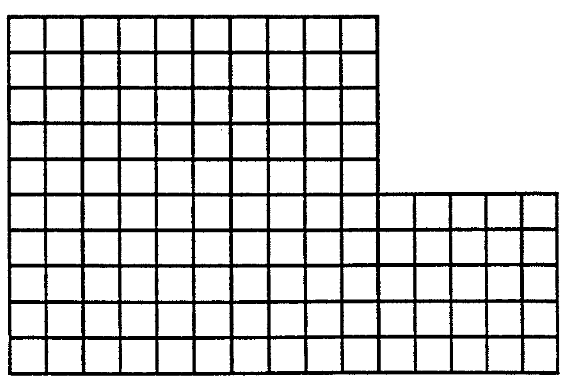


III. Shade in 72% of the area of the following regions.



15

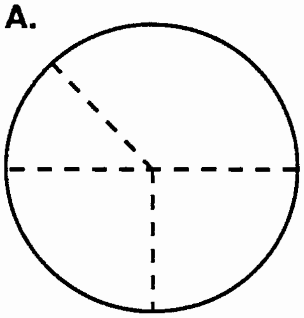
10



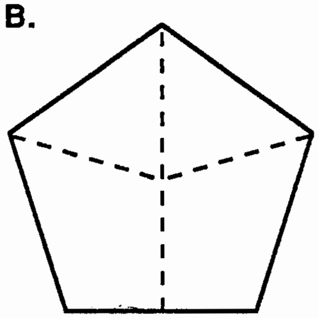
15

What percent is the area of each subregion of the area of the entire region?

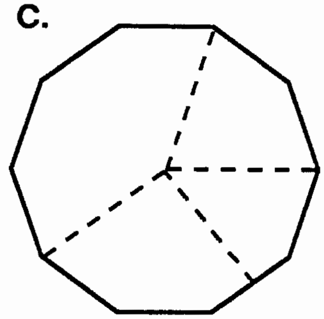
A.



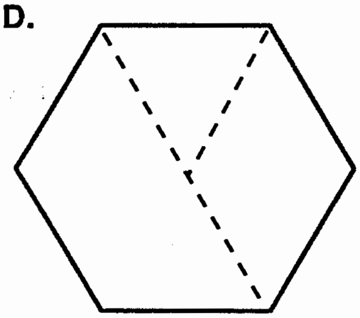
B.



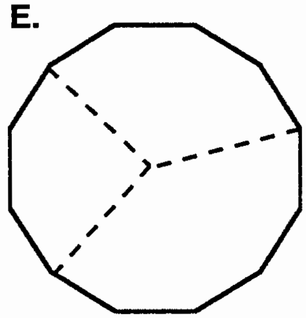
C.



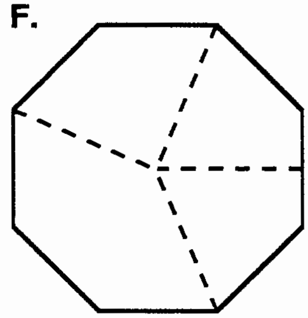
D.



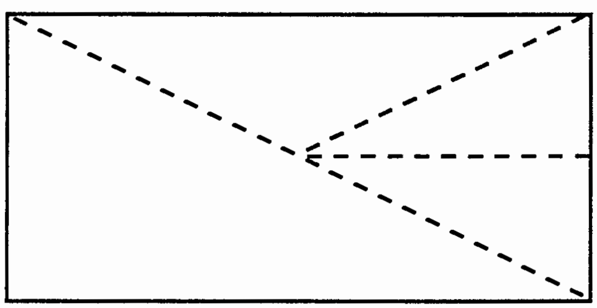
E.



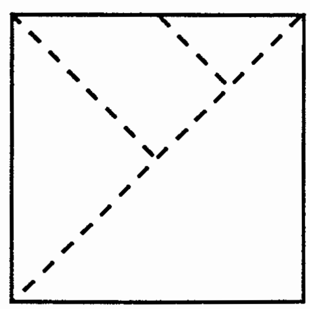
F.



G.

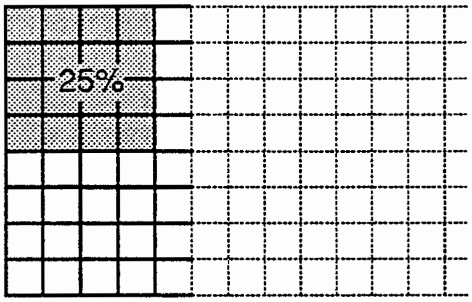


H.

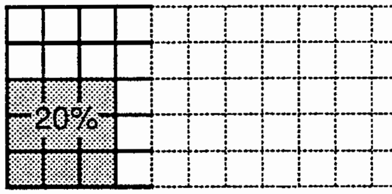


I. Complete the rectangle so the shaded region is the given percentage of its area.

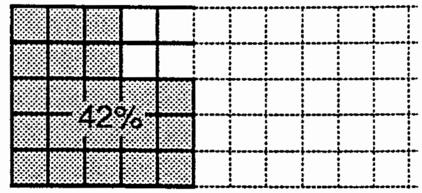
A.



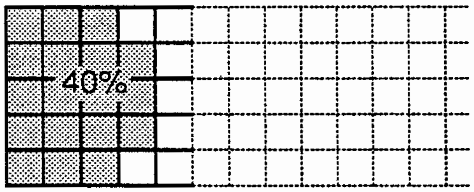
B.



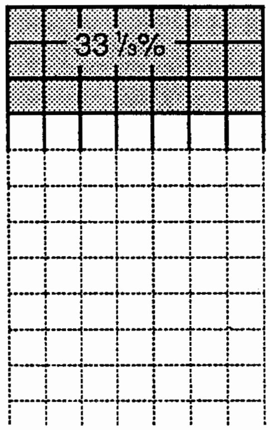
C.



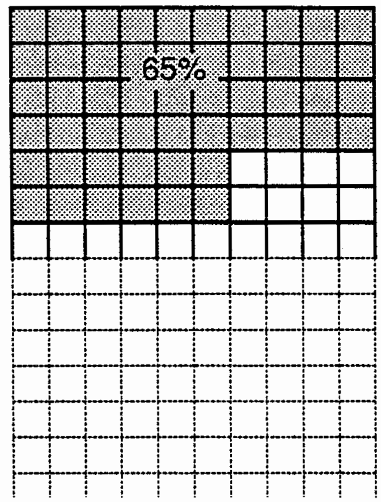
D.



E.



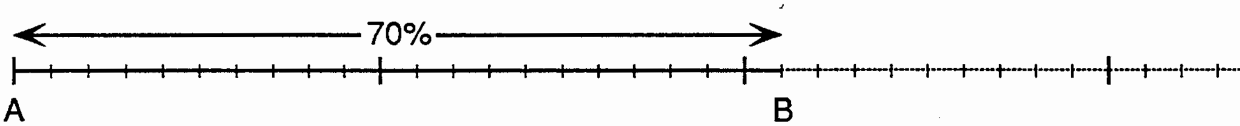
F.



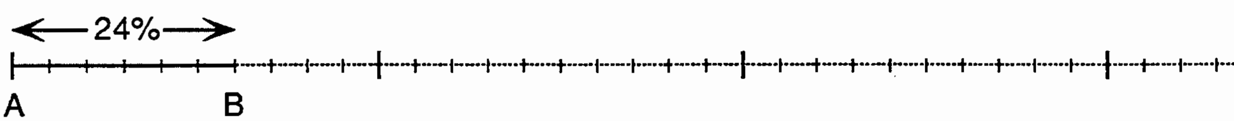
II. In each of the following, AB is part of a line segment. Complete the line segment if AB is the given percentage of its length.

A.

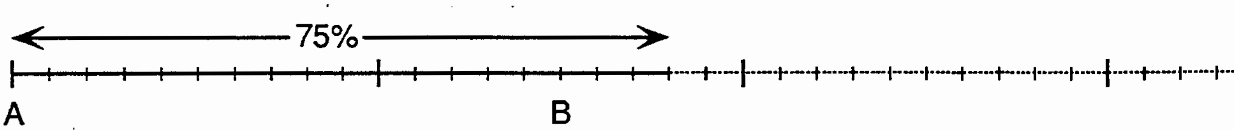
B.



C.

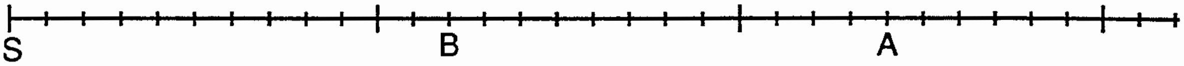


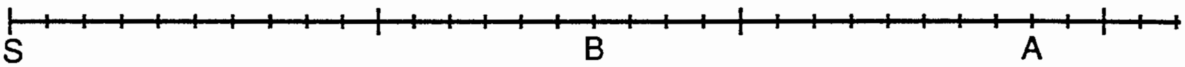
D.

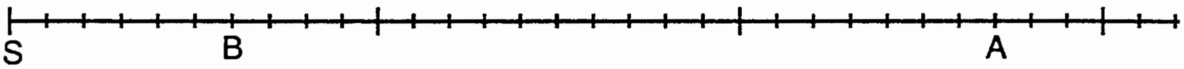


E.

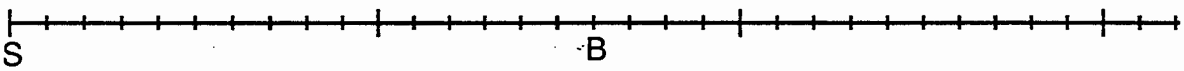
1. For each of the following find what percent length SA is of length SB.

a. 

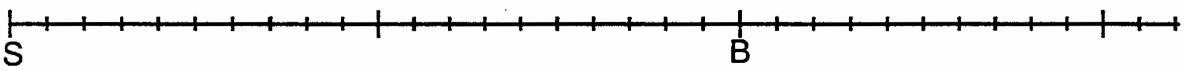
b. 

c. 

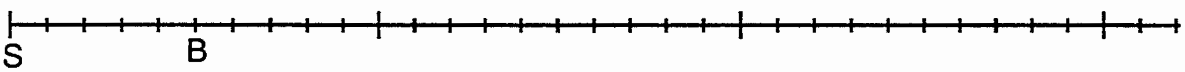
2. Find point A if the length of SA is 150% of the length of SB.



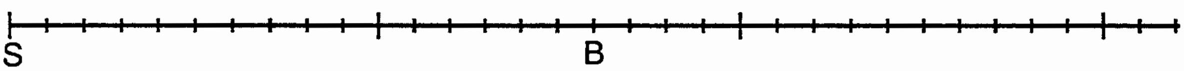
3. Find point A if the length of SA is 135% of the length of SB.



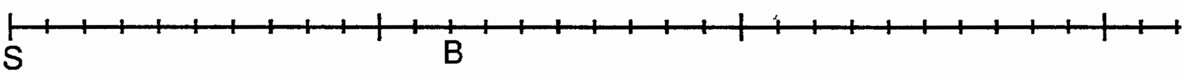
4. Find point A if the length of SA is 460% of the length of SB.



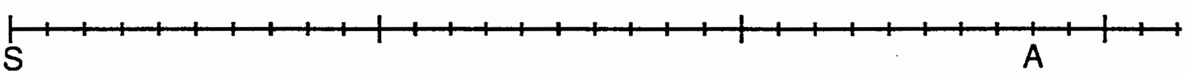
5. Find point A if SA is 25% longer than SB.



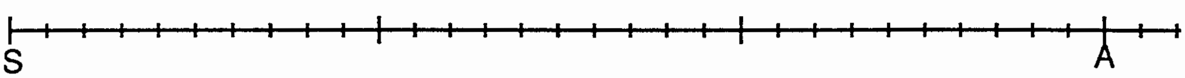
6. Find point A if the length of SA is the length of SB increased by 125%.

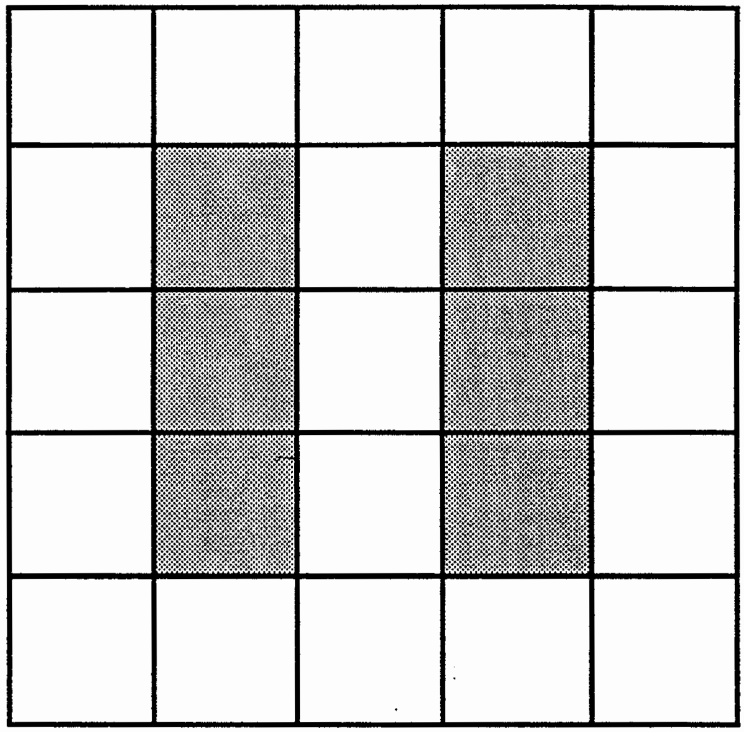


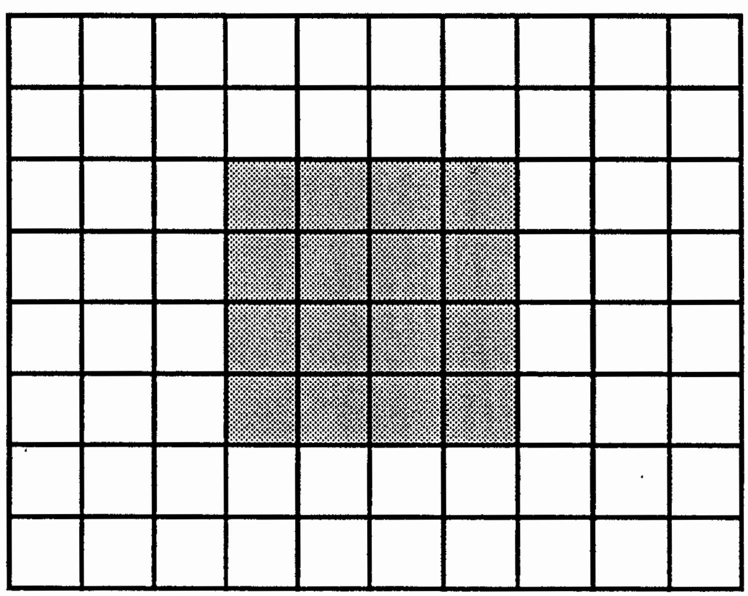
7. Find point B if the length of SA is 200% of the length of SB.



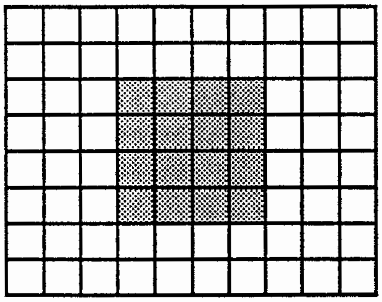
8. Find point B if the length of SA is 120% of the length of SB.



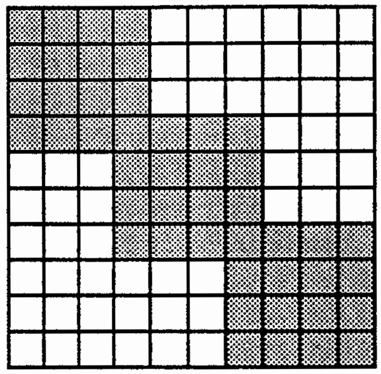




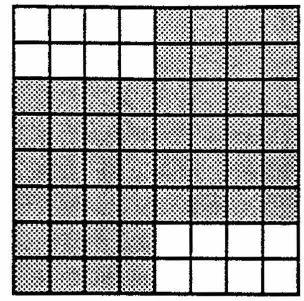
A.



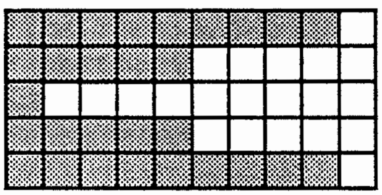
B.



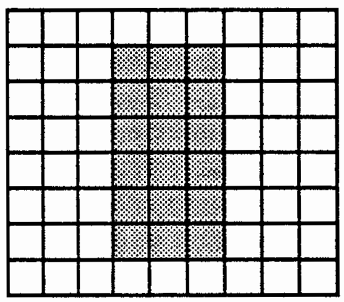
C.



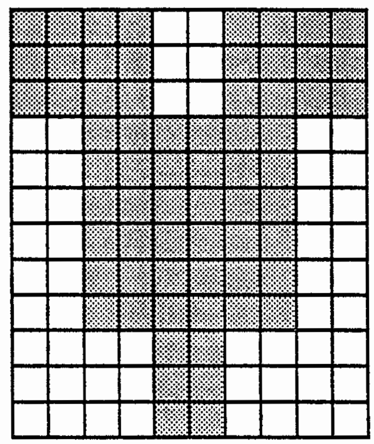
D.



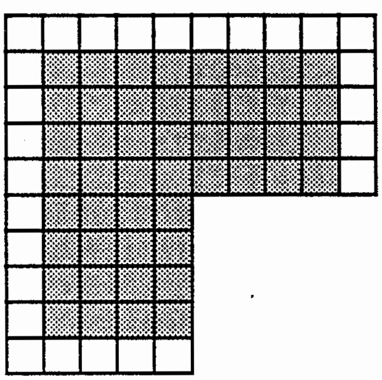
E.



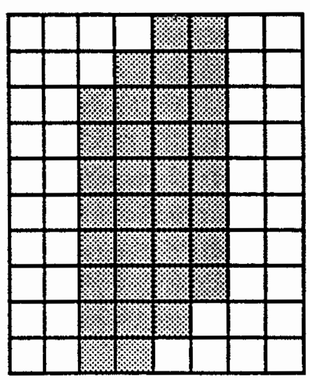
F.



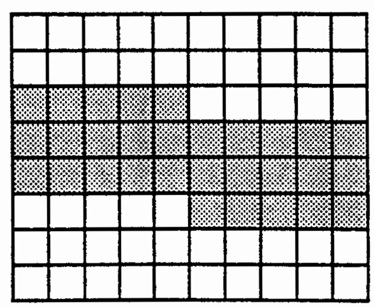
G.



H.

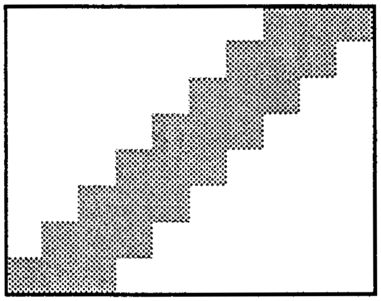


I.



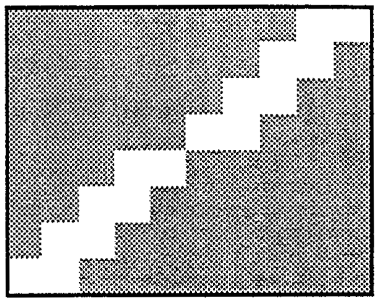
Activity Sheet VII-2-A

A.



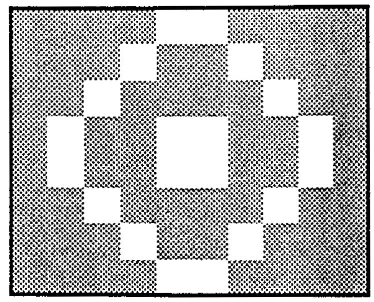
.3 of the area is shaded.

B.



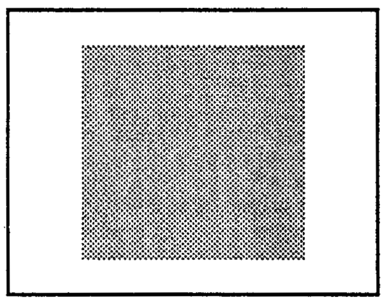
80% of the area is shaded.

C.



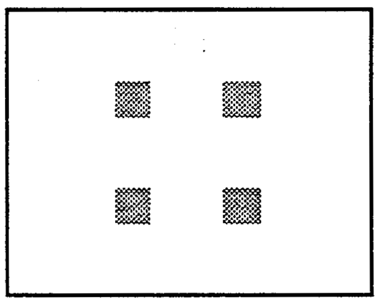
75% of the area is shaded.

D.



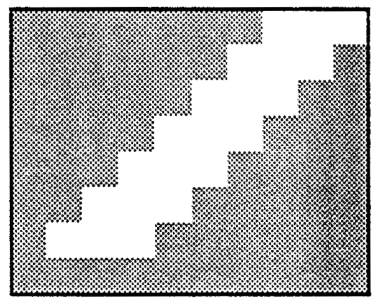
.45 of the area is shaded.

E.



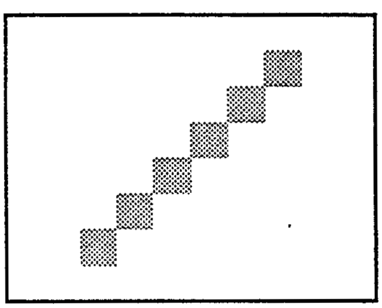
5% of the area is shaded.

F.



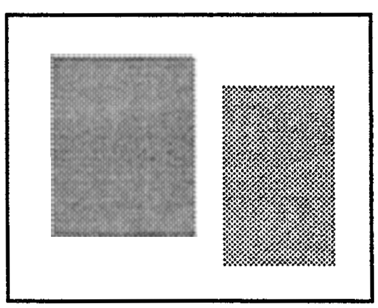
.725 of the area is shaded.

G.



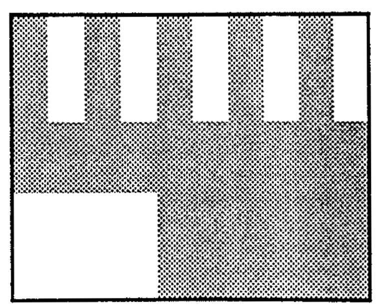
.075 of the area is shaded.

H.



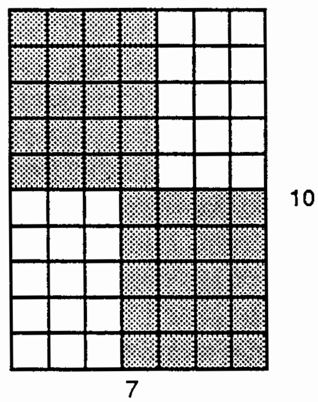
.4375 of the area is shaded.

I.

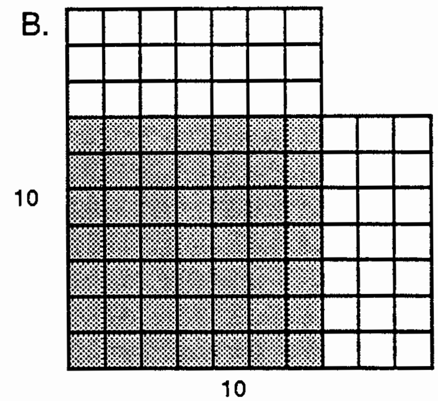


66 ¼% of the area is shaded.

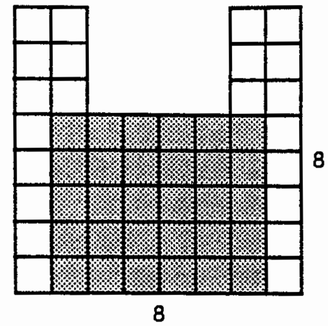
A.



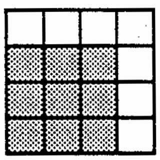
B.



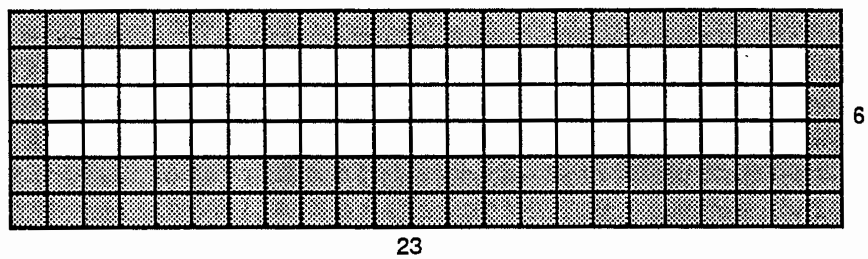
C.



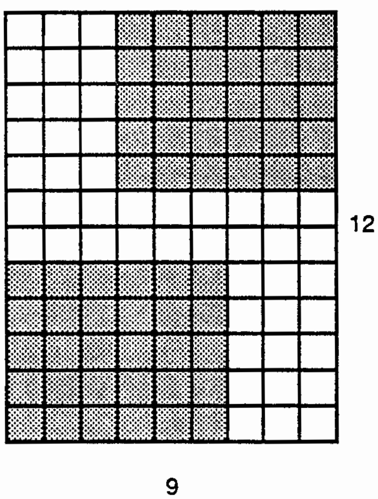
D.



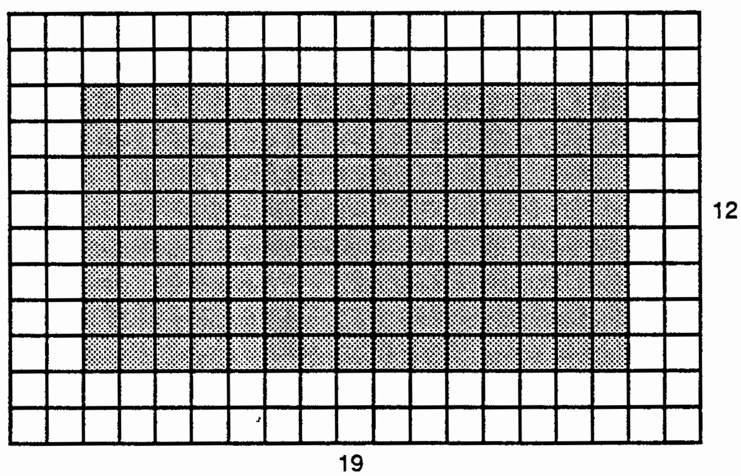
E.



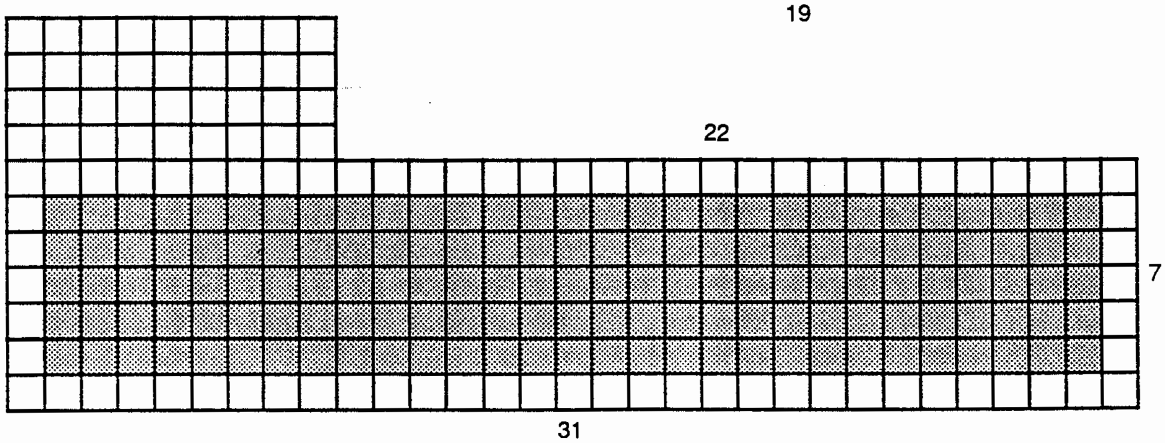
F.



G.



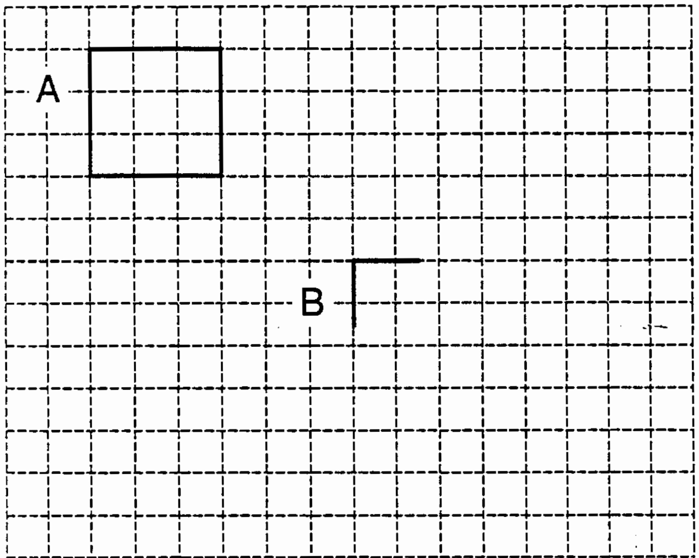
H.



Activity Sheet VII-2-C

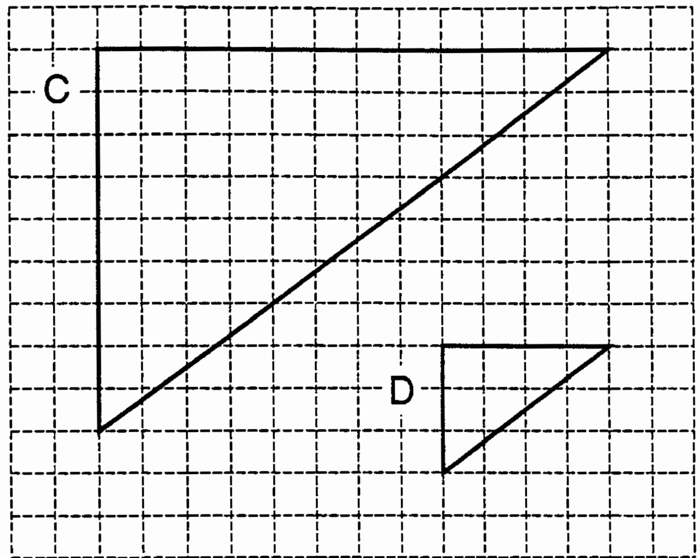
	Ratio of Black to Red Pieces	Number of Black Pieces	Number of Red Pieces	Total Number of Pieces
1.	2 to 3			20
2.	to	10		30
3.	3 to 4		12	
4.	to	6	14	
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				

1. Complete square B so the ratio of the length of its side to the side of square A is 2 to 1. What is the ratio of the area of square B to the area of square A?



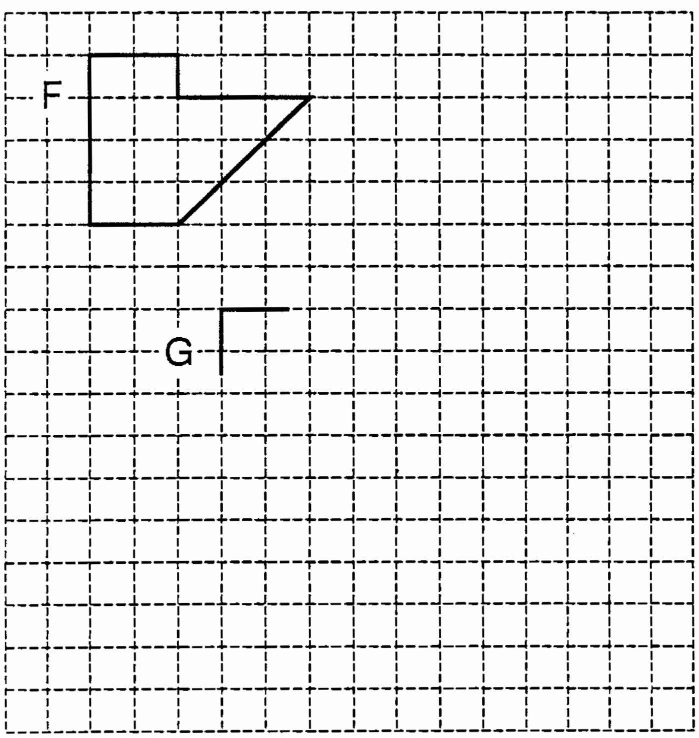
2. What is the ratio of the perimeter of triangle C to the perimeter of triangle D?

What is the ratio of the area of triangle C to the area of triangle D?



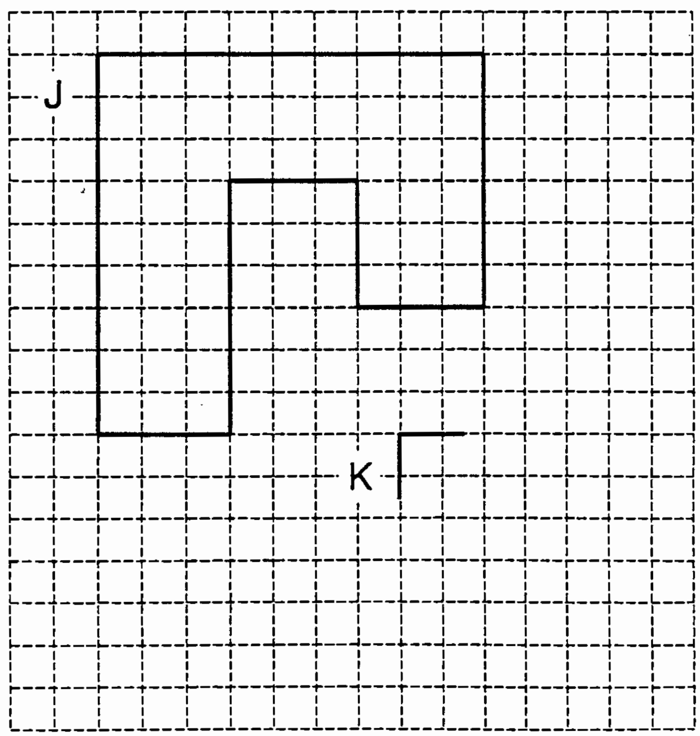
3. Complete figure G so it has the same shape as figure F and the ratio of the perimeter of F to the perimeter of G is 1 to 2.

What is the ratio of the area of F to the area of G?



4. Complete figure K so it has the same shape as figure J and the ratio of the perimeter of J to the perimeter of K is 3 to 2.

What is the ratio of the area of J to the area of K?



ONE
At a clearance sale, the price of a VCR was discounted 40%. The discounted price was $375. What was the original price?

TWO
The ratio of children to adults at the circus is 2 to 1. If the total attendance is 285, how many children are present?

THREE
The price of a large chocolate bar was increased by 20%. It now costs $1.50. What did it cost before the increase?

FOUR
Two numbers are in the ratio 3 to 4. One number is 17 more than the other. What are the numbers?

FIVE
In a certain town, 16% of the population lives in apartment buildings. If 2800 people live in apartments, what is the town's population?

SIX
A telethon for a local charity raised $45,000. This was 125% of the goal. What was the goal?

SEVEN
The ratio of two numbers is 8 to 5. Their difference is 45. What are the numbers?

EIGHT
Sandra sold 40% more Campfire Girls cookies than her sister. Together they sold $408 worth. How much did each girl sell?

NINE
Three numbers are in the ratio 5 to 6 to 7 and their total is 720. Find the numbers.

TEN
A length of string that is 180 cm long is cut into 3 pieces. The second piece is 25% longer than the first and the third piece is 25% shorter than the first. How long is each piece?

ONE
Jane paid $17.49 for a book, including tax. If the sales tax is 6%, what was the price without tax?

TWO
If 5 gallons of a mixture with a 30% concentration of ammonia is added to 20 gallons of water, what is the concentration of ammonia?

THREE
The ratio of the length of a rectangle to its width is 4 to 3. Its area is 300 square inches. What are its length and width?

FOUR
In a collection of dimes and nickels worth $6.40, the ratio of the number of dimes to the number of nickels is 3 to 2. Find the number of each type of coin.

FIVE
The population of Multnomah County is 20% of the population of Oregon. Also, 65% of the population of Multnomah County is within the Portland city limits. What percent of the Oregon population is in Multnomah County within the Portland city limits?

SIX
The ratio of Sue's age to her father's age is 2 to 7. In three years their ages will total 60. How old is each now?

SEVEN
If 50 gallons of cream with 20% butterfat is mixed with 150 gallons of milk with 4% butterfat, what percent butterfat is the mixture?

EIGHT
Clara is 31 years old. Her sister Molly is 47. In how many years, will their ages be in the ratio of 4 to 5?

NINE
Mrs. Howe drove 320 miles in 6 hours. If she continues at that rate, how many hours of driving will it take her to complete a trip of 1400 miles?

TEN
Of 50 students in a school who took part in a survey, 14 said they had moved within the last year. If this is representative of the school as a whole and there are 425 students in the school, how many of them moved within the last year?

ONE
During its January clearance sale, a gift shop offered all its merchandise at a 20% discount. Arnold paid $33.39 for an item, including a 5% sales tax. What was the original price of the item?

TWO
Last week, a state park employee caught 50 fish, tagged them and threw them back. Today he caught 40 fish from the same lake and 6 were tagged. About how many fish are there in the lake?

THREE
How much milk with 4% butterfat should be added to 10 gallons of cream with 20% butterfat to obtain a mixture that is 8% butterfat?

FOUR
There are 2820 cars in Smithtown. The ratio of intermediates to compacts is 7 to 5 and the ratio of compacts to full-size cars is 8 to 9. How many full-size cars are there?

FIVE
A car is sold with a $1000 rebate and a 15% discount. Which gives the better price for the buyer: (a) rebate and then discount the price after rebate, (b) discount the price and then rebate?

SIX
The ratio of two numbers is 3 to 4. If the smaller number is increased by 18, the ratio becomes 4 to 5. What are the numbers?

SEVEN
The population of the United States is 60% of the population of the North America and 5% of the population of the World. What percent is the population of North America of the population of the World?

EIGHT
A 15-gallon radiator contains 40% coolant and 60% water. How much of this mixture should be replaced by coolant to increase the percentage of coolant to 50%.

NINE
A baseball team has played 3/4 of its season and won 45% of its games. What percent of its remaining games must it win to have a break-even season?

TEN
The ratio of Alex's cycling speed to John's is 6:5. John leaves school 3:00 p.m. and Alex leaves at 3:10 p.m. By 3:30, Alex is only 2 km behind John. How fast is each boy going?