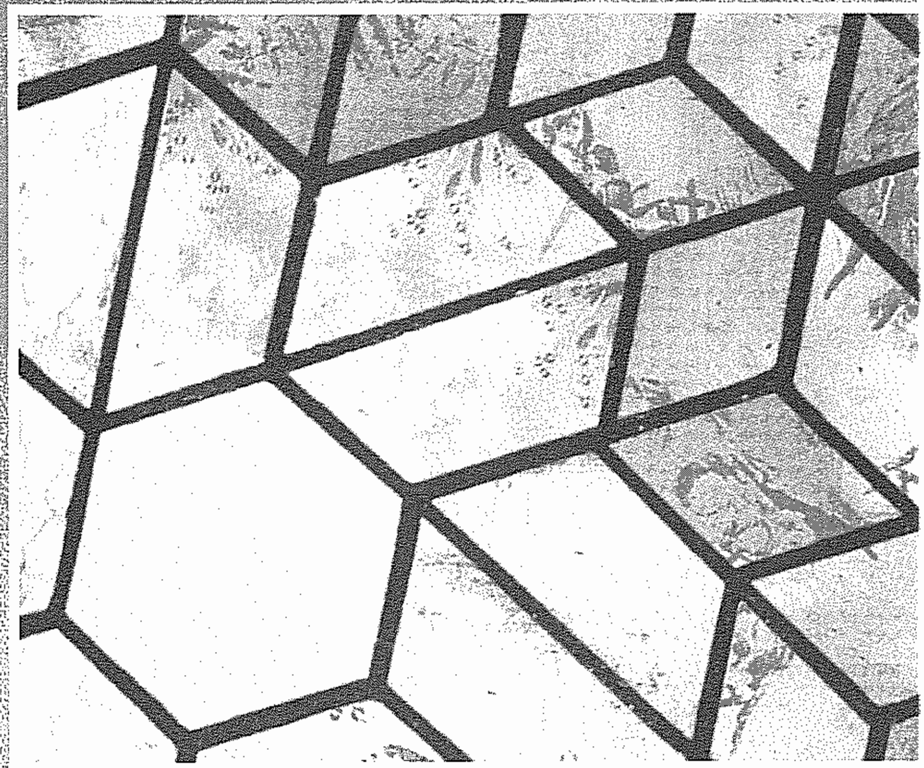
Unit I / Math and the Mind's Eye Activities



Seeing Mathematical Relationships

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Seeing Mathematical Relationships

1 The Handshake Problem

The handshake problem is used to illustrate the use of visual thinking in mathematical problem solving. In Part I, an expression is obtained for the number of handshakes if everyone in the classroom shakes hands with one another. The expression is evaluated in Part II.

2 Cube Patterns

The beginning buildings in a sequence of cube patterns are constructed. Students are asked to use visual observations and mental images to describe other buildings in the sequence and determine the number of cubes needed to construct them.

3 Pattern Block Trains and Perimeters

"Trains" of pattern blocks exhibiting certain geometric patterns are constructed. Students are asked to describe other trains which exhibit the same patterns. These descriptions are then used as a basis for determining the perimeters of the trains.

4 Diagrams and Sketches

Students are asked to create mental images of situations described in story problems. They are then asked to draw sketches or diagrams, based on their images, that lead to solutions of the problems

Math and the Mind's Eye materials are intended for use in grades 4-9. They are written so teachers can adapt them to fit student backgrounds and grade levels. A single activity can be extended over several days or used in part.

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Math and the Mind's Eye

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Unit I • Activity 1

The Handshake Problem

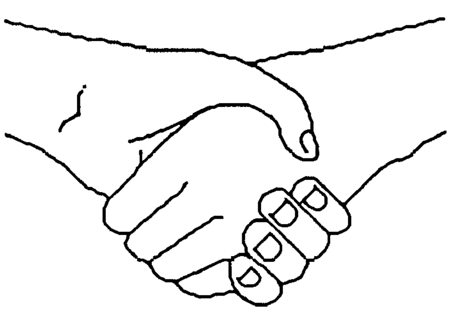
O V E R V I E W	
This activity illustrates the use of visual thinking in mathematical problem solving. In Part I, an expression is obtained for the number of handshakes if everyone in the classroom shakes hands with one another. This expression is evaluated in Part II.	***Prerequisite Activity*** None ***Materials*** Cubes or tiles for the students (see Comment 9)

Actions

Comments

Part I

1. Mention that when people get together, they often shake hands with one another. Ask the students to each guess the number of handshakes there would be if everyone in the room shook hands with everyone else, and to record their guess on a slip of paper. Make certain the students understand what constitutes a single handshake.



1. Asking for individual guesses will encourage the students to formulate their own thoughts about the problem.

In this activity, 2 persons shaking hands is counted as 1 handshake. You can illustrate this by shaking hands with a student while saying, "This is one handshake."

2. Collect the guesses and, without comment, record them on the chalkboard or overhead.

2. This action is not essential for what follows—it may increase interest since people are often curious about others' guesses. Avoid comments on guesses that might be construed as value judgements. This will encourage students who are reluctant to make guesses for fear of being wrong, and will also discourage those who make outlandish guesses for theatrical effect.

3. Pick a student, or ask for a volunteer, to assist you. Explain to your assistant that you want her or him to help check the guesses against the actual number of handshakes.

3. The assistant will be asked to do an impossible job—this may influence who you pick.

Actions

4. Tell the students that the actual number of handshakes will now be determined. Ask them to get up and shake hands with one another. Ask your assistant to count the handshakes. Instruct your assistant to tell you if he or she has difficulty counting the handshakes.

5. Get the students' attention. Ask them to suggest procedures for shaking hands that will allow your assistant to count the number of handshakes.

Comments

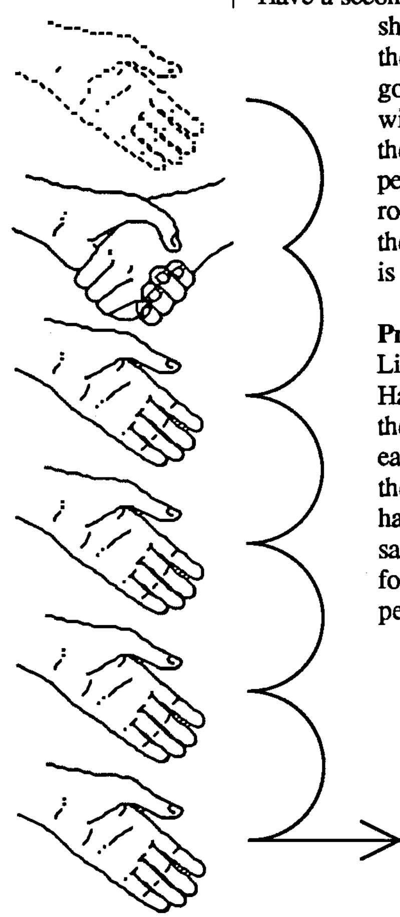
4. The students may be hesitant to start shaking hands with each other. Encourage them by moving around the room, randomly shaking hands with students. The intent is to create a setting in which it is impossible for your assistant to count all the handshakes taking place. This provides a graphic picture of the need for a systematic procedure.

If, in a minute or so, your assistant does not inform you of the hopelessness of their task, you can ask her or him if they are counting all the handshakes.

5. A number of procedures may be suggested. You may need to clarify some of the suggestions, but avoid judging one better than another.

Following are two procedures suggested by students:

Procedure A
Have a person go the center of the room. Have a second person go to the center and shake hands with the person there. Then have a third person go to the center and shake hands with the two persons already there. Continue having one person go to the center of the room and shake hands with all the people there until everyone is in the center of the room.



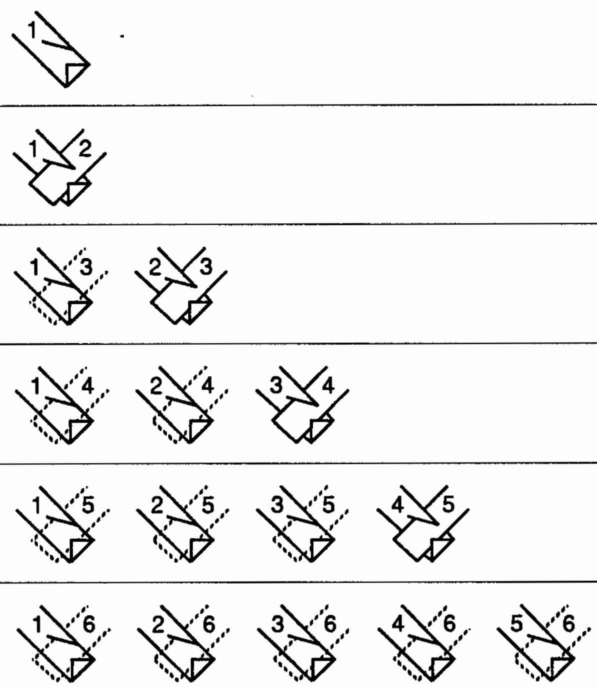
Procedure B
Line up everyone in a row. Have the first person walk down the row, shaking hands with each person, then sit down (see the illustration at left). Then have the second person do the same. Continue with the third, fourth, etc. until only one person is left in the row.

Actions

6. Use one of the procedures suggested to determine the number of handshakes if everyone in a group of 6 students shakes hands with one another. Make certain the students see that the number of handshakes for a group this size is the sum of the counting numbers from 1 through 5.

Comments

6. You may want to let the students pick a procedure to try. You can have the student who suggested the procedure carry it out with a group of students, while the rest of the class observes. The size of the group is not important, it should be large enough so students can see how the procedure would be carried out if everyone in the room participated.



	No. of Shakes
First Student	
Second Student	1
Third Student	+ 2
Fourth Student	+ 3
Fifth Student	+ 4
Sixth Student	+ 5
Total	**15**

Shown here is a diagram for 6 students carrying out procedure A in Comment 5. The first student goes to the center of the room. A second student goes to the center of the room and shakes hands with the student there. Then a third student goes to the center and shakes hands with the two students who are already there. This process continues until the sixth student goes to the center and shakes hands with the 5 students who are there.

As each student shakes hands, you can record the number of handshakes on the chalkboard or overhead. In this procedure, for 6 students, the number of handshakes is 1 + 2 + 3 + 4 + 5 = 15.

If procedure B in Comment 5 is carried out with a group of 6 students, the first student in the row will shake 5 hands before sitting down (see illustration on previous page). The next person will shake hands with the 4 other students remaining in the row—he or she has already shaken hands with the person sitting down. The next person will shake 3 hands, the next 2. Finally, the next to last person will shake hands once (with the last person). There is no one left for the last person to shake hands with. Thus, for 6 students, the number of handshakes is 5 + 4 + 3 + 2 +1 = 15.

Notice, in both procedures, the number of handshakes for 6 students is the sum of the first 5 positive whole numbers.

Actions

7. Discuss with the students how the number of handshakes could be determined if everyone in the room shook hands with one another.

8. (Optional.) Ask the students to answer the following questions, imagining the handshaking in their mind's eye.

When the 9 justices of the Supreme Court convene, they each shake hands with one another.

(a) How many handshakes will there be?
(b) Five of the judges shake hands with each other; then the other 4 arrive. How many more handshakes will there be?
(c) The judges form two groups. The 6 in one group have shaken hands with each other; so have the 3 in the other group. How many more handshakes will there be?

Comments

7. You can ask the students to imagine carrying out the procedure you used in Action 6 with everyone in the room participating. If the procedure used was the first one described in Comment 6 and there are 32 people in the room, the number of handshakes will be $1 + 2 + 3 + ... + 31$. (If you write an expression like this on the chalkboard or overhead, point out that '...' is a standard punctuation mark, the *ellipsis*, which, when used in mathematics, indicates something obvious has been omitted.)

Notice that for 32 people the number of handshakes is the sum of the whole numbers from 1 through 31. At this point, it is not important that students compute this sum. This will be done in Part II.

The students should see that, in general, the number of handshakes is the sum of the whole numbers beginning with 1 and ending with 1 less than the number of people shaking hands.

8. This problem can be deferred until Part II has been completed.

If students have a difficult time imagining the actions described in the problem, you can have a group of students carry them out.

(a) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$.

(b) The first of the 4 to arrive will shake 5 hands, the next 6, the next 7 and the last to arrive will shake 8 hands. So there will be $5 + 6 + 7 + 8$, or 26, more handshakes.

Students may have other ways of arriving at the answer.

(c) Each of the 6 will shake 3 hands. So there will be 6×3, or 18, more handshakes.

Actions

Comments

Part II

9. Distribute tile to each student. Explain to the students that they will be using tile to find sums like those encountered in the handshake problem.

9. Each student will need at least 15 tile (cubes will also work).

If your supply of materials is limited, you can carry out the following actions as a demonstration using tile on the overhead or on some surface that all the students can see.

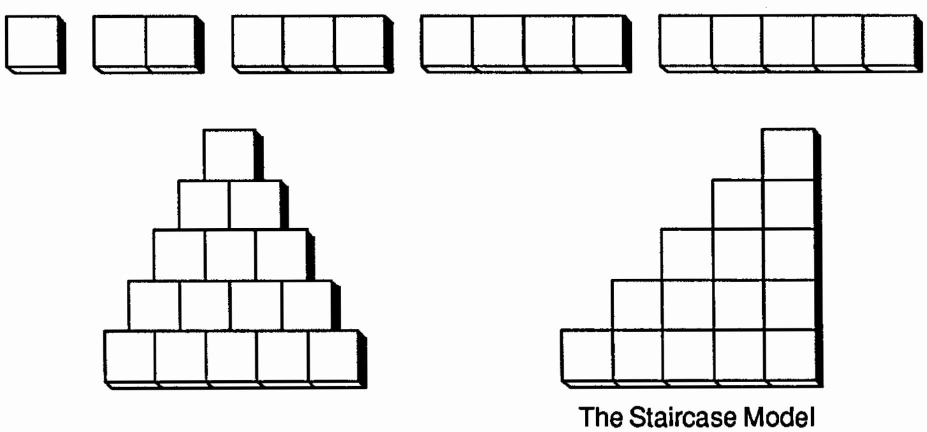
10. Write '1 + 2 + 3 + 4 + 5' on the chalkboard. Ask the students to think for a few moments how they would arrange the tile to model this sum. Then ask them to make whatever arrangement came to mind. Emphasize that there is no right or wrong way to do this and you anticipate a variety of models.

10. Asking the students to think for a few moments before arranging the tile, will help them focus on the task and not wait to see what a neighbor does.

11. Acknowledge, without judgement, the different models. Discuss what you see with the students. Find a "staircase" and ask the students to focus their attention on this model.

11. Seeing the models will give you an idea of how the students relate numbers and objects. For example, some may form numerals with the tile. These students may associate school mathematics with symbols and their manipulation.

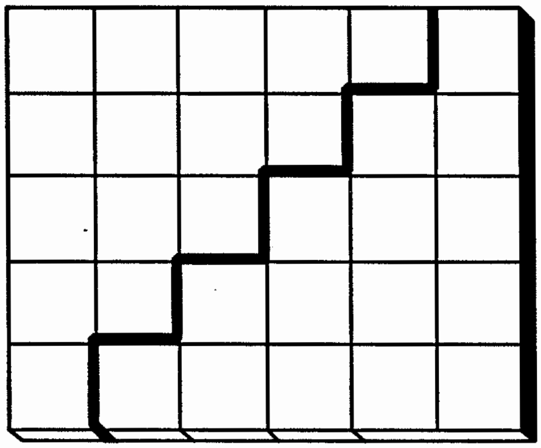
If no one forms a staircase, offer it as your model. Indicate that models are neither right nor wrong and, for particular purposes, one model may be more helpful than another. In this case, the staircase model is useful in finding the sum of consecutive whole numbers.



Models of 1 + 2 + 3 + 4 + 5

Actions

12. Have the students work in pairs. Ask each member of a pair to form a staircase model for the sum 1 + 2 + 3 + 4 + 5. Then ask each pair of students to form a rectangle with their two staircases. Discuss how the sum of 1 + 2 + 3 + 4 + 5 can be determined from the number of tile in this rectangle.

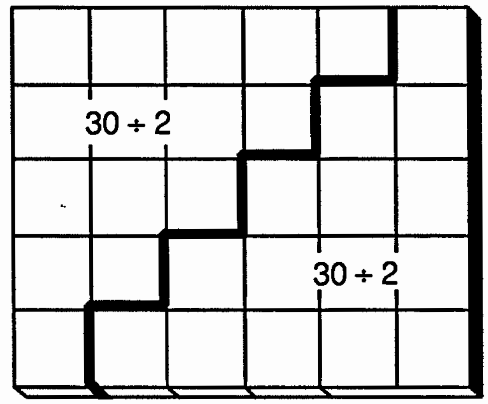


Two Staircases Form a Rectangle

13. Ask the students to imagine a staircase for the sum of the first 10 positive whole numbers, 1 + 2 + 3 + ... + 10. Ask them for the number of tile in a rectangle made from 2 of these staircases. Then ask for the number in each staircase.

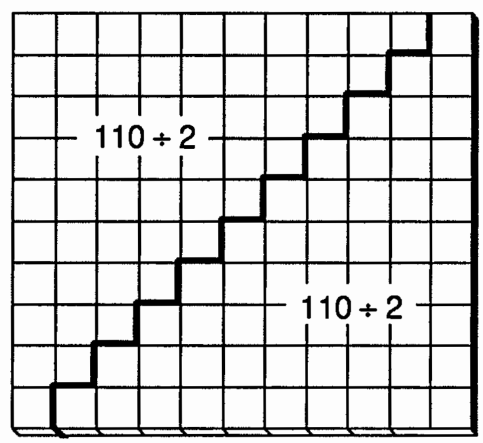
Comments

12. The rectangle contains 5 x 6, or 30, tile. Since the rectangle is comprised of 2 staircases, each staircase contains 30 ÷ 2, or 15, tile. Also, each staircase was built to contain 1 + 2 + 3 + 4 + 5 tile. Hence, 1 + 2 + 3 + 4 + 5 = 15.



1 + 2 + 3 + 4 + 5 = 30 ÷ 2 = 15

13. Two staircases form a 10 x 11 rectangul containing 110 tile. Hence each staircase contains 55 tile. Thus 1 + 2 + 3 + ... + 10 = 55.



1 + 2 + 3 + ... + 10 = 110 ÷ 2 = 55

Actions

14. Ask the students to use the 'staircase' method to find the number of handshakes if everyone in the room shakes hands with each other.

15. (Optional.)
(a) Ask the students for the number of handshakes if everyone in a room of 50 people shakes hands.
(b) Ask the students to imagine going to the classroom next door and counting the number of people in the room. Then ask them to describe how they could use this information to determine how many handshakes there would be if everyone in the classroom next door shook hands. Discuss.

Comments

14. If there are 32 people in the room, the number of handshakes will be the sum 1 + 2 + 3 + ... + 31. Two staircases, each representing this sum, will form a 31 x 32 rectangle. The number of handshakes will be half this amount: 31 x 32 ÷ 2 = 496. This computation can be done with a calculator or mentally (31 x 32) ÷ 2 = 31 x 16 = 30 x 16 + 1 x 16 = 480 +16 = 496.

15. (a) There are (49 x 50) ÷ 2 = 1225 handshakes. This computation is easily done with a calculator.

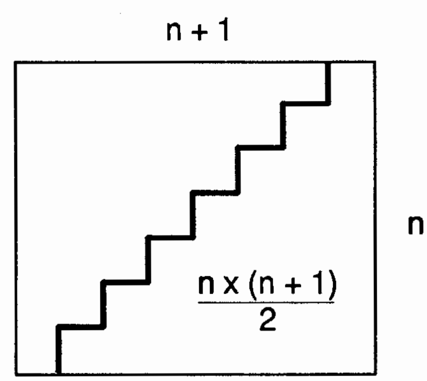
(b) You may want to ask the students to write instructions for computing the number of handshakes. They will have various ways of describing how to do this. Work with the students to arrive at descriptions which give correct answers and are unambiguous. Refrain from judging one correct method better than another; allow the students to make their own judgements.

You can use this situation to show the students how formulas evolve from written descriptions. For example, [the number of handshakes] = 1 + 2 + 3 + ... + [one less than the number of people next door]. Now represent the phrases in brackets by letters: Let h stand for 'the number of handshakes' and let n stand for 'one less than the number of people next door'. Then

$$h = 1 + 2 + 3 + ... + n.$$

This formula can be written in a simpler form: if the sum 1 + 2 + 3 + ... + n is thought of as a staircase, two of them will form an n x (n + 1) rectangle. Thus

$$h = \frac{n \times (n + 1)}{2}$$



$$h = 1 + 2 + 3 + \ldots + n = \frac{n \times (n + 1)}{2}$$

If the students are unfamiliar with the use of parentheses, explain how their use eliminates ambiguities.

Unit I • Activity 2

Cube Patterns

OVERVIEW

Students will be asked to arrive at conclusions based on their visual observations and mental images. The beginning buildings in a sequence of cube patterns are constructed. Students are asked to describe how a certain building in the sequence would look, if constructed, and to determine the number of cubes needed to construct it.

Prerequisite Activity

None

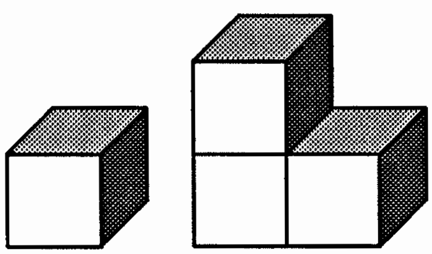
Materials

Approximately 40 cubes for each student or group of students (or a set of 60 to 100 demonstration cubes; see comment #1).

Actions

1. Give each pair of students 40 cubes.

2. Construct the two buildings below. Tell the students that the third building must have 5 cubes. It must also continue the pattern they see in the first two. Have them construct buildings 1 and 2 and then construct building 3.

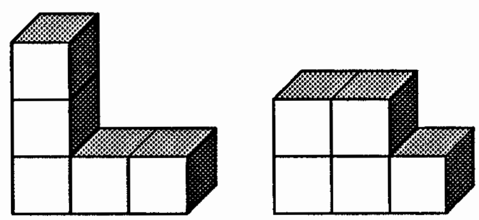


Comments

1. Any size cubes will work; cubes 2 cm on a side or 3/4 inch on a side are easy to handle.

Note: The following actions and commments assume each student has 40 cubes. Students can also work in small groups. Another approach is to have a teacher-directed demonstration where students are encouraged to come forward to manipulate the cubes and also imagine manipulating them in their minds.

2. Usually building 3 will look like one of these. Either extends the pattern satisfactorily. Be open-minded to other designs from students.

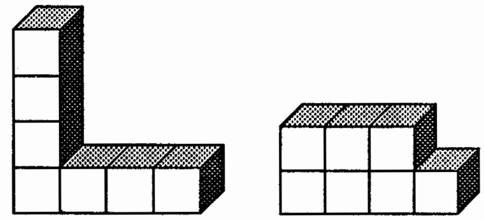


Actions

3. Ask them to construct a fourth building that extends the pattern.

Comments

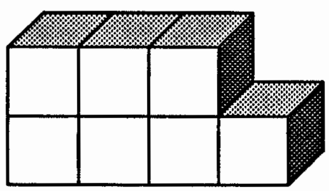
3. Most will immediately use seven cubes and construct one of the following, depending upon their building 3. If some students are having trouble, help them build the next building and then see if they can continue the process.

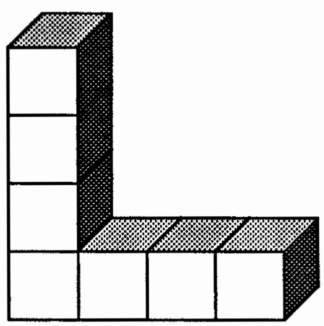


4. Ask your students to imagine constructing the 20th building. Tell them you would like them to be able to describe that building and to determine the number of cubes needed to construct it.

4. The directions at this point are crucial. Students are not being asked to construct 20 buildings but to *imagine* what the 20th building would look like *if* constructed.

5. Ask for volunteers to describe their mental picture of building 20 and the number of cubes it contains.





5. This part is interesting because of the variety of approaches. Here are some examples:

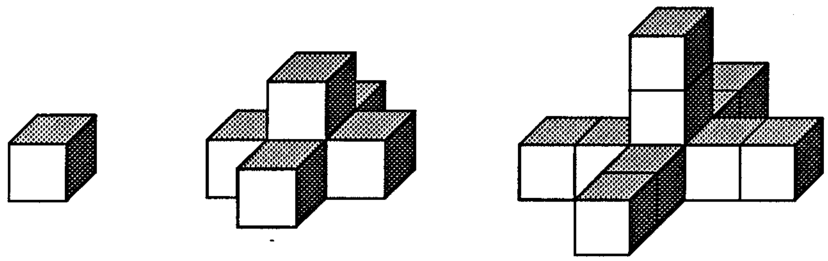
- *"There is 1 cube in the 1st building, and you add 2 cubes each time for 19 times."*
- *"My 4th building has 4 cubes with 3 on top so I thought the 20th building would have 20 + 19 or 39."*
- *"The 4th one has 4 down and 4 across or 8—but I had to subtract 1 because I counted the corner twice. So the 20th building would have 20 + 20 - 1 or 39."*
- *"My 4th building had 3 on top and 3 on the bottom plus one corner. So my 20th building would have 20 on top—no wait, 19 on top and 19 on the bottom plus the corner."*

It is worthwhile to take time on this activity so students can make the visual connection. Depending upon the nature of your class, you might have asked for the number of blocks in the 50th building instead of the 20th.

For more advanced students, one may ask for the number of cubes in the nth building.

Actions

6. Have your students construct these buildings and determine the number of cubes in buildings 2 and 3. Ask them each to write down their conjecture about the number of cubes in buildings 4 and 5 and then build them to check their conjectures.



7. Ask your students to imagine constructing the 20th building and to determine the number of cubes needed to construct it. Discuss the results.

Comments

6. The act of making a conjecture and recording promotes personal involvement.

7. These buildings are growing in several directions. Here are a few ways students have generated number patterns from mental images of this building:

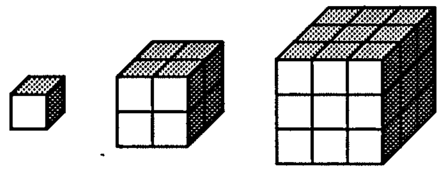
- *"There are 20 cubes in the middle and 19 sticking out each arm."*
- *"There are 20 cubes in each of the 5 arms so 100 cubes, but I've counted the center cube 5 times and have to subtract 4."*
- *"There are $5 \times 19 + 1$ because each arm has 19 and there is one in the middle."*
- *"You start with one cube and then add 5 nineteen times."*

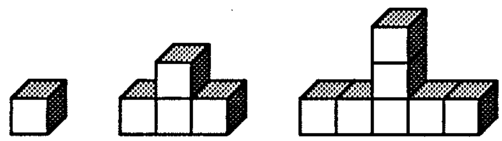
For more advanced students one may ask for the number of cubes in the 50th, 100th or nth building.

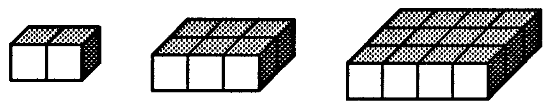
Actions

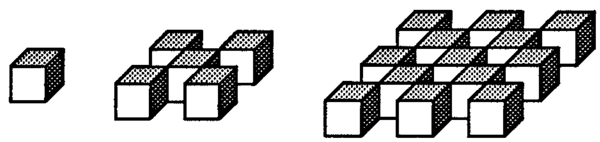
8. Have students explore patterns selected from the following list.

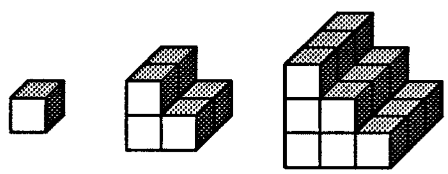
a) 

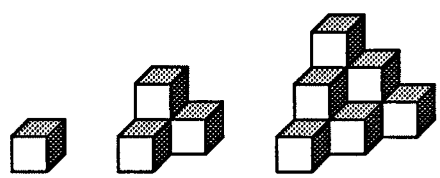
b) 

c) 

d) 

e) 

f) 

g) 

Comments

8. Your selection will depend upon the level of your class. You, or your students, can also create your own patterns.

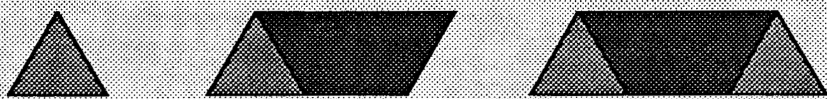
Note: These patterns are ordered according to level of difficulty. You may wish to ask students to describe and determine the number of cubes in the 10th building.

Patterns f and g are the most difficult. If you do these you may wish to concentrate on the description of the 10th building.

Pattern Block Trains and Perimeters

OVERVIEW

This activity uses pattern blocks to encourage students to visualize geometric patterns and make conjectures about the resulting number patterns. *Trains* of pattern blocks are built according to some geometric pattern.



In this example the first train has 1 car (the triangle), the second train has 2 cars (the triangle with the trapezoid) and the third train has 3 cars. If the geometic pattern continues the next train will have 2 triangles and 2 trapezoids. After examining this sequence the 10th train in the pattern can be built or described without constructing the intervening trains.

There are two parts to this activity. In Part I the number of blocks used in each of the trains is recorded and conjectures are made concerning the shape and number of blocks in the 10th (or 15th or 20th) train. In Part II perimeters of trains are determined. For most classes Parts I and II will each constitute one lesson.

Prerequisite Activity

None

Materials

Classroom set of pattern blocks.

Actions

Part I **Trains**

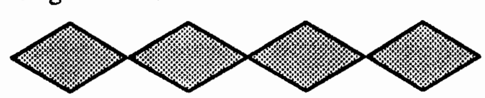
1. Using blue parallelograms form the first three trains of this sequence. Explain that you have made three trains: a 1-car train, a 2-car train and a 3-car train.

2. Tell the students that you see a pattern from the first three trains and are picturing the fourth train in your mind. Ask if anyone can guess your pattern. Have a volunteer build the fourth train.

Comments

1. It works nicely to show these three trains on a demonstration table or on an overhead so everyone can see them.

2. You want the student to place four parallelograms in a line.



Asking them to guess the pattern you are thinking about allows you to dismiss variations you don't want by saying "That's a nice pattern, but not the one I was thinking of," or you could extend an interesting pattern suggested by a student.

Actions

6. Discuss reasons why students believe the 10th train looks as it does.

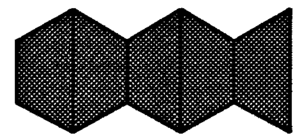
7. Continue an examination of trains by selecting appropriate patterns from the following list (page 4).

Comments

6. It is helpful to have students share their reasons in an open and accepting atmosphere. Students tend to see things in different ways even though the final answer is the same. For example, here are some responses for determining the 10th train in the discussion above.

• *"There are 10 red trapezoids which make 5 hexagons."*

• *The 5th train looks like this:*



so the 10th train must be twice as big."

• *"The 2nd train is one complete hexagon; the 4th train is 2 hexagons; so, the 10th must be 5 hexagons."*

Some may *see* the answer for reasons they can't verbalize.

7. Be sure the patterns you choose are appropriate for the level of your class.

These train patterns can be handled in several ways:

A. Continue to demonstrate and ask for volunteers to complete patterns.

B. Give each student some pattern blocks. Start the patterns on the demonstration table or overhead and have students build the next pattern (or the 10th pattern) individually on their desks.

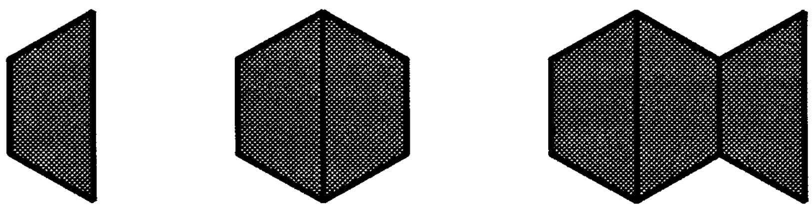
C. Give small groups of students pattern blocks to finish patterns you have started.

No matter how you approach this activity there is value in talking about the different ways students arrived at their patterns. This may help students realize that there is more than one way of viewing the same structure.

Actions

3. Ask another volunteer to come forward to build the 10th train in the sequence without building the intervening trains first. Urge the volunteer to discuss his or her reason for building that particular train.

4. Announce that you would like to try a more interesting sequence of trains and show them this sequence of red trapezoids.



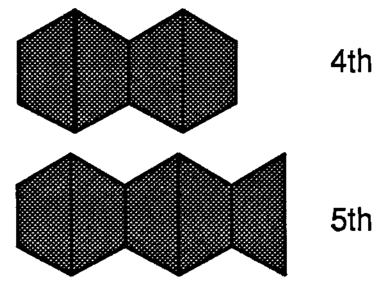
Ask for volunteers to build the 4th and 5th trains in this sequence.

5. Ask if someone can build the 10th (or 15th or whatever is appropriate to your class) train in this sequence without building the intervening trains.

Comments

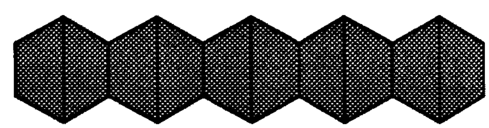
3. Some students may have trouble verbalizing their reasons and that is alright. Many will have reasons and these reasons (for the same structure) may vary. It is often times informative to have the entire class listen to a variety of reasons. This relatively simple first example may not prompt much discussion but more complicated patterns will.

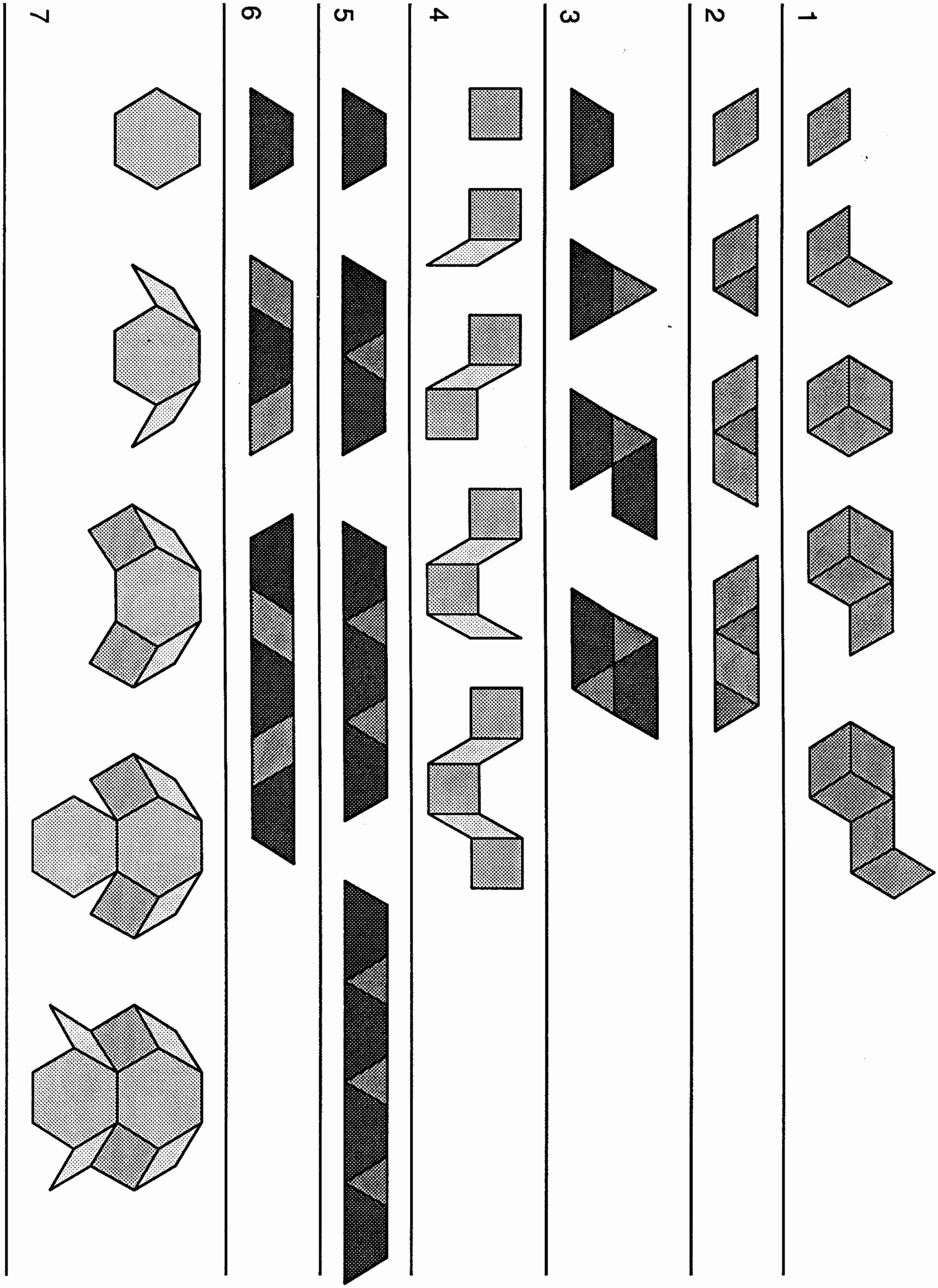
4. The 4th and 5th trains look like this.



Once again, be receptive to other *correct* patterns that were not the one you had in mind.

5. The 10th train will look like this:

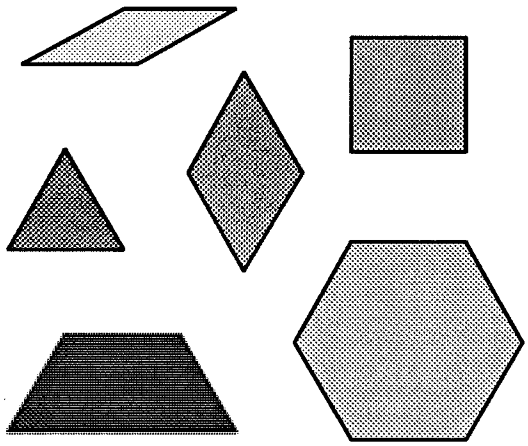



1
2
3
4
5
6
7

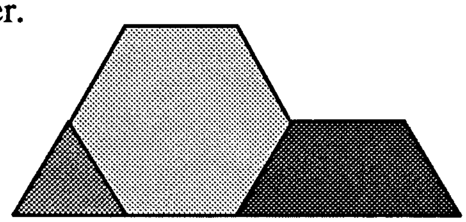
Actions

Part II **Perimeters**

8. Discuss the idea of perimeter. Give each student (or group of students) pattern blocks and ask them to find the perimeter of each piece.

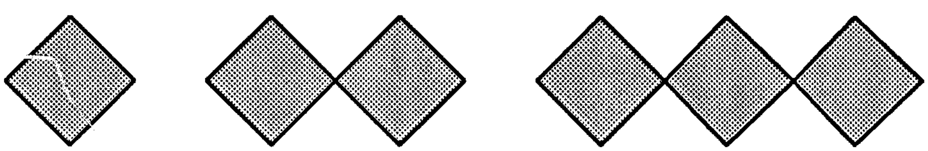


9. a) Ask each student to form a figure like the following and compute its perimeter.



b) Have students form other figures and find their perimeters.

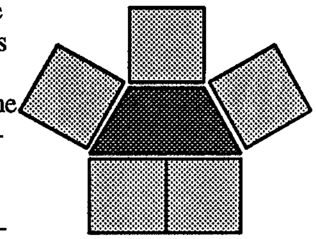
10. On the overhead projector or demonstration table form the following sequence of trains.



11. Have students compute the perimeter of the first three trains. Ask them to predict the perimeter of the fourth train and then build the fourth and confirm their conjecture.

Comments

8. Assume that the side of the square is one unit long. (The square pattern block in the commerical set measures 1 inch on each side.) Because the perimeter is the total distance around the shape, the perimeter of the square is 4 units. By comparing the side of the square to the sides of the other pieces their perimeters can be found.



9. a) Make sure this is viewed as one figure of perimeter 10 units rather than three distinct figures with total perimeter 14 units. Students must know this before proceeding.

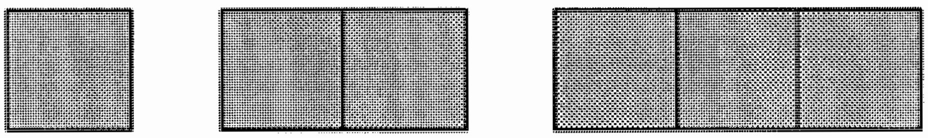
10. Many students will call these diamond shapes. It could be pointed out that they are also squares.

11. The perimeter of the first four trains are 4, 8, 12 and 16, respectively.

Actions

12. What will be the perimeter of the 10th train? (The 20th?)

13. Construct the following sequence of trains on the overhead or demonstration table.



14. Have students find the perimeters of the first four trains in the sequence. Then ask them to predict the perimeter of the 10th train (or 20th depending upon the class).

15. Continue finding perimeters of trains for train sequences chosen from the following list (page 7). Ask students to imagine that they are constructing the 10th train and to look for easy ways to find each perimeter.

Comments

12. Discussion is important. From previous work with trains many will say that the 10th train has ten squares each with a perimeter of four units. Others may *see* it differently. Someone could say, "I see 9 dents on the top and bottom and each dent is 2 units. Then there are 2 units left on each end so 18 + 18 + 2 + 2 = 40. Variety is fun!

13. This time the squares share an edge.

14. This problem is interesting because of the different ways it can be approached:

- Some students may sketch 10 squares and count the perimeter.

- Noticing that the first four perimeters are 4, 6, 8 and 10, some students may count by two's until reaching the 10th perimeter.

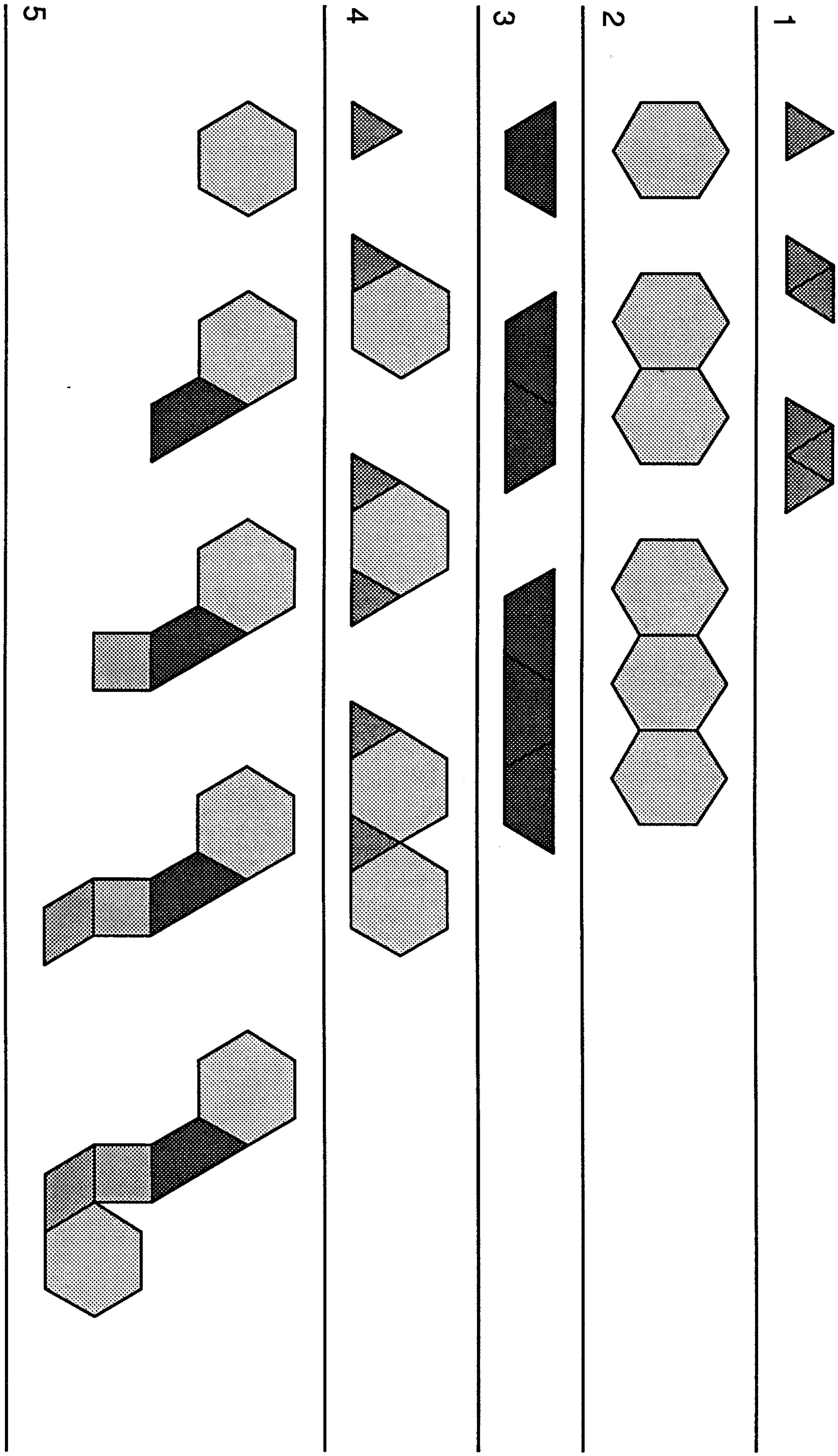
- Students who can picture the 10th train may see 10 units along the top, 10 units along the bottom and 1 unit on each end.

- A few may visualize the 10 squares, having a total perimeter of 40 units, joining together in nine places. In each of the nine places two sides of the square come together reducing the 40 units by 18 to leave a perimeter of 22.

Through a discussion of various approaches students can learn from each other. They may also discover that some approaches work more easily than others when finding perimeters of trains too large to build or draw, like the 50th or 100th.

15. You may wish to do a few more sequences with the whole class. However, they can be done individually or in small groups and then discussed in a large group session.

Pattern Block Perimeter Trains



Diagrams and Sketches

O V E R V I E W

Puzzles and story problems are solved by creating images and drawing sketches and diagrams.

Prerequisite Activity

None

Materials

Blank transparencies for student drawings (see Comment 3). Copies of Picture Puzzle Stories and More Picture Puzzle Stories (see Comments 4 and 6).

Actions

1. Read the following to the students and ask them to picture the story in their minds as they listen. Read slowly, giving the students time to absorb and visualize the story.

"You are standing on the middle rung of a ladder painting the house. As you paint you climb three rungs up the ladder, but then some hornets force you to descend five rungs. When the hornets leave, you climb seven rungs to finish painting and then climb the remaining six rungs to get on the roof."

Ask them to think about the following questions but not to answer out loud.

a) What rung of the ladder were you standing on at the beginning of the story?

b) Was your next move to go up or down? How many rungs?

c) What did the hornets force you to do?

d) What did you do after the hornets left?

Comments

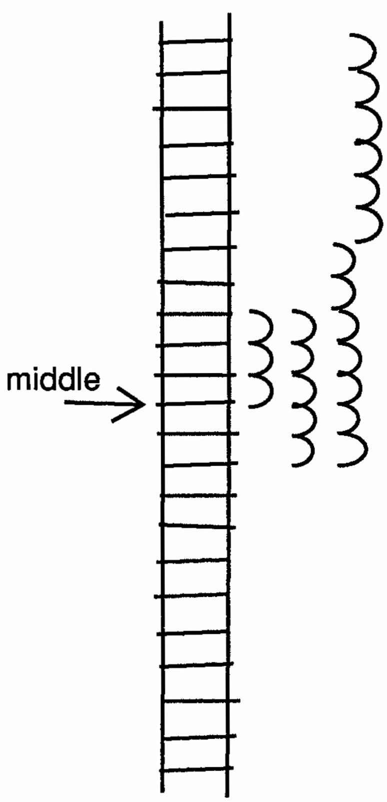
1. The goal of this action is to encourage students to create mental images of a story. Create a relaxed setting. Some teachers ask the students to fold their hands and close their eyes so they will not be distracted while listening.

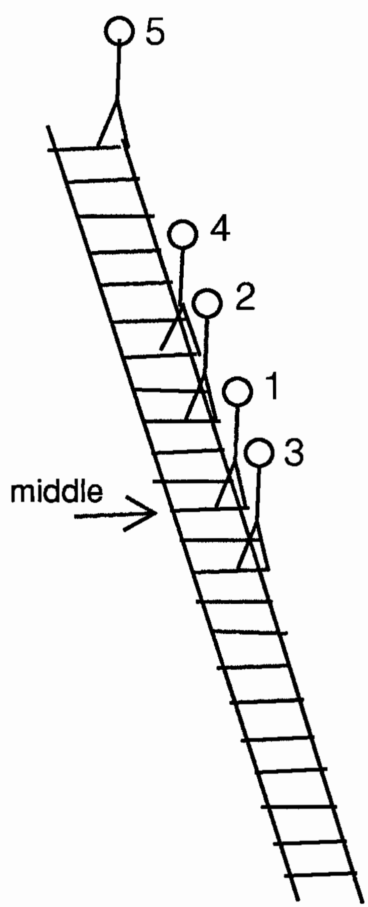
The questions following the story are meant to stimulate the students to recreate their mental image of the story. These questions are not meant to be answered, other than in each student's own mind.

Actions

2. Ask the students to draw a sketch or diagram, without words, that describes the essence of the story as they heard it. Tell them that, for this drawing, it is not important that the number of steps up or down the ladder be exact. Ask for volunteers to show their diagram to the class.

3. Write the story on the chalkboard or overhead projector. Let the students read the story. Ask them to use diagrams and sketches to determine the number of rungs in the complete ladder. Discuss.





Comments

2. In the last action, the students created a mental image of the story and now they are being asked to represent the image in a diagram or sketch. A diagram or sketch is a paper and pencil drawing which represents, to the *drawer*, the essence of the story as imagined. These representations can vary greatly. There is no right or wrong way to represent a story and students usually enjoy seeing how others have represented the same story.

As the students draw, you may wish to circulate among them and acknowledge their work. Privately asking students if they would be willing to show their diagram during the class discussion helps "break the ice" when sharing time arrives.

3. Allowing the students to read the story will enable them to recapture facts they have forgotten.

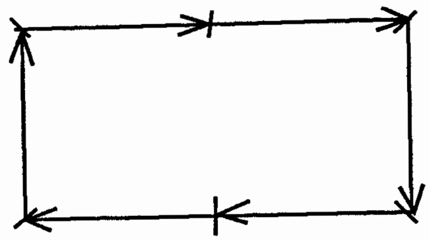
Having students share their drawings with the class acknowledges their work. It also lets students see a variety of ways to represent with diagrams and sketches.

Some students will feel comfortable in front of the class presenting their drawing on the overhead or chalkboard. Others may prefer to put sketches on an overhead transparency at their desk and then bring the completed drawing to the overhead projector.

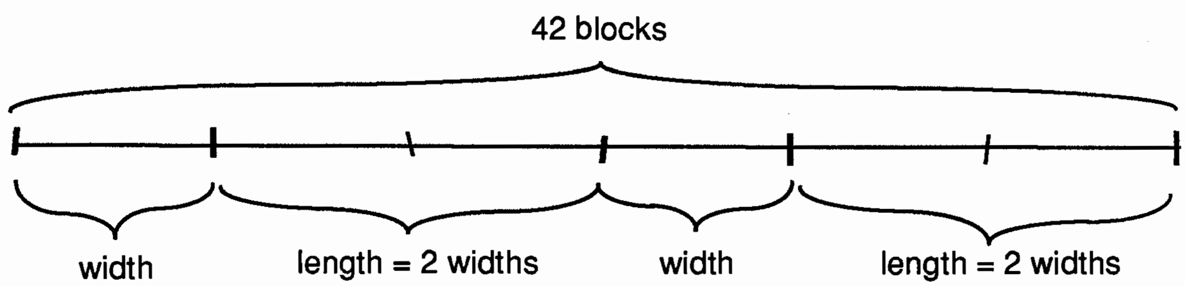
At left are two possible sketches. The numbers by the stick figures indicate the successive positions of the painter.

Actions

4. Write story ONE from the Picture Puzzle Stories activity sheet on the chalkboard or overhead. Have the students read the story. Ask them to visualize this story in their mind's eye, and then draw a sketch or diagram that enables them to determine the length and width of the rectangular path. Discuss.



"The total length of the path is 42 blocks, so each of the 6 arrows represents 7 blocks. The width is 1 arrow or 7 blocks. The length is 2 arrows or 14 blocks."



"The 42 block path is the same length as 6 'widths' of the rectangle, so the width is 7 blocks and the length is 2 widths or 14 blocks."

Comments

4. A master for Picture Puzzle Stories is included with this activity.

Alternatively, story ONE can be cut from copies of the page, Picture Puzzle Stories, and distributed so that each student gets the story on a slip of paper. Avoid presenting students with the whole page of story problems at once.

Allow enough time for everyone to work on this puzzle problem. Perhaps the students could start it one day and then discuss it the next day. On this page are two ways that students have approached this problem. You are likely to see other ways.

In the first visual solution, the rectangular path is viewed as 6 consecutive arrows of the same length. In the second solution, the rectangular path is "unfolded" to form a linear path 42 blocks long.

These sketching activities take time at the beginning. Most students have not done this before. Practice, and observing others' sketches, helps students improve their ability to devise visual solutions.

Actions

5. Have the students create sketches and diagrams which enable them to solve the remaining Picture Puzzle Stories. Discuss the diagrams and the solutions.

6. Have the students read story ONE from the activity sheet, "More Picture Puzzle Stories." Ask the students to devise a sketch or diagram which represents the story and helps them determine the length of each piece of string. Discuss.

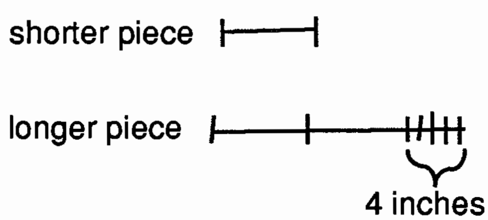
Comments

5. Rather than assign all the problems on the activity sheet at once, you may wish to assign one each day or one every few days, while the students are working on other mathematical topics. Classroom presentation of drawings that lead to visual solutions will help students generate ideas for other visual solutions.

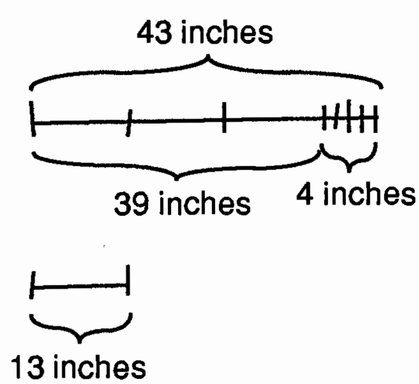
It is beneficial to have several students present visual solutions to the same story problem. Rarely do two individuals create exactly the same diagram for the same reasons. Even though one student may say that another student has given their solution, the reasoning is likely to be different.

Pages 6 and 7 contain sample sketches leading to visual solutions for the remaining stories on the Picture Puzzle Stories activity sheet.

6. A master for More Picture Puzzle Stories is included with this activity.



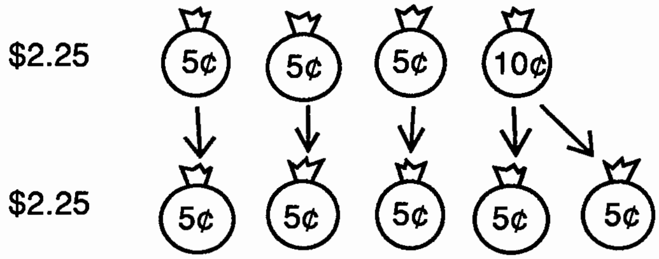
In the visual solution to story ONE shown here, the shorter piece is represented by a line segment. If the shorter piece is doubled and increased by 4 inches, the longer piece is obtained.



If the two pieces are placed end-to-end the resulting length is 43 inches. Subtracting 4 from 43, one sees that the total length of the 3 remaining segments is 39 inches long. So the shorter piece is 13 inches long and the length of the longer piece is 13 inches + 13 inches + 4 inches = 30 inches.

Actions

7. Have the students read story SIX on the activity sheet, "More Picture Puzzle Stories." Ask for a visual solution to this story.



"Each of the bags of nickels have the same value, so each bag contains 45¢ or 9 nickels. Thus Andrea has 27 nickels and 9 dimes."

8. Have students work on visual solutions for the remaining stories on "More Picture Puzzle Stories."

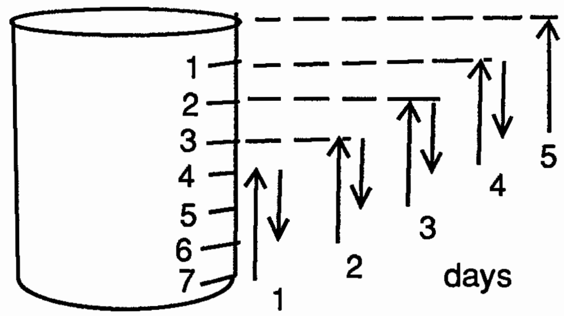
Comments

7. In coin stories, both the number of coins and their value must be considered.

In this solution of story SIX, Andrea's coins are placed into bags each containing the same number of coins. When the bag of dimes is exchanged for 2 bags of nickels, there are 5 bags of nickels, each containing the same number of coins.

8. Assigning story puzzles one at a time and sharing and discussing solutions in class, is a productive and enjoyable method for learning to solve problems visually. Pages 8 and 9 contain sample sketches leading to visual solutions for the remaining stories on the activity sheet, "More Picture Puzzle Stories."

TWO



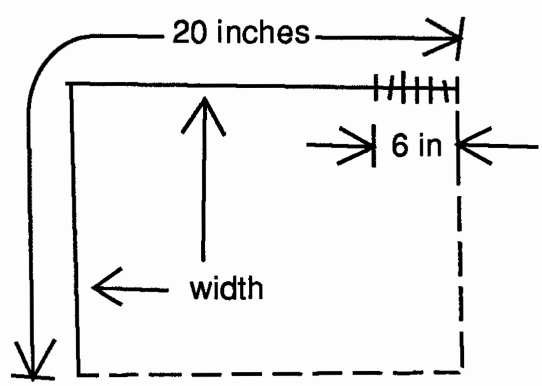
THREE

"If all had two legs:

This is a total of 10 legs, so 6 more are needed:

There are 3 sheep and 2 ducks."

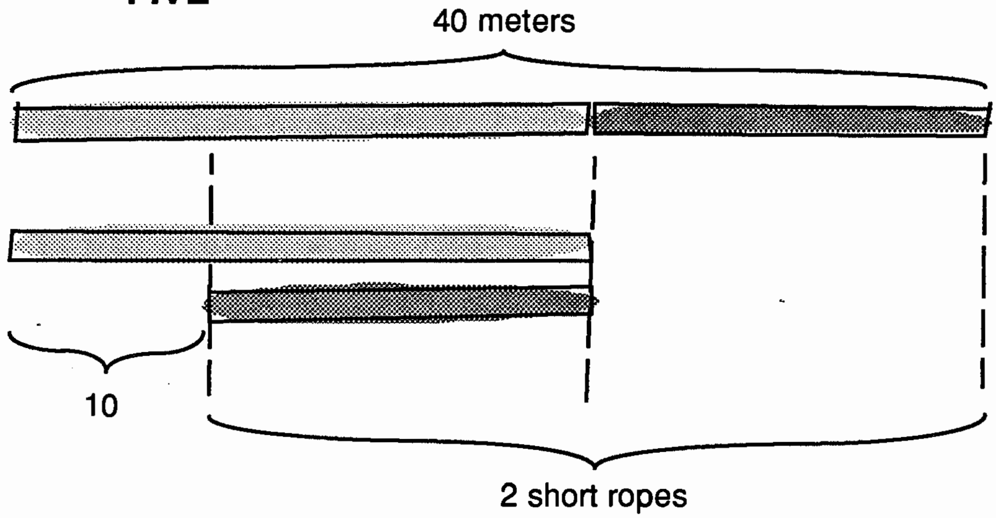
FOUR



"Twice the width is 14 inches.
So width = 7 inches, length = 7 + 6 = inches."

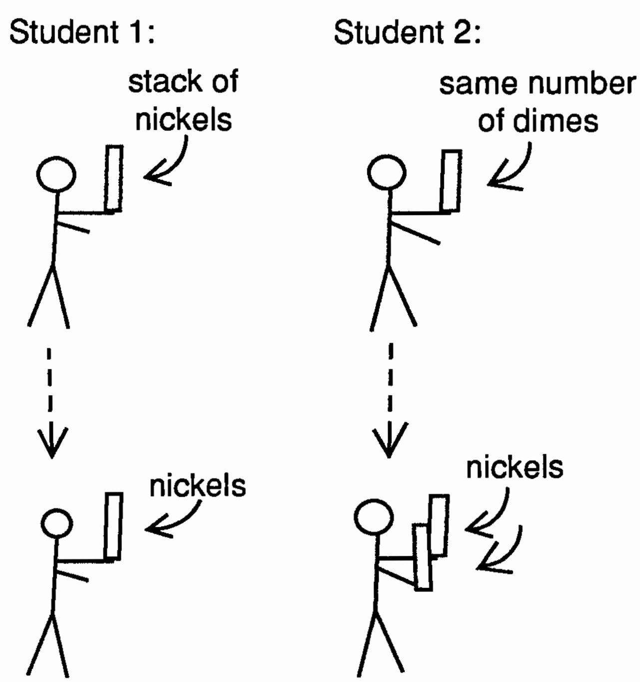
Continued next page

FIVE



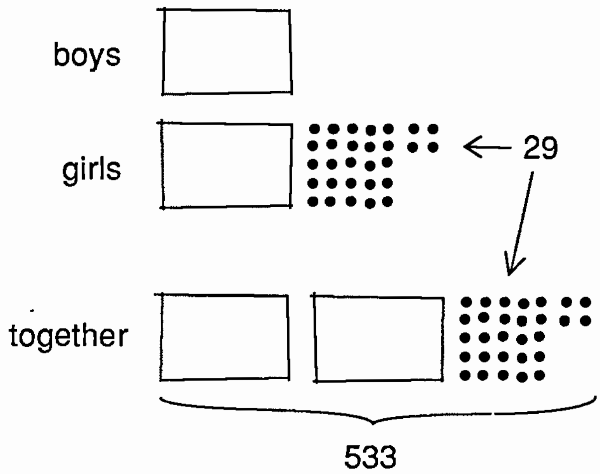
"Twice the shorter rope is 30 meters. Hence the shorter rope is 15 meters. The longer rope is 15 + 10 = 25 meters."

SIX



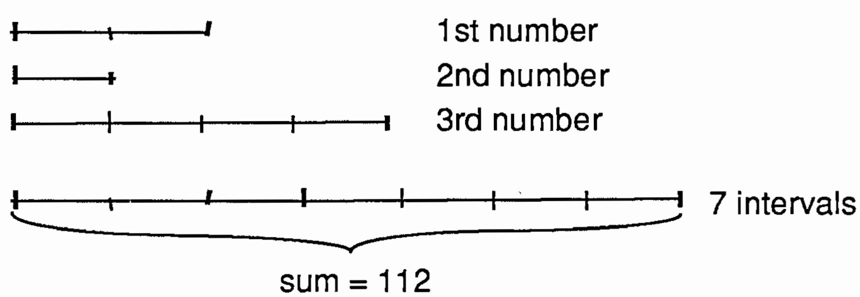
"The value of each stack of nickels = $1.05 ÷ 3 = 35¢.
So, there are 7 coins in each stack."

TWO



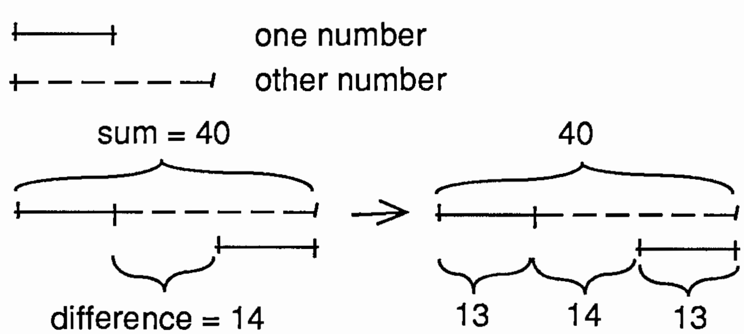
"The two boxes represent 533 − 29 = 504 students, so each represents 252 students. Number of boys = 252; number of girls = 252 + 29 = 281."

THREE



"Each interval represents 112 ÷ 7 = 16.
The numbers are 32, 16 and 64."

FOUR

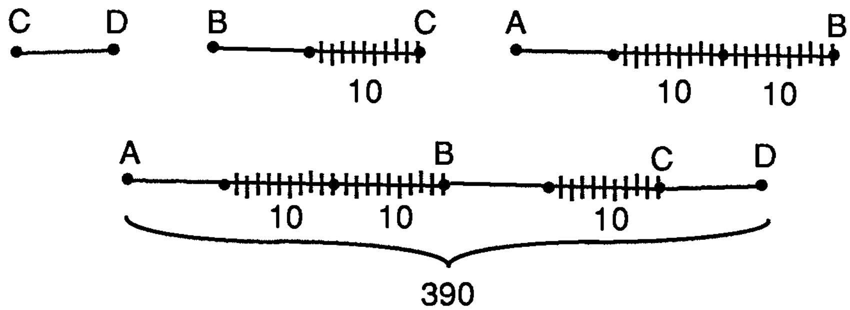


"One number = 13.
The other number = 14 + 13 = 27."

Continued next page

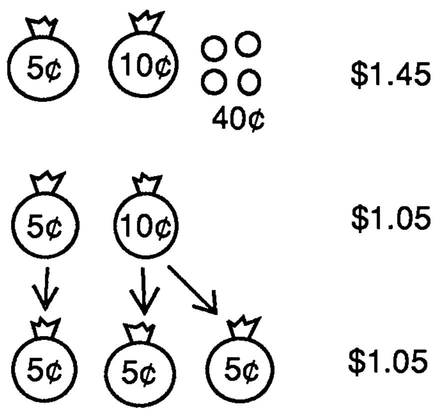
Sample Sketches • More Picture Puzzle Stories
continued

FIVE



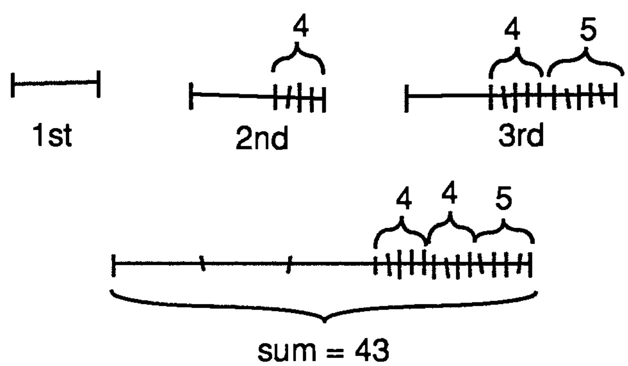
"Together, the three unmarked intervals represent 360 miles. So each unmarked interval represents 120 miles. The distance from A to B is 120 + 10 + 10 = 140 miles."

SEVEN



"Since each bag contains the same number of coins, there must be 35¢ or 7 coins in each bag."

EIGHT



"The length of the first side is (43 - 13) ÷ 3 = 10."

ONE

Each morning a math teacher runs a total distance of 42 city blocks and always chooses a rectangular running path. This morning her rectangular path was twice as long as it was wide. How many blocks long and how many blocks wide was this morning's rectangular path?

TWO

A snail at the bottom of a well 7 feet deep started climbing to the top. Each day it would climb up 3 feet but at night, when sleeping, it would slide down 2 feet. How many days will it take the snail to reach the top?

THREE

The tall girl looked over the fence into a barnyard containing geese and sheep and announced that there were five heads altogether. Her little brother looked under the fence and said he saw 16 legs. How many geese and how many sheep were in the barnyard?

FOUR

The length of a rectangle is six inches longer than the width. If the perimeter is 40 inches, what is the width?

FIVE

When two pieces of rope are placed end-to-end they measure 40 meters in length. When the pieces are laid side-by-side one is 10 meters longer than the other. How long is each piece of rope?

SIX

One student has only nickels in her hand and a second student has exactly the same number of dimes and no other coins. Together they have a total of $1.05. How much money and how many coins does each student have?

ONE

How would you cut a piece of string 43 inches long into two parts so that one part is 4 inches longer than twice the other part?

TWO

There are 29 more girls than boys in the middle school. The school has 533 students. How many girl students are there?

THREE

Find the secret numbers. The first number is twice the second number. The third number is twice the first number. Their sum is 112.

FOUR

The sum of two numbers is 40. The difference between the two numbers is 14. What are the numbers?

FIVE

To drive from town A to town D, one must drive through town B and then town C. It is 10 miles farther from town A to town B than it is from town B to town C. And, it is 10 miles farther from town B to town C than it is from town C to town D. It is 390 miles from town A to town D. How far is it from town A to town B?

SIX

Andrea has three times as many nickels as dimes. If she has $2.25 altogether, how many dimes does she have?

SEVEN

Bob has four more dimes than nickels. He has $1.45 in all. How many nickels does he have?

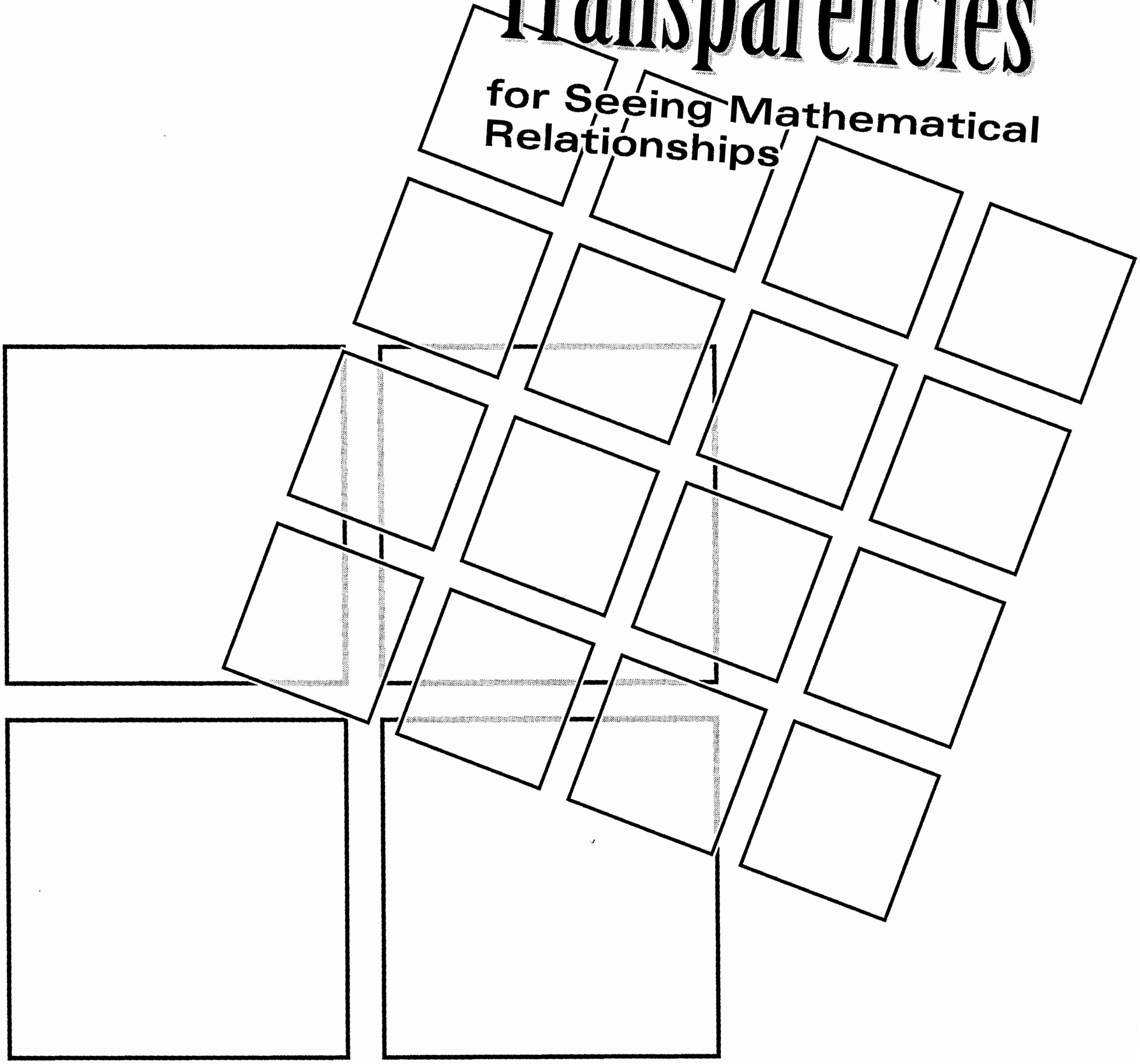
EIGHT

Determine the lengths of the three sides of a triangle whose perimeter is 43 inches. The second side of the triangle is 4 inches longer than the first side, and the third side is 5 inches longer than the second side.

Unit I / Math and the Mind's Eye

Transparencies

for Seeing Mathematical Relationships



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PO Box 3226
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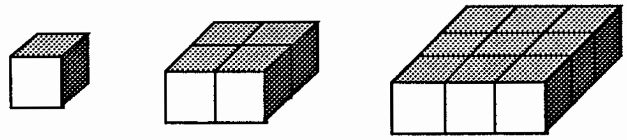
Catalog #MET1

a)



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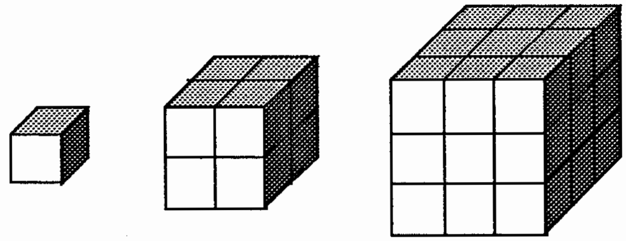

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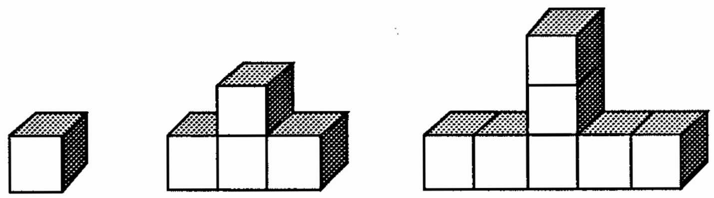

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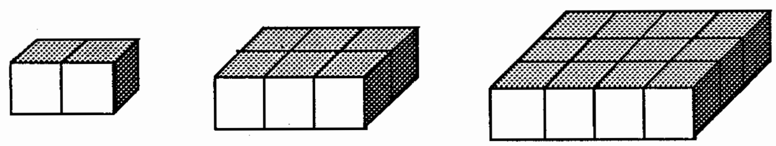

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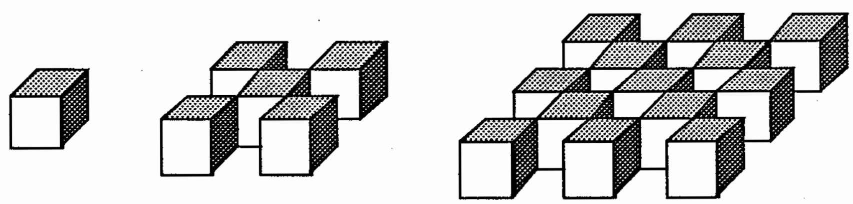

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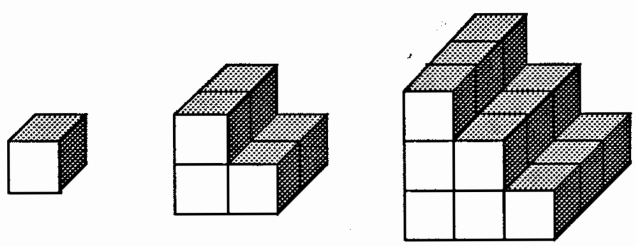

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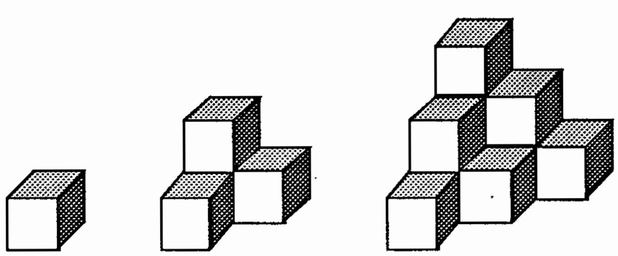
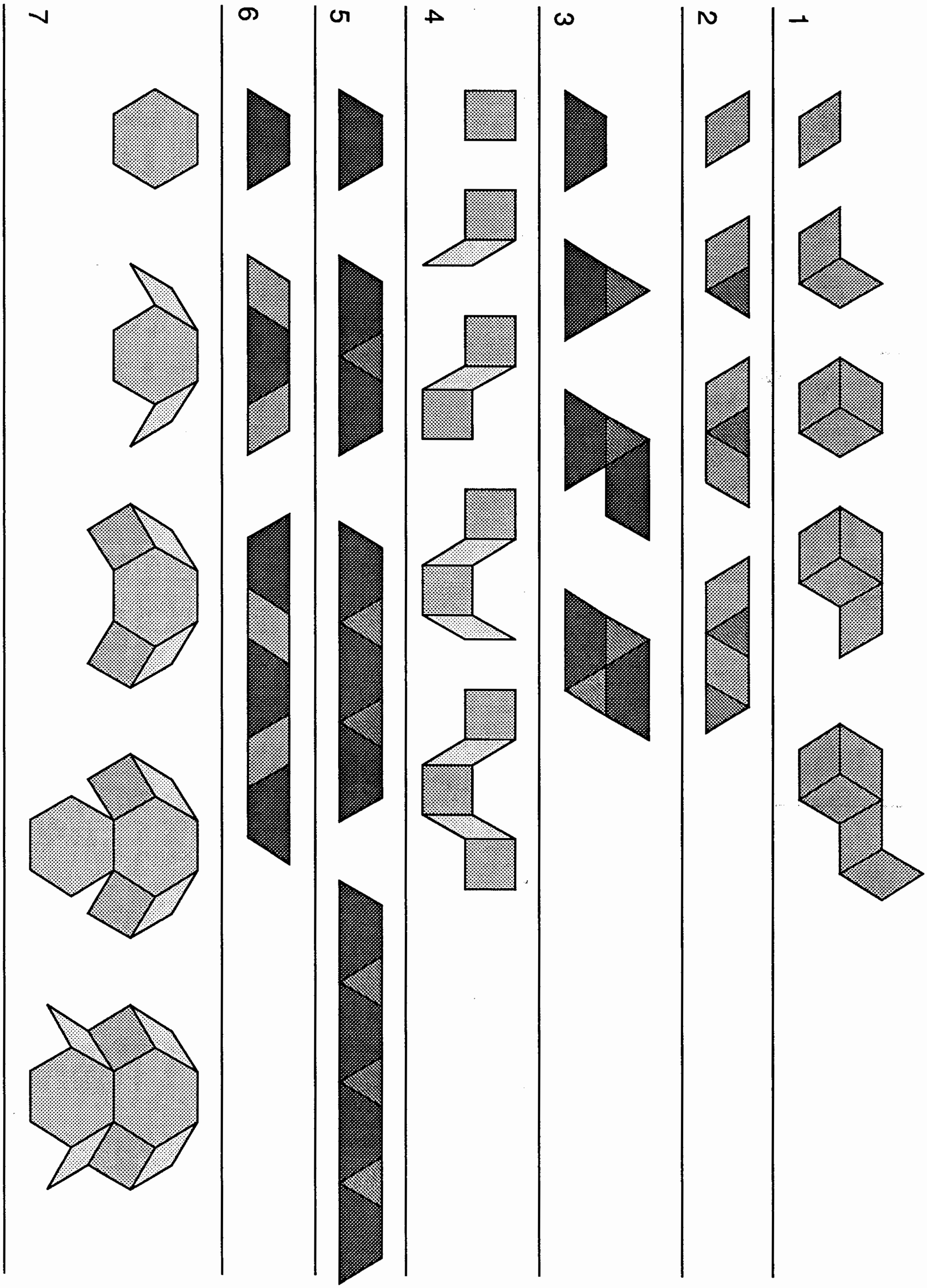
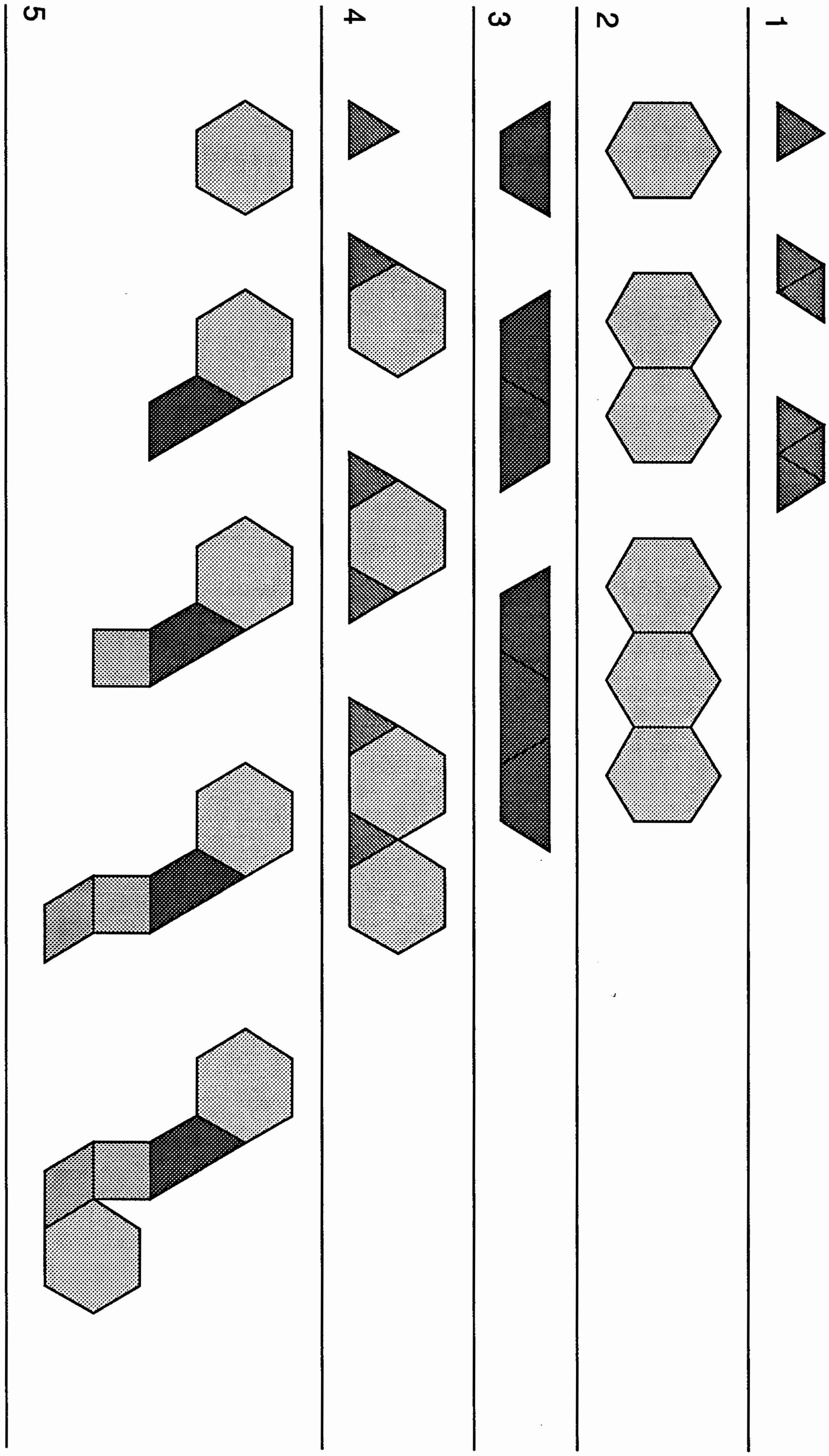


Pattern Block Trains



Pattern Block Perimeter Trains



ONE

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FIVE

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