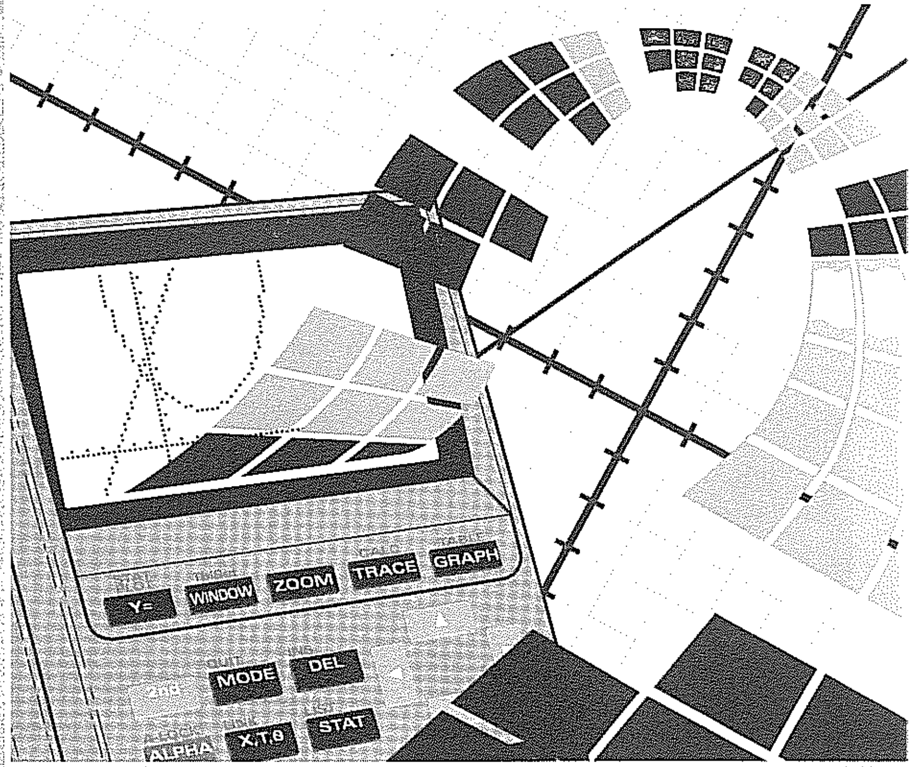
Unit XI / Math and the Mind's Eye Activities



Graphing Algebraic Relationships

Eugene Maier &
Michael Shaughnessy

Graphing Algebraic Relationships

1. **An Introduction to Graphs, Part I**
The values of the arrangements in extended sequences are graphed. The graphs are examined for information about these values.

2. **An Introduction to Graphs, Part II**
Extended sequences of arrangements are augmented so their graphs become continuous.

3. **An Introduction to Graphs, Part III**
Further investigations with continua of arrangements.

4. **Introduction to Graphing Calculators, Part I**
Graphing calculators are introduced to provide an alternative to the "by-hand" method of plotting graphs, and as a way to represent continua of arrangements in more detail. Connections between Algebra Piece, graphing, and symbolic representations of patterns are developed.

5. **Introduction to Graphing Calculators, Part II**
We continue our explorations with graphing calculators, investigating systems of equations. Connections among the Algebra Piece, graphing, tabular, and symbolic representations of equations are reinforced.

Math and the Mind's Eye materials are intended for use in grades 4-9. They are written so teachers can adapt them to fit student backgrounds and grade levels. A single activity can be extended over several days or used in part.

A catalog of Math and the Mind's Eye materials and teaching supplies is available from The Math Learning Center, PO Box 3226, Salem, OR 97302, 503-370-8130. Fax: 503-370-7961.

Math and the Mind's Eye

These materials were prepared with the support of National Science Foundation Grant MDR-840371.

ISBN 1-886131-39-2

An Introduction to Graphs, Part I

O V E R V I E W

The values of the arrangements in extended sequences are graphed. The graphs are examined for information about these values.

Prerequisite Activity

Unit IX, *Picturing Algebra.*

Materials

Red and black bicolored counting pieces, Algebra Pieces, masters and activity sheets as noted.

Actions

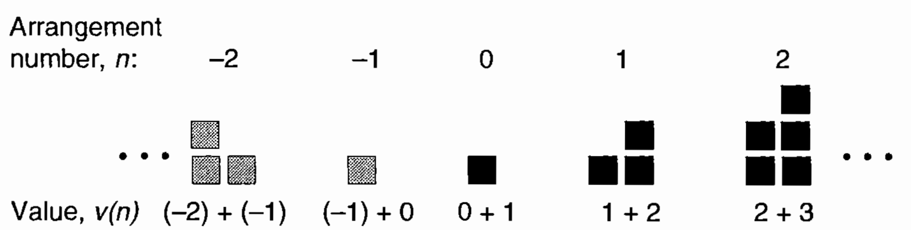
1. Distribute counting pieces to the students. Have them form an extended sequence of counting piece arrangements which fits the following data, where n is the arrangement number and $v(n)$ is the value of arrangement n.

n	-2	-1	0	1	2
$v(n)$	-3	-1	1	3	5

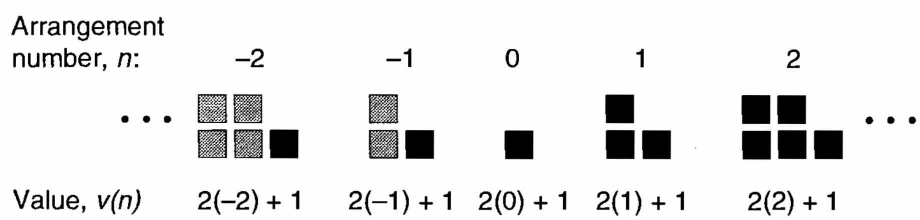
Ask the students to write a formula for $v(n)$. Discuss.

Comments

1. Various extended sequences of arrangements are possible. Shown below are two possibilities with the formulas for $v(n)$ they suggest. Be sure the students indicate how they are numbering the arrangements in their sequences.



$v(n) = n + (n + 1)$

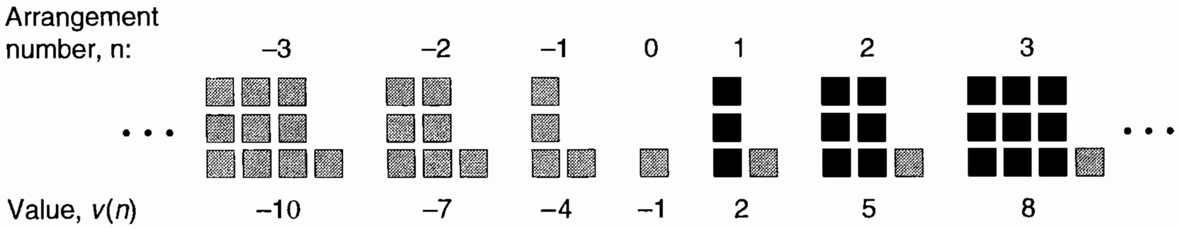


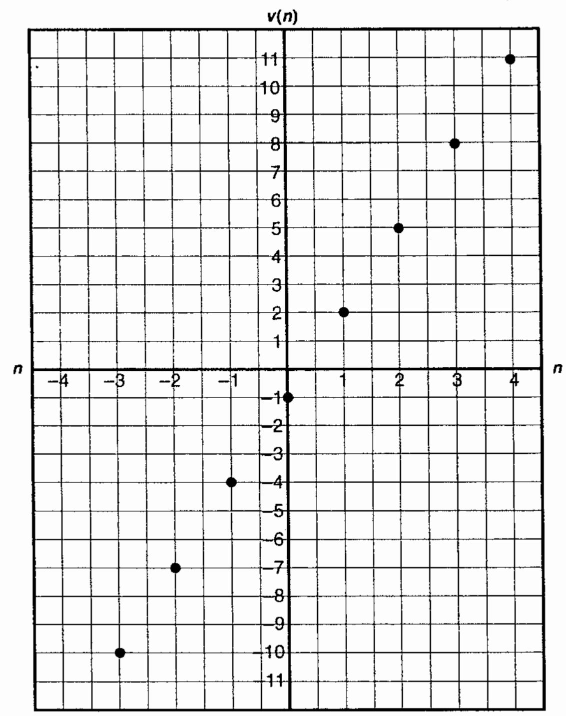
$v(n) = 2n + 1$

Actions	Comments

2. Give each student a copy of Activity Sheet XI-1-A. Have the students form an extended sequence of counting piece arrangements which fits the data displayed in graphical form on the sheet. Ask them to determine $v(-4)$, $v(-3)$, $v(3)$ and $v(4)$ for their sequence and, if possible, add this information to their graph.

2. Shown below is one extended sequence that fits the data. For this sequence, $v(-4) = -13$, $v(-3) = -10$, $v(3) = 8$ and $v(4) = 11$. In the latter three cases, this information has been added to the graph shown below. The value for $v(-4)$ lies outside the range of the graph.





$v(n) = 3n - 1$

3. Ask the students to find a formula for $v(n)$ for the sequence they constructed in Action 2 and record it in the space provided on the activity sheet. Ask the students for their observations about the graph. Discuss.

3. The form of the arrangements shown in Comment 2 suggests the formula $v(n) = 3n - 1$. The students may have other formulations.

Some possible observations:

- The points of the graph lie on a straight line.
- The points are equally spaced.
- To get from one point to the next, go 1 square to the right and 3 up.
- The increase from point to point is always the same.
- There are only points on the graph when n is an integer.

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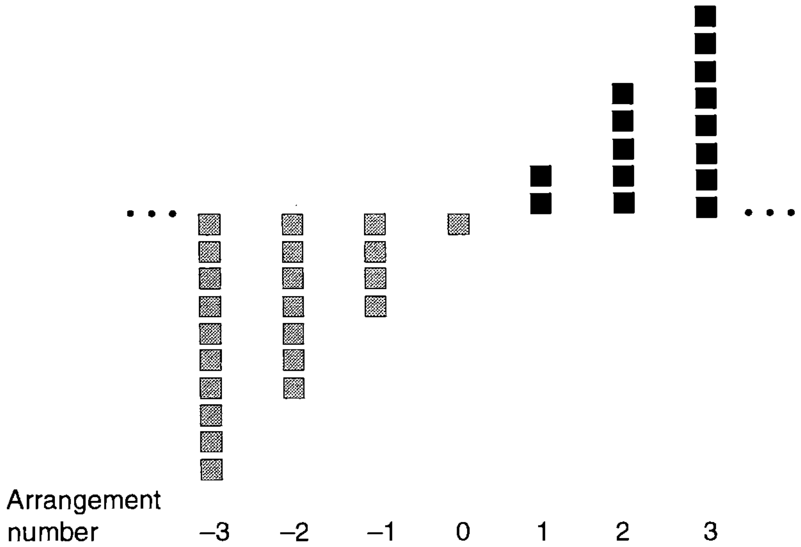
Actions



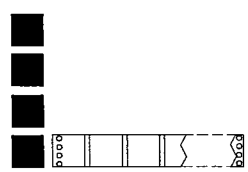
Comments

3. *Continued.* It may be instructive to compare the graph with the sequence obtained when the arrangements of the original sequence are rearranged into columns, as shown below. The columns contain a minimal number of tile (so no column contains both red and black tile). Black columns extend above a base line and red columns extend below.

You may want to ask the students how the numbers in their formula for $v(n)$ relate to the graph. In the above formula, 3, the coefficient of n, is the amount the height increases as n increases by 1. The constant term, –1, is the value of the 0th arrangement. It indicates where the graph coincides with the vertical axis.

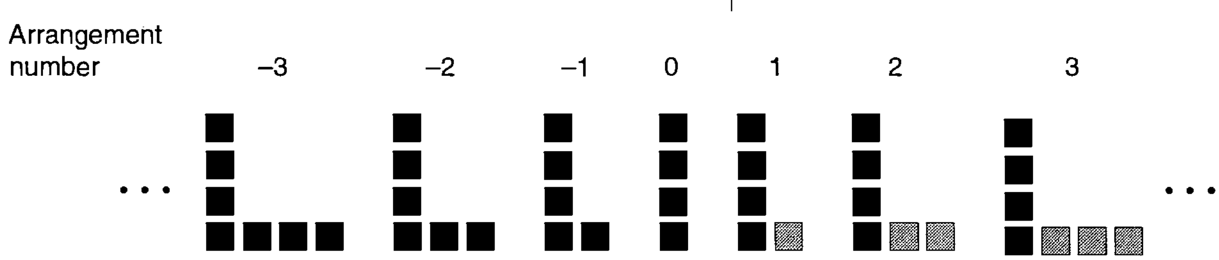
Some students may draw a line connecting the points of the graph, implying there are arrangements for non-integral values of n. The students may suggest ways for constructing such arrangements. (See the next activity, *An Introduction to Graphs, Part II.*) However, for the extended sequence shown above, there are only points on the graph for integral values of n.

4. Show the students the following Algebra Piece arrangement. Tell them it is the nth arrangement for an extended sequence of tile patterns. Ask the students to form the –3rd to 3rd arrangements of this sequence.



4. A transparency master of the arrangement is attached (Master 1, top half). If you use this transparency, do not show the students the bottom half of the transparency until they have built several arrangements.

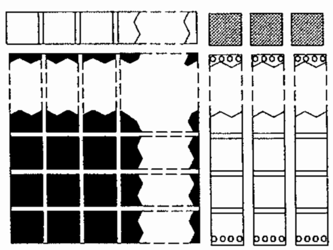
Below are arrangements number –3 through 3. A master for a transparency of these arrangements is attached (Master 1, bottom half). Recall that a $-n$-frame contains red tile if n is positive and black tile if n is negative. It contains no tile if n is 0.



Actions

5. Distribute copies of Activity Sheet XI-1-B to the students. For the sequence of Action 4, ask the students to record a formula for $v(n)$, construct its graph and record their observations about the graph. Discuss.

6. Repeat Actions 4 and 5 for the following Algebra Piece arrangement, using Activity Sheet XI-1-C in place of Activity Sheet XI-1-B.

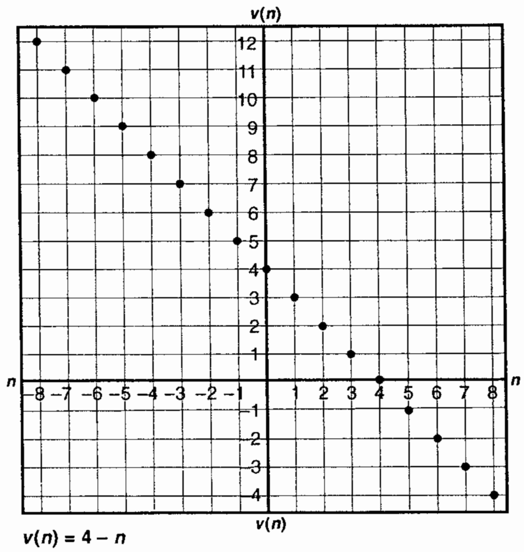


Comments

5. A master for the Activity Sheet is attached.

The formula for $v(n)$ can be written in various forms. One possibility is $v(n) = 4 - n$. Another is $v(n) = 4 + (-n)$.

The completed graph is shown below.



6. A transparency master of the arrangement is attached (Master 2, top half). If you use this transparency, do not show the students the bottom half of the transparency until they have built several arrangements.

Below are arrangements number −3 through 3. A master for a transparency of these arrangements is attached (Master 2, bottom half).

Continued next page.

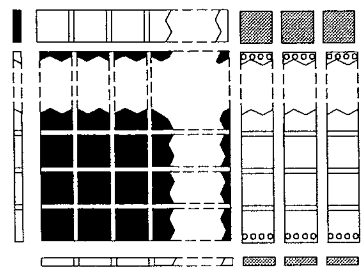
Actions

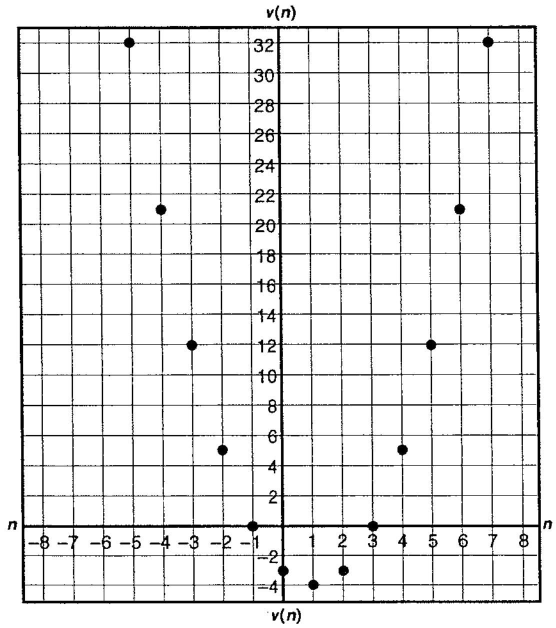
Comments

6. *Continued.* Here are two possibilities for $v(n)$:

$$v(n) = n^2 - 2n - 3$$
$$v(n) = (n + 1)(n - 3)$$

Edge pieces may help the students see the latter formulation:





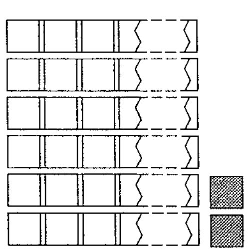
$v(n) = (n + 1)(n - 3)$

Some observations about the graph:

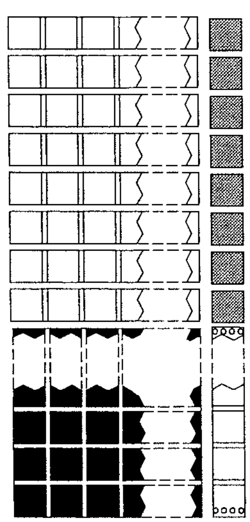
- The points of the graph do not lie on a straight line.
- The graph is symmetric about $n = 1$, that is, if the graph where folded along the vertical line that goes through $n = 1$, the points to the right of the fold would coincide with those to the left of the fold.
- The smallest value for $v(n)$ is -4. It occurs when $n = 1$.

7. Show the students arrangements I and II below. Tell them they are the nth arrangements of two different extended sequences. Ask them to write formulas for $v_1(n)$, the value of the nth arrangement of the first sequence and $v_2(n)$, the value of the nth arrangement of the second sequence.

I



II



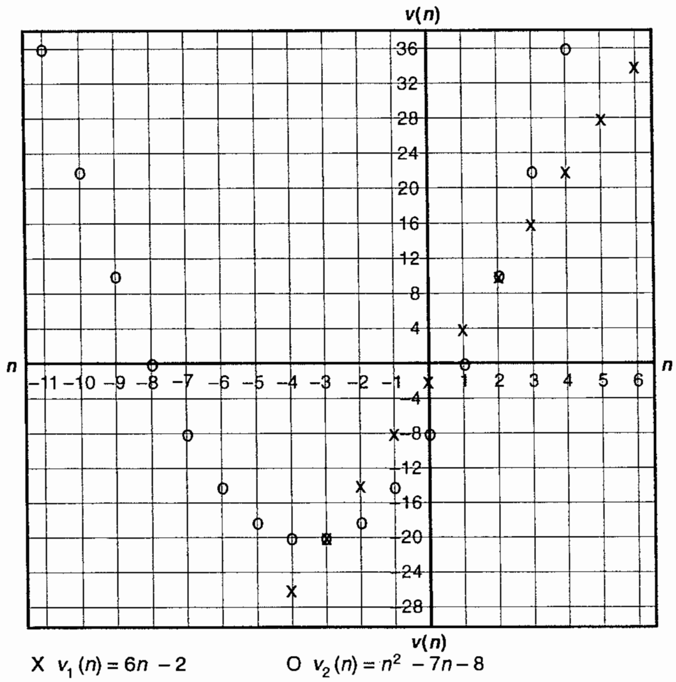
7. A transparency master of the arrangements is attached (Master 3).

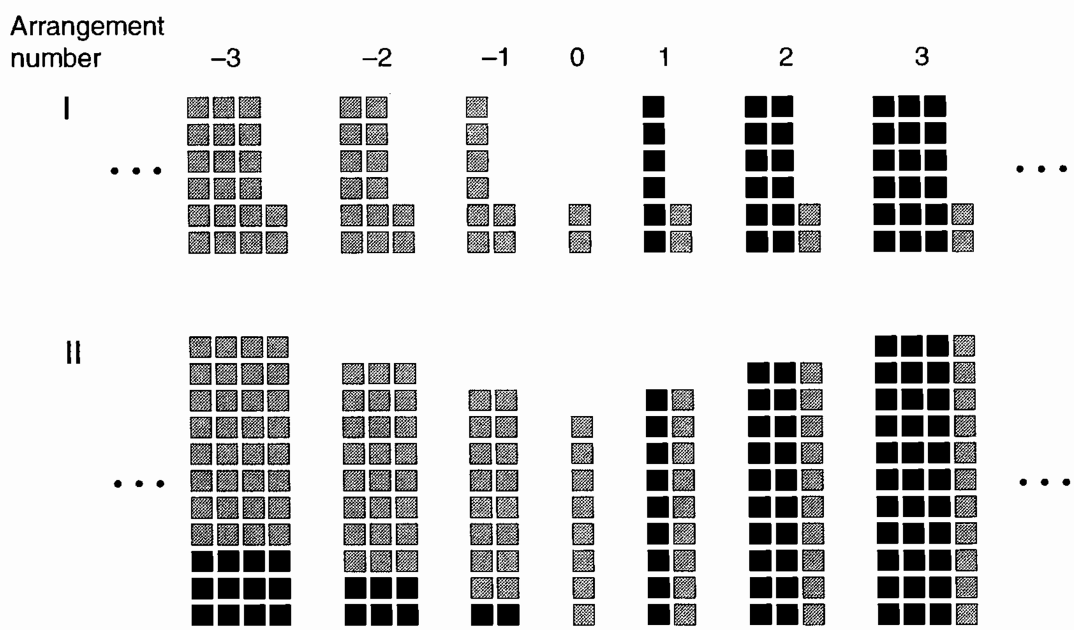
$$v_1(n) = 6n - 2$$
$$v_2(n) = n^2 + 7n - 8$$

Other formulations are possible. For example, $v_2(n) = (n + 8)(n - 1)$. Edge pieces may help the students see this formulation.

Actions

8. Ask the students to graph $v_1(n)$ and $v_2(n)$ on Activity Sheet XI-1-D. Have them indicate the points on the graph of $v_1(n)$ with an x and those on the graph of $v_2(n)$ with an o. Ask the students to examine the graphs and record their observations. Discuss.





Comments

8. A master of Activity Sheet XI-1-D is attached. The completed graph is shown below.

It may facilitate discussion to refer to points of the graphs by their *coordinates*. The *coordinates* of a point of a graph are a pair of numbers the first of which locates the point horizontally, the second vertically. For example, (–4, –20), (0, –2) and (4, 22) are coordinates of points on the graph of $v_1(n)$.

Notice that two points, (2, 10) and (–3, –20), are on both graphs. This tells us that the 2nd arrangements of the two sequences have the same value, namely 10, and the –3rd arrangements also have the same value, namely –20. It also tells us the equation $6n - 2 = n^2 + 7n - 8$ has two solutions, $n = 2$ and $n = -3$.

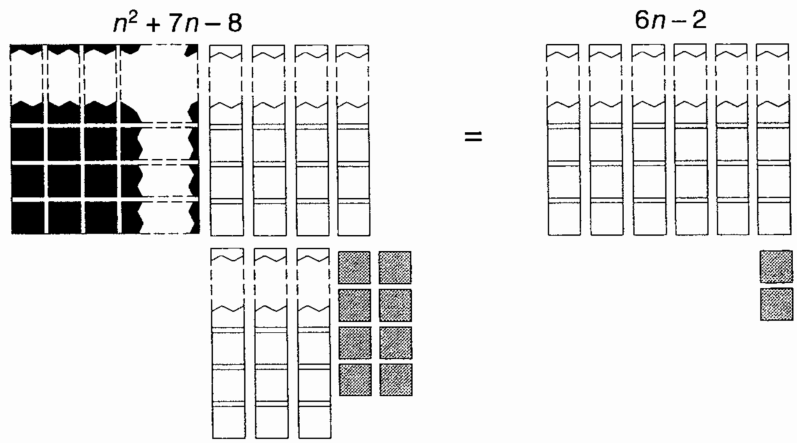
The students may wish to form the 2nd arrangement of each sequence and verify that they have the same value. Likewise for the –3rd arrangement. Arrangements number –3 through 3 for both sequences are shown below. A transparency of these arrangements is attached (Master 4).

Actions

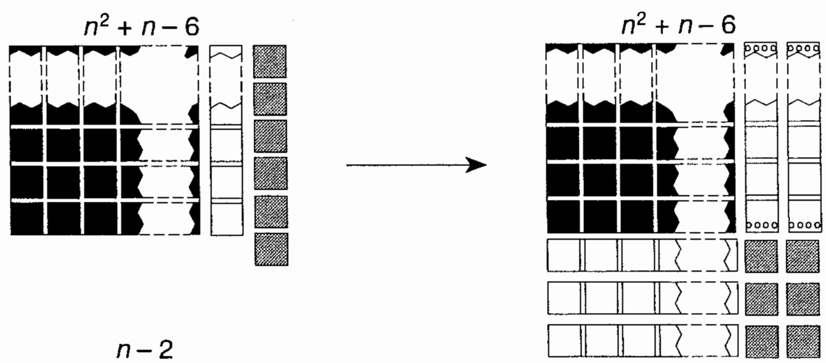
9. Ask the students to use the pieces to build the nth arrangement of both extended sequences, $v_1(n)$ and $v_2(n)$, and to then solve the equation $6n - 2 = n^2 + 7n - 8$ using the pieces. Ask students to share their solution.

Comments

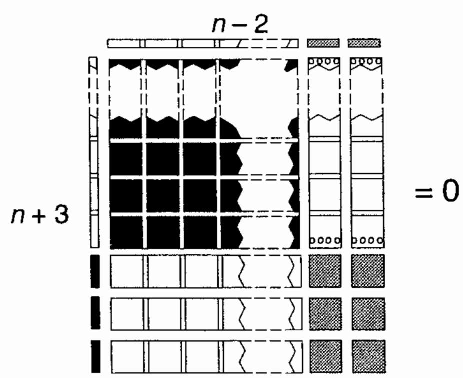
9. One possibility is given below. (See also Unit IX, Activities 6 and 7). Thus the students will have three different ways to represent the solution to an equation like $6n - 2 = n^2 + 7n - 8$: using the pieces, using a graph, or comparing arrangements of extended sequences. You might point out these three representations to the students and ask them which one they like best at this point, and why.



Remove 6 n-strips and 2 shaded tiles from each arrangement. Then we obtain the equivalent equation: $n^2 + n - 6 = 0$



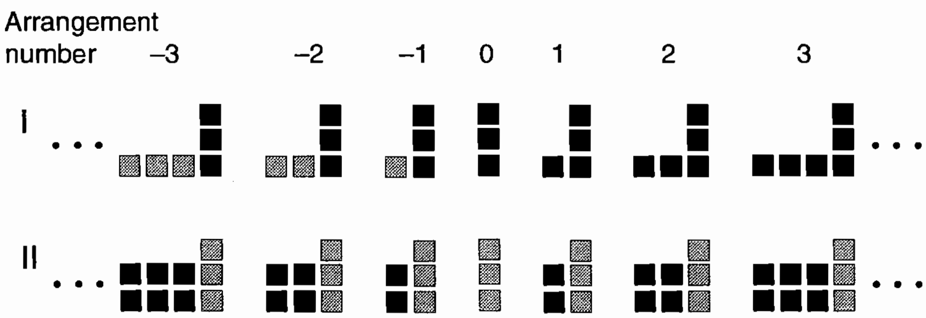
Note: We can complete a rectangle by including 2 opposite frames and 2 more n-frames in our arrangement and maintain a net value of $n^2 + n - 6$.



This rectangle has edge piece values $n - 2$ and $n + 3$. Thus, $(n - 2)(n + 3) = 0$ when $n = -3$ or $n = 2$. (The area is 0 if and only if one of the edges has value 0.)

Actions

10. (Optional) Show the students the following portions of extended sequences I and II. Ask them to write formulas for $v_1(n)$, the value of the nth arrangement of sequence I and $v_2(n)$, the value of the nth arrangement of sequence II. Then have the students graph $v_1(n)$ and $v_2(n)$ on Activity Sheet XI-1-E. Ask the students to record their observations. Discuss.



Comments

10. Masters for Activity Sheet XI-1-E and a transparency of the two extended sequences (Master 5) are attached.

From sequence I, one sees that $v_1(n) = n + 3$.

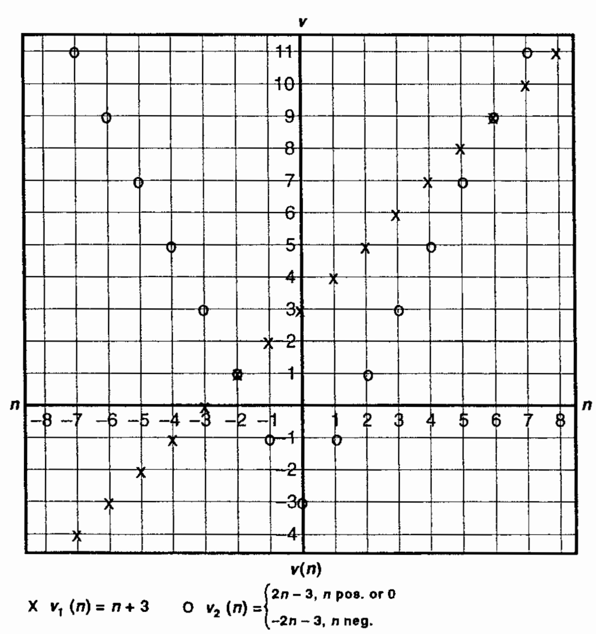
The students may readily see the pattern of sequence II, but have difficulty in writing a formula for $v_2(n)$. You can suggest to the students that they write the formula in two parts, one part for non-negative arrangement numbers and one part for negative arrangement numbers:

$$v_2(n) = \begin{cases} 2n - 3, & n \text{ non-negative} \\ 2(-n) - 3, & n \text{ negative.} \end{cases}$$

Notice that the case $n = 0$ is included in the first part.

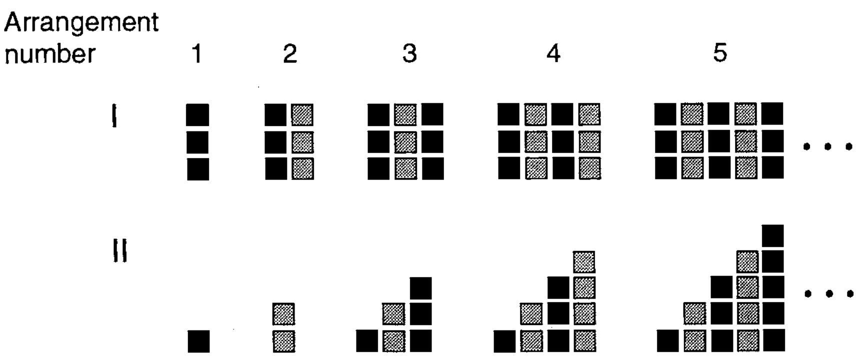
You may want to tell the students about the mathematical symbol $|n|$, read *absolute value of n,* which is defined to be n, if n is non-negative (e.g., $|3| = 3$) and $-n$ if n is negative (e.g., $|-3| = 3$). Using this symbol, one has $v_2(n) = 2|n| - 3$ for all n.

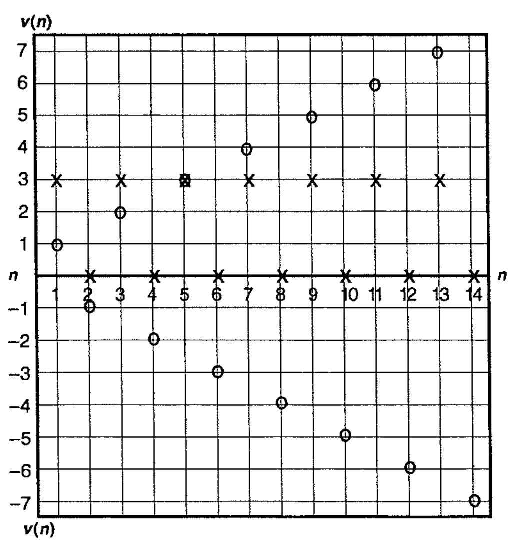
The graphs of $v_1(n)$ and $v_2(n)$ are shown below. The points of the graph of $v_1(n)$ lie on a straight line. The points of the graph of $v_2(n)$ lie on a V whose vertex is the point (0, –3). Note that the points (–2, 1) and (6, 9) lie on both graphs.



Actions

11. (Optional) Repeat Actions 9 for the following two sequences, using Activity Sheet XI-1-F in place of Activity Sheet XI-1-E.





X $v_1(n) = \begin{cases} 3, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$ O $v_2(n) = \begin{cases} \frac{(n+1)}{2}, & n \text{ odd} \\ \frac{-n}{2}, & n \text{ even} \end{cases}$

Comments

11. Masters for Activity Sheet XI-1-F and a transparency of the two sequences (Master 6) are attached.

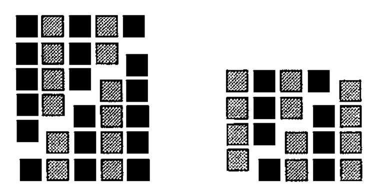
Notice that these two sequences are not extended—in both cases, the set of arrangement numbers is the positive integers.

If students have difficulty in writing formulas for $v_1(n)$ and $v_2(n)$, you may want to suggest they write the formulas in two parts, one part for n odd and one part for n even.

$$v_1(n) = \begin{cases} 3, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

$$v_2(n) = \begin{cases} (n+1)/2, & n \text{ odd} \\ -n/2, & n \text{ even} \end{cases}$$

One way to see the above formula for $v_2(n)$ is to notice that, if n is odd, two copies of the nth arrangement form a rectangle whose value is $n + 1$ (an example for $n = 5$ is shown below) and, if n is even, two copies of the nth arrangement form a rectangle whose value is $-n$ (an example for $n = 4$ is shown below).

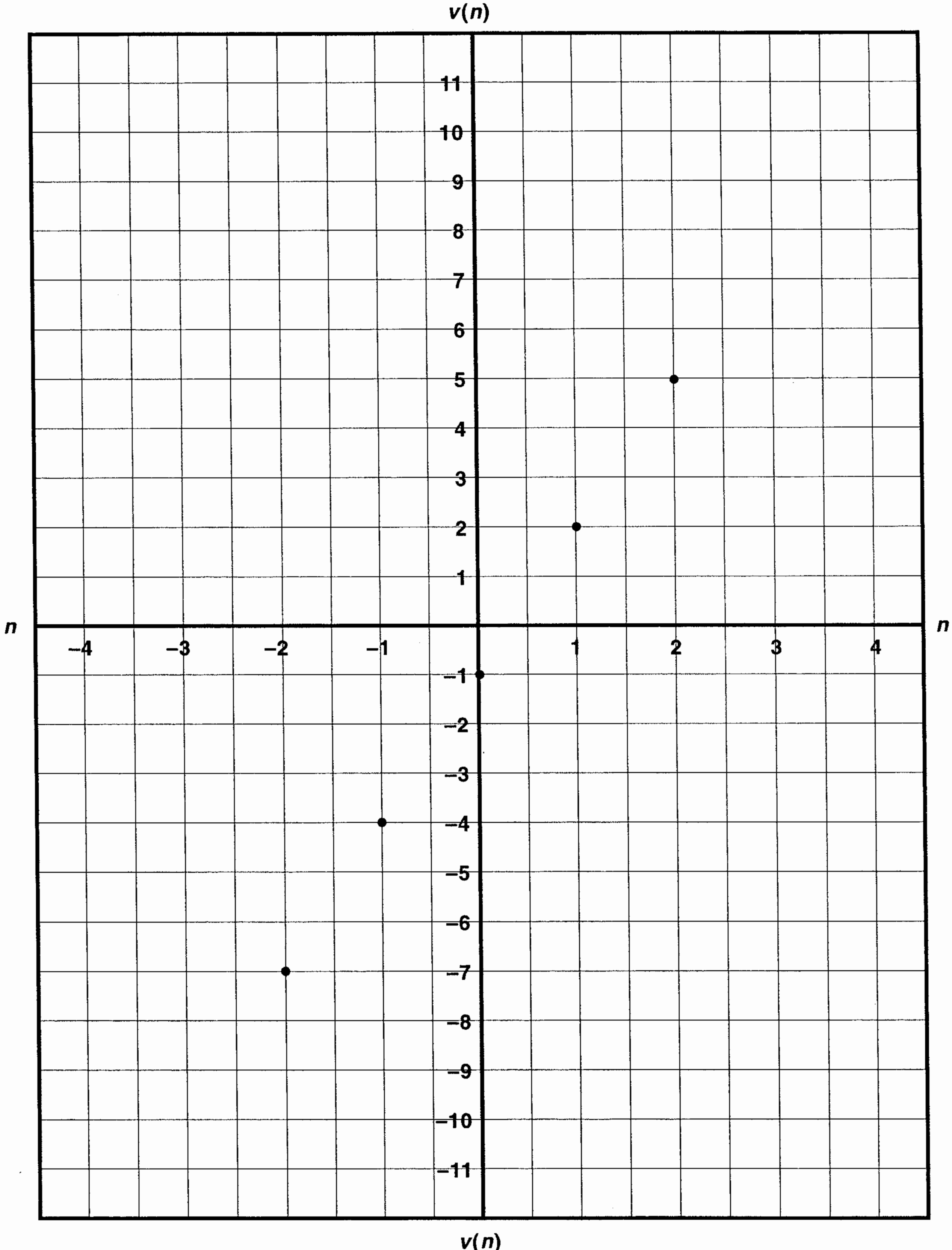


Two copies of 5th arrangement. Two copies of 4th arrangement.

The graphs are shown to the left. Notice that the points on the graph of $v_1(n)$ alternately lie on the line parallel to and 3 units above the n-axis and on the n-axis. The points on the graph of $v_2(n)$ alternately lie on two lines, one sloping upward and one sloping downward..

The only point that lies on both graphs is (5, 3).

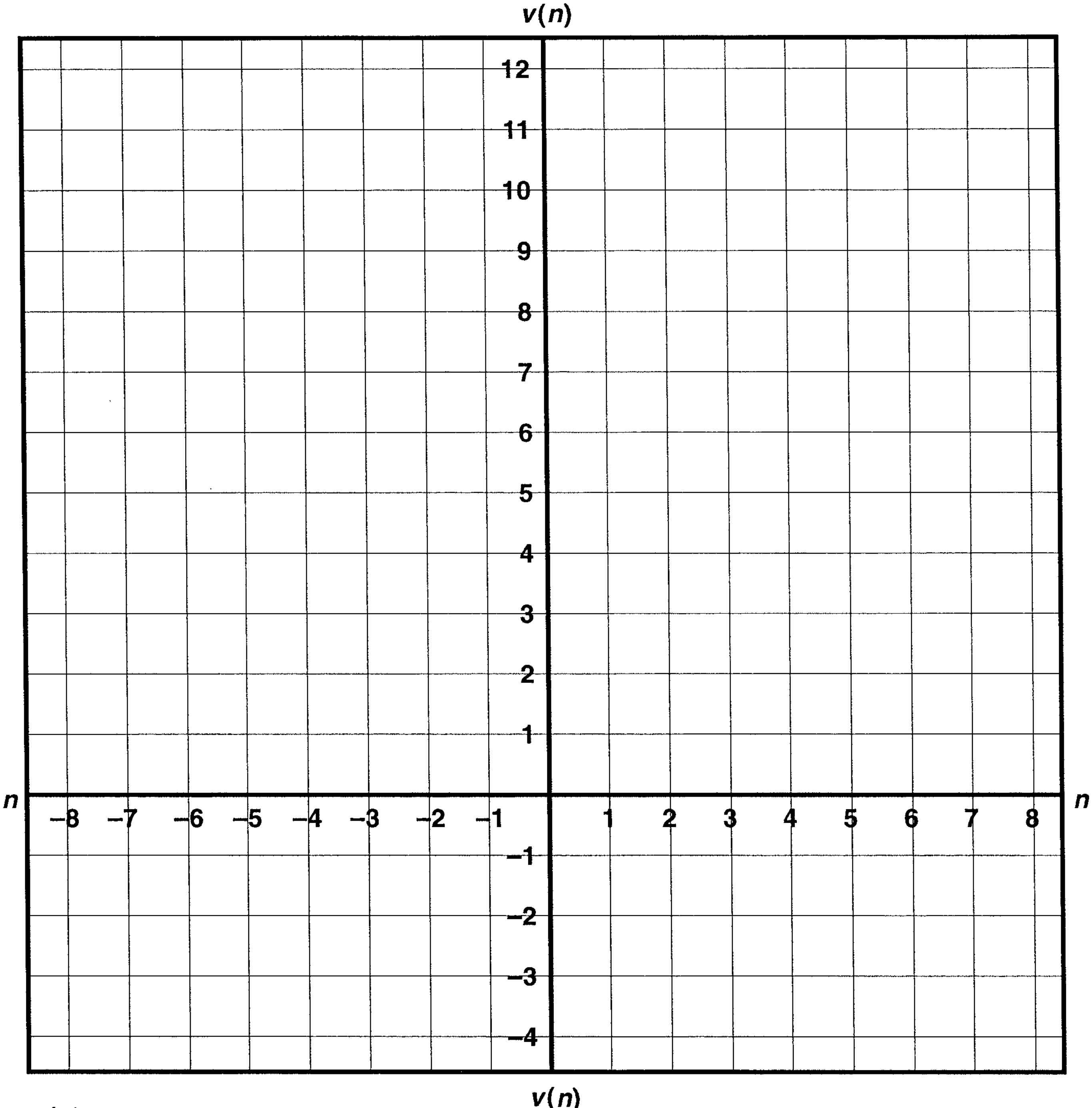
Name ______________________________



$v(n) =$

Activity Sheet XI-1-B

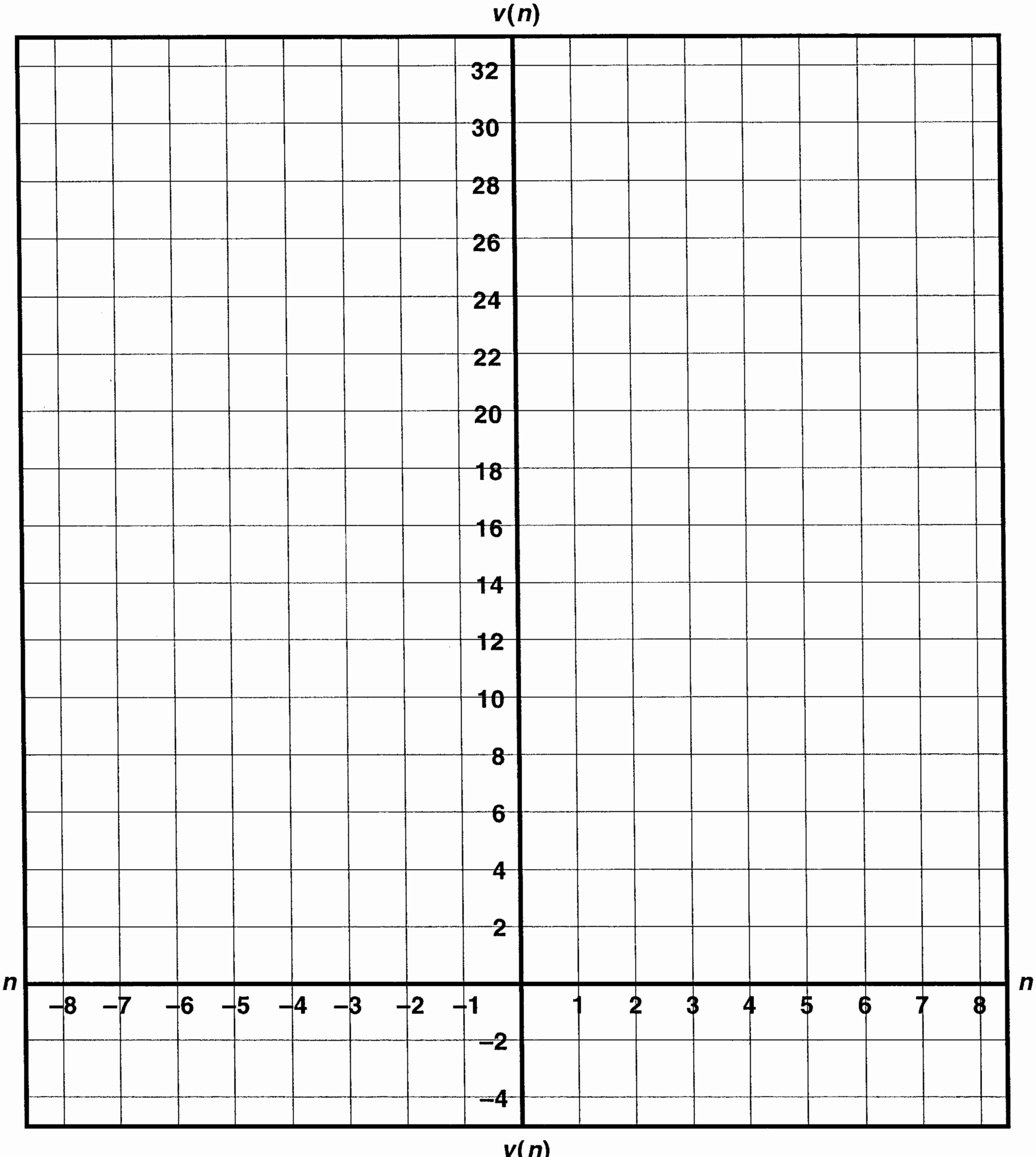
Name ______________________________



$v(n) =$

Observations about the graph:

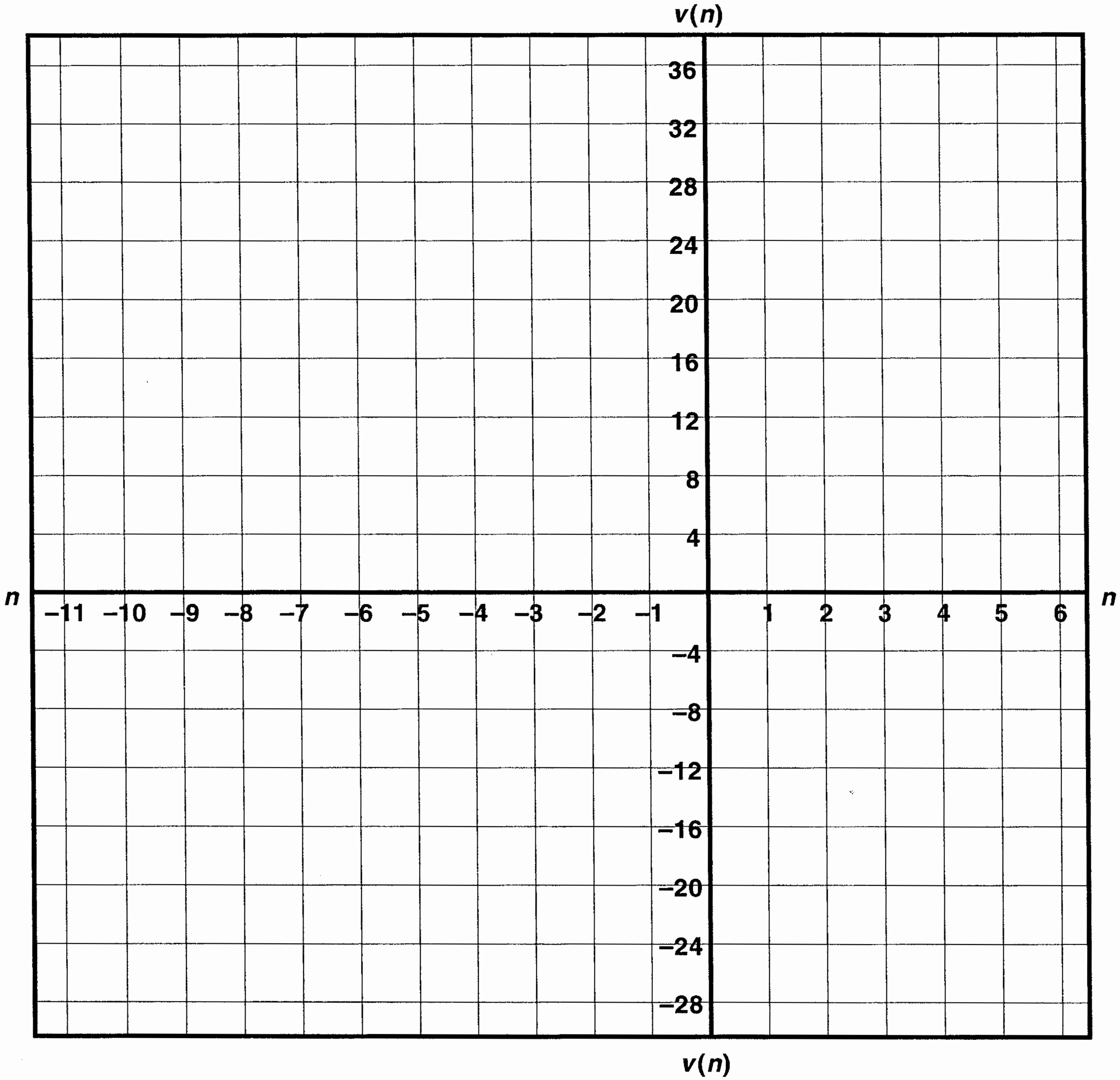
Name ______________________________



$v(n) =$

Observations about the graph:

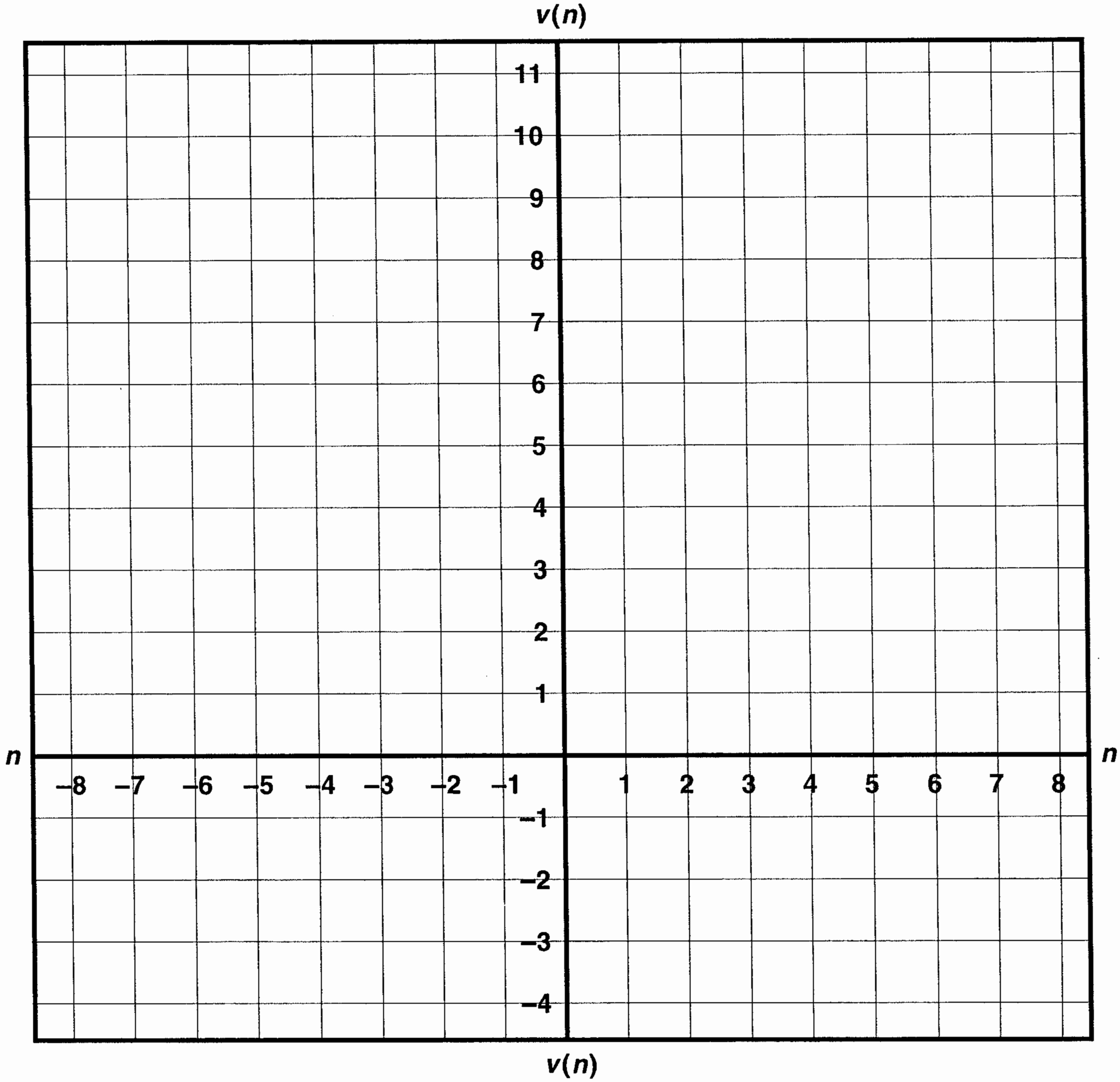
Name ______________________________



X $v_1(n) =$ O $v_2(n) =$

Observations:

Name ______________________________

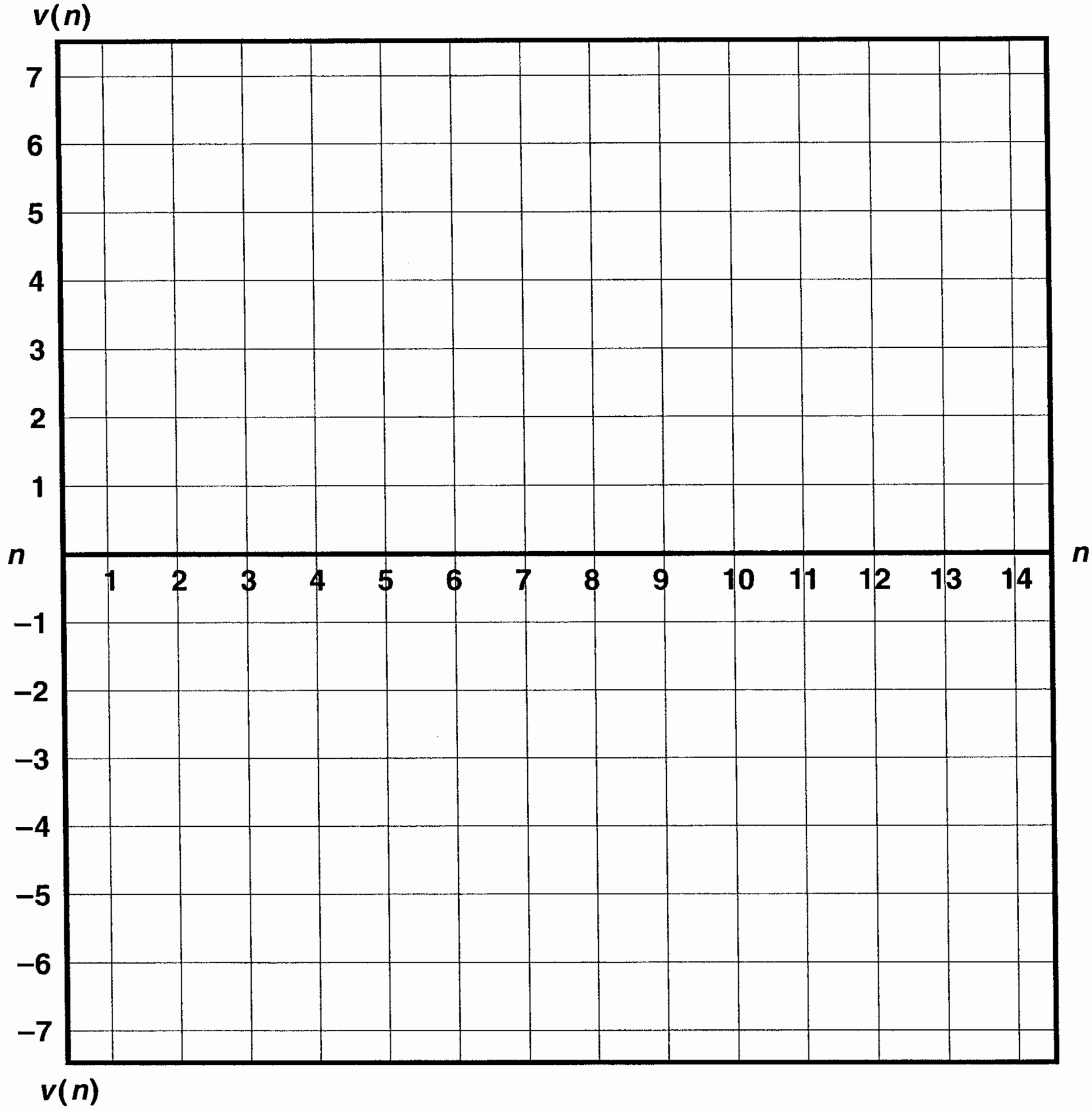


X $v_1(n)$ =

O $v_2(n)$ =

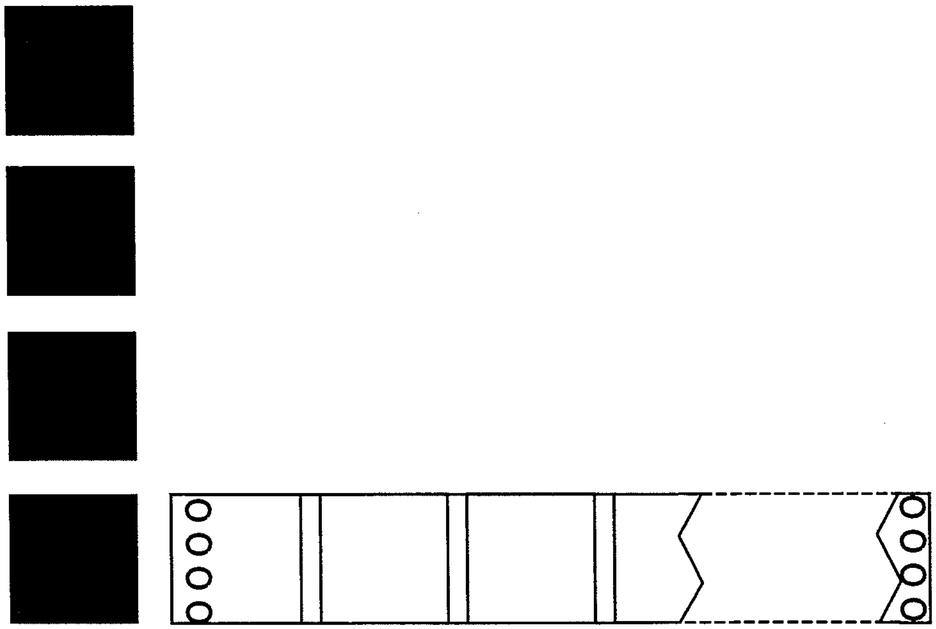
Observations:

Name ______________________________



X $v_1(n) =$ O $v_2(n) =$

Observations:



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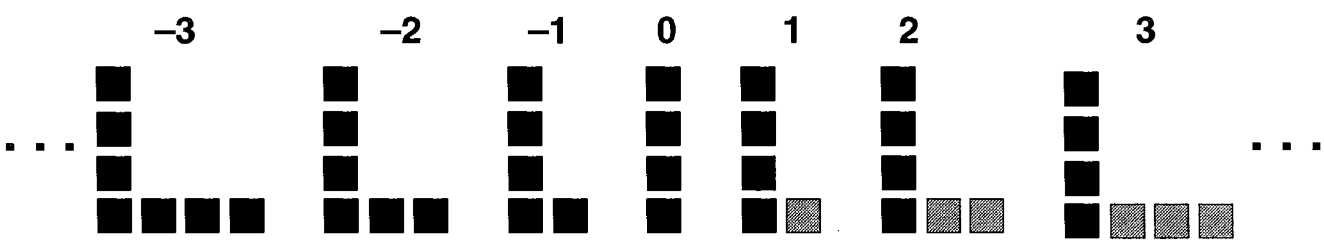

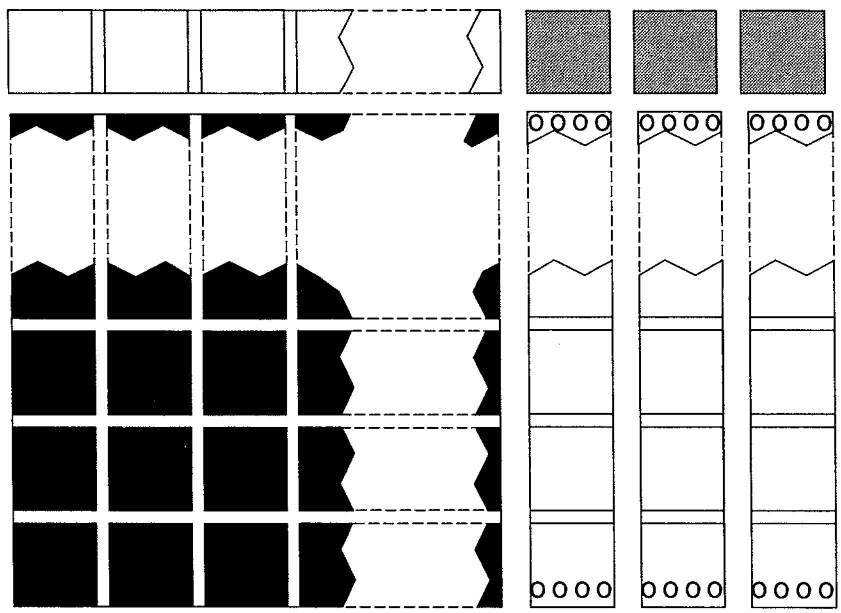

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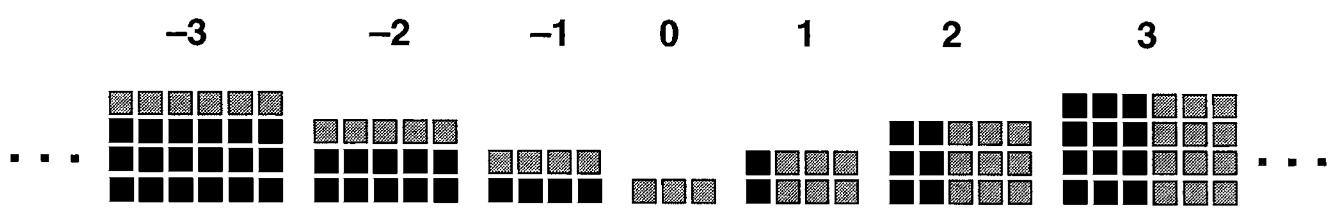

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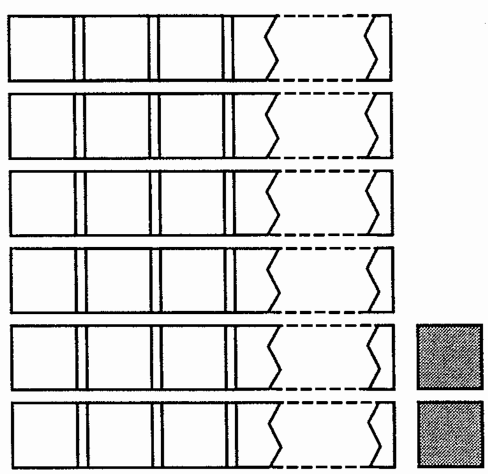
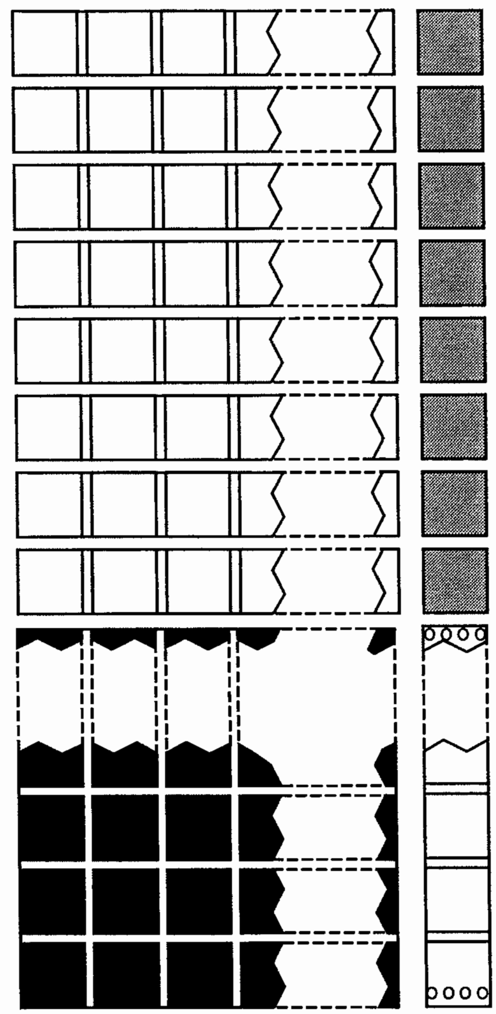


II



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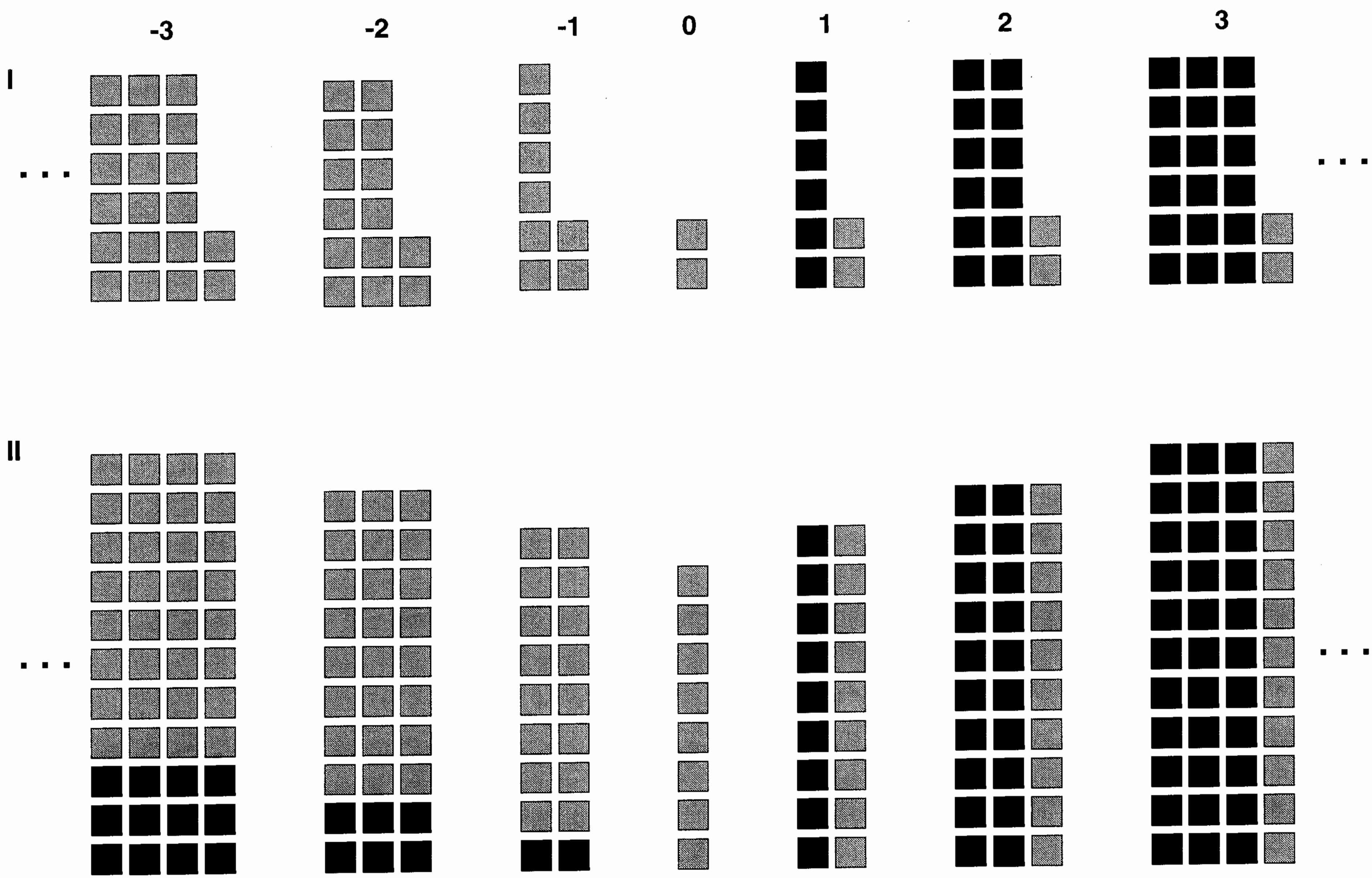

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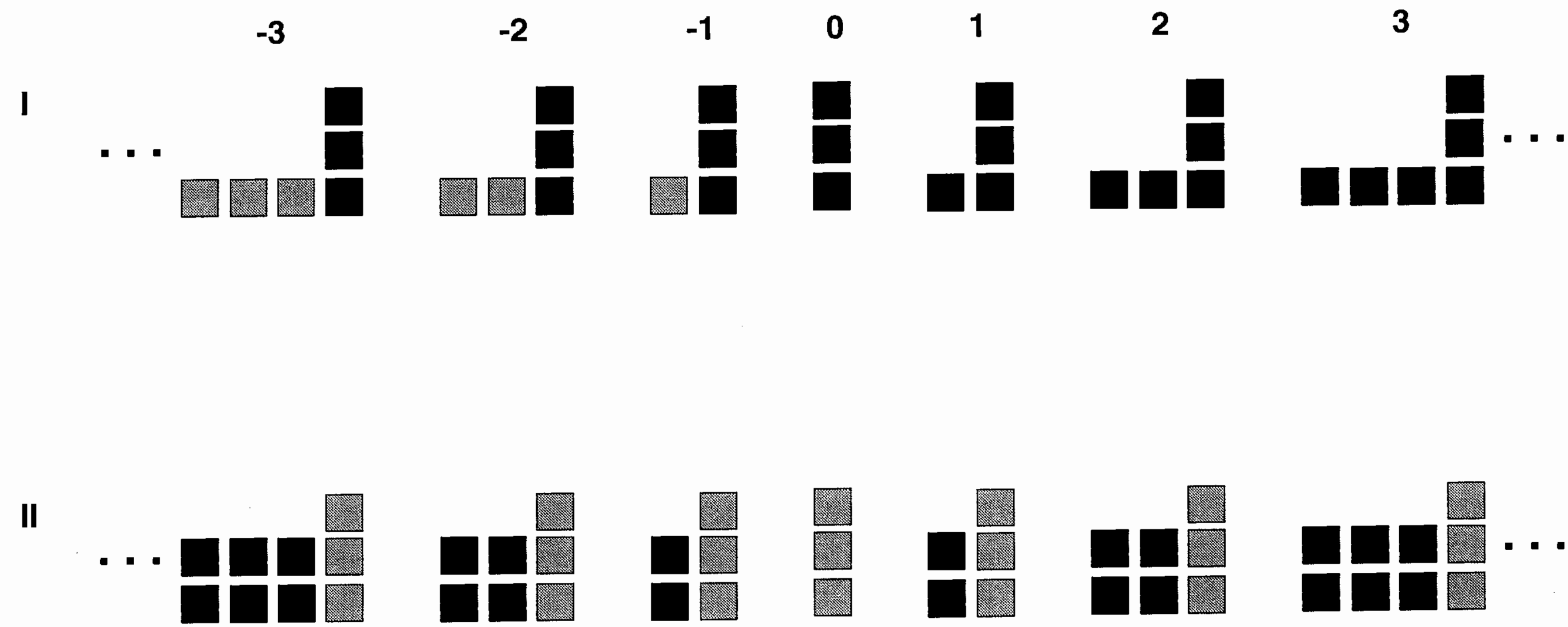
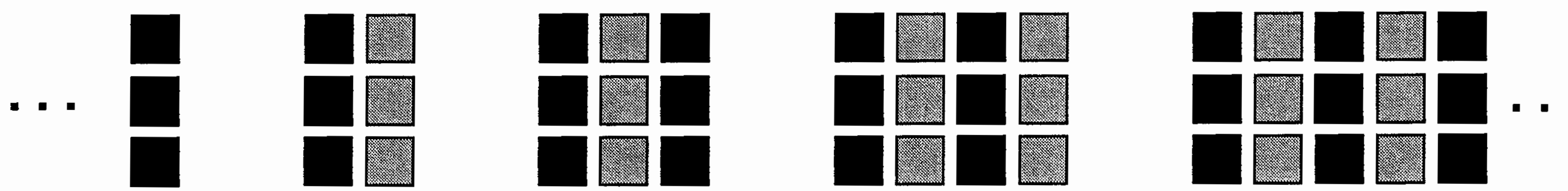


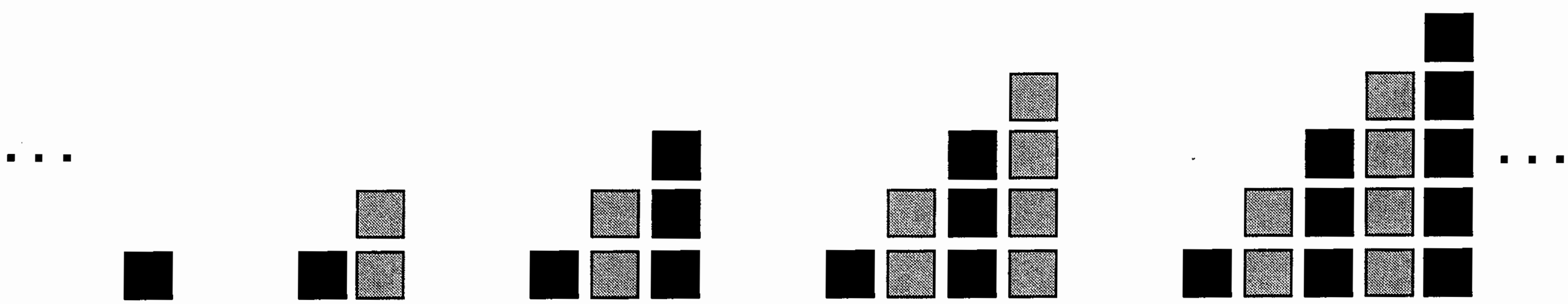
Arrangement number:

1 2 3 4 5

I



II



An Introduction to Graphs, Part II

O V E R V I E W

Extended sequences of arrangements are augmented so their graphs become continuous.

Prerequisite Activity

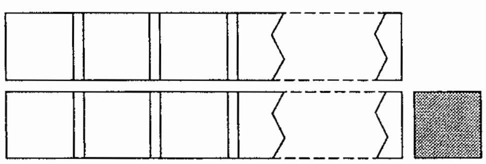
Unit XI, Activity 1, *An Introduction to Graphs, Part I.*

Materials

Bicolored counting pieces, Algebra Pieces, n-frames, centimeter grid paper, masters and activity sheet. Also, red and black pens or markers, if available (see Comment 1).

Actions

1. Distribute centimeter grid paper to the students. Show them the following Algebra Piece arrangement. Tell them it is the nth arrangement of an extended sequence of tile arrangements. Ask them to sketch the –3rd through 3rd arrangements of the sequence on a sheet of centimeter grid paper, representing tile by grid squares.



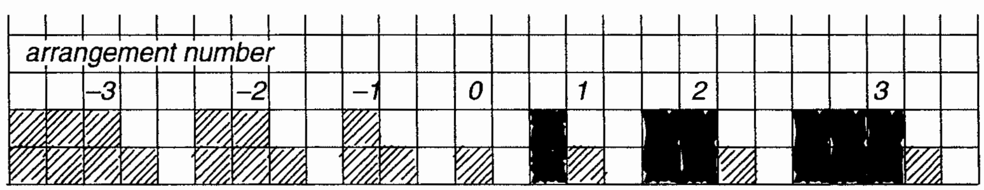
Comments

1. A master for centimeter grid paper is attached. Also attached is a master for an overhead transparency of the arrangement (Master 1, top half). The bottom half of the transparency is used in Action 7.

It is intended that the students draw grid paper sketches of the arrangements. If students use tile to form the arrangements, ask them to also draw sketches.

If the grid paper is oriented so the horizontal dimension is the longest, the –3rd through 3rd arrangements can be sketched side by side.

The students can use red and black pens or markers, if available, to fill in squares to represent red and black tile. Otherwise, they can devise ways of indicating red and black tile. In the sketch below, the lighter-hatched squares represent red tile.



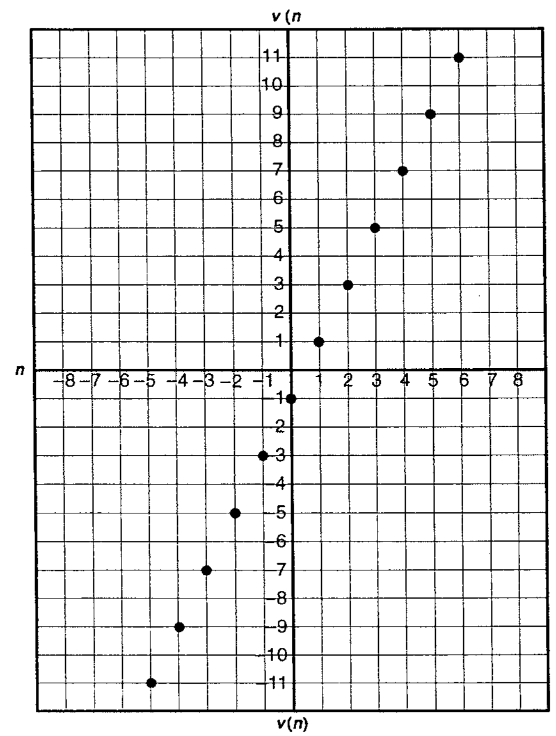
centimeter grid

Actions

2. Distribute Activity Sheet XI-2. For the extended sequence introduced in Action 1, ask the students to record a formula for $v(n)$ in the space provided and then construct its graph.

Comments

2. Make several overheads of Activity Sheet XI-2. The completed graph is shown below.

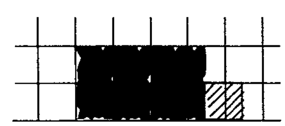


$v(n) = 2n - 1$

3. Point out to the students that there is no point on the graph when $n = 3\frac{1}{2}$ since there is no $3\frac{1}{2}$th arrangement. Tell the students to imagine that the sequence has been augmented to contain such an arrangement. Ask them to draw a grid paper sketch of how it might look. Have them compute the value of their arrangement and add the corresponding point to their graph. Repeat for $n = -2\frac{3}{4}$.

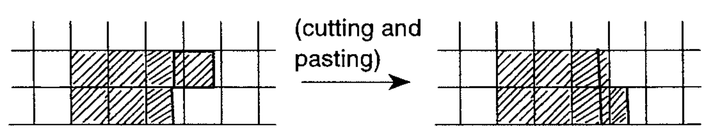
4. Have the students choose some non-integer point on the

3. Below is a sketch of a $3\frac{1}{2}$th arrangement based on the pattern of the arrangements in the original sequence. Its net value is 6. Thus, $(3\frac{1}{2}, 6)$ is the point on the graph corresponding to this arrangement.



$v(3\frac{1}{2}) = 6$

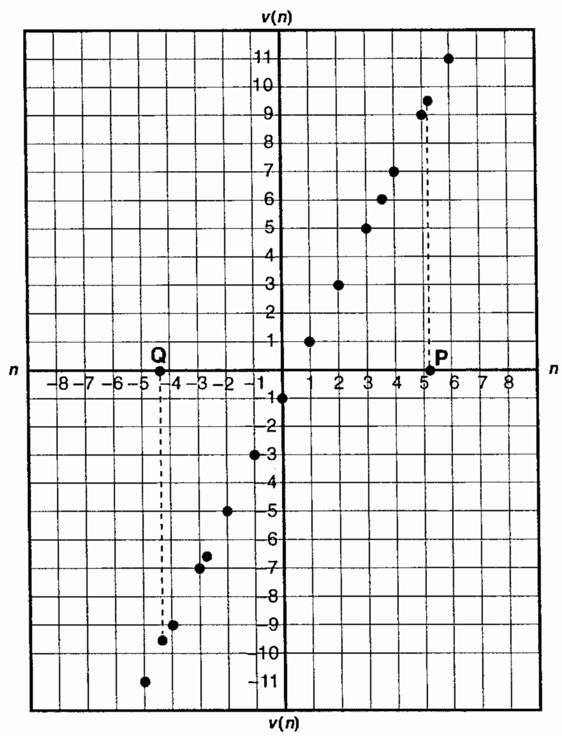
The net value of the $-2\frac{3}{4}$ arrangement, shown below, is $-6\frac{1}{2}$. Its corresponding point is $(-2\frac{3}{4}, -6\frac{1}{2})$. A sequence of sketches may help students see this.



$v(-2\frac{3}{4}) = 2(-2\frac{3}{4}) - 1$ $\quad$ $v(-2\frac{3}{4}) = -6\frac{1}{2}$

Actions

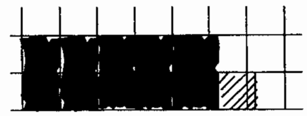
4. Have the students choose some non-integer point on the positive part of the n-axis and label it P. Then have them choose a non-integer point on the negative part of the n-axis and label it Q. Ask them to sketch Pth and Qth arrangements, determine their values and add the corresponding points to their graph. Ask for volunteers to show their sketches and discuss with the students how they determined the location of the points on the graph.



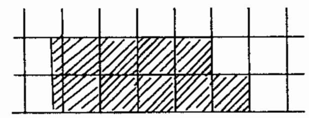
$v(n) = 2n - 1$

Comments

4. Shown below are sketches of the Pth and Qth arrangements for the choices of P and Q shown on the graph. The corresponding points on the graph are also shown.



$v(P) = 2P - 1$



$v(Q) = 2Q - 1$

The location of the points on the graph can be determined by measuring. For example, one can mark off on the edge of a piece of paper a segment whose length is the distance between 0 and P and adjoin to it a segment whose length $P - 1$. The sum of these two lengths will be the distance of the point $[P, v(P)]$ above the n-axis. Some of the students may locate the points by noting that all the points of the graph are colinear and locate $[P, v(P)]$ and $[Q, v(Q)]$ so that colinearity is maintained.

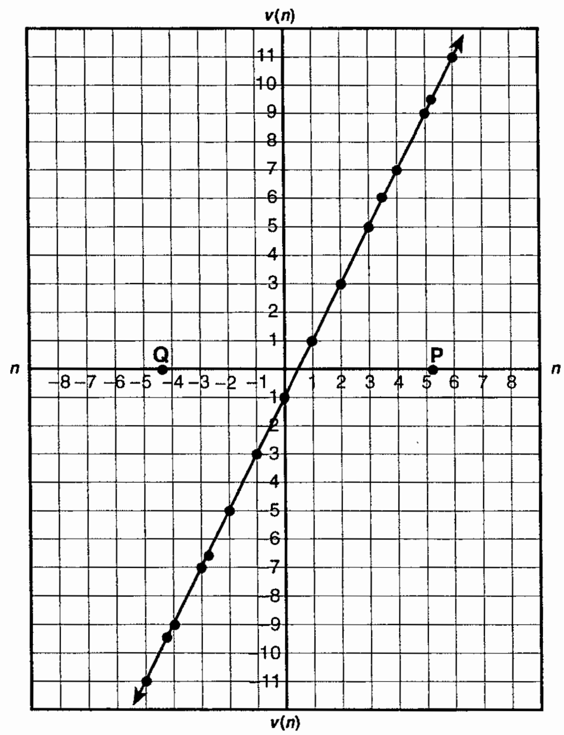
Actions

Comments

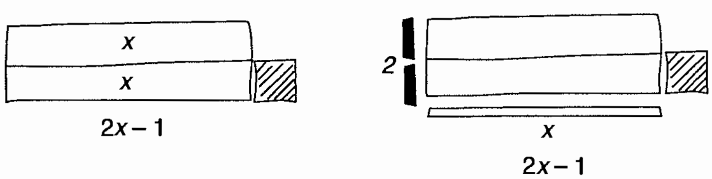
5. Ask the students to imagine that the sequence of arrangements has been augmented so there is an arrangement corresponding to every point on the n-axis. Ask them how they could show this on their graphs. Discuss.

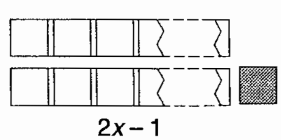
5. The resulting graph is a straight line, only a portion of which shows in the graph the students have constructed. The actual graph extends indefinitely in both directions.

A complete collection of arrangements—where there is an arrangement corresponding to every point on an axis, that is, every real number—will be referred to as a *continuum* of arrangements.



6. Suppose a generic point x of the n-axis is selected. Ask the students to create a representation of the xth arrangement. Discuss their representations.





6. The students may suggest a variety of ways to represent the xth arrangement. It might be represented by a sketch. The sketch on the left consists of two unshaded strips, each labeled x, and a single red tile. It is understood that each strip is to be filled with a collection of tile whose value equals its label. Thus, if x is positive, the strip is filled with black tile, if x is negative it is filled with red tile and if x is 0, it is empty. The sketch on the right uses edge pieces. Here the edge pieces have the value of their labels.

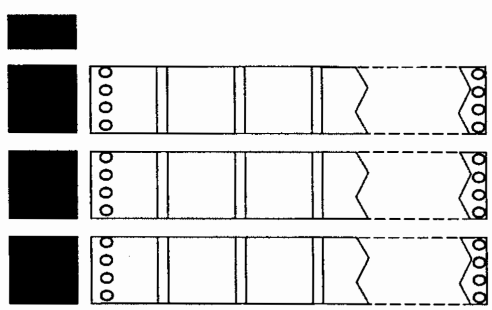
Alternatively, frames might be used to form a representation, as shown here. The frames are to be thought of as x-frames rather than n-frames, that is, each frame represents a strip of tile whose value is x rather than n.

Continued next page.

6. *Continued.* Henceforth, the letter x will be used to refer to a generic arrangement when the collection of arrangements under consideration is a continuum (real numbers). The letter n will be used when the collection of arrangements is a sequence or extended sequence (integers). In the former case, frames will be designated as x-frames or $-x$-frames and represent strips of tile whose value is x and $-x$, respectively. In the latter case, frames will be referred to as n-frames or $-n$-frames. It may help to call $-x$-frames, "opposite x-frames".

The above usage follows the customary, but not universal, practice of using letters like x, y and z to represent quantities that can take on any value, integral or not, (i. e., *continuous* variables) and using letters like k, m and n to represent quantities that have integral values (i. e., *discrete* variables). The choice of a letter to represent a generic arrangement is arbitrary. For example, one might refer to the zth arrangement and write $v(z) = 2z - 1$. In this case, if frames were used to represent the zth arrangement, they would be referred to as z-frames or $-z$-frames and have values z or $-z$, respectively.

7. Show the students the following xth arrangement from a continuum of arrangements. Ask them to write a formula for $v(x)$. Then distribute Coordinate Graph Paper to the students and have them construct a graph consisting of all points (x, y) such that $y = v(x)$.



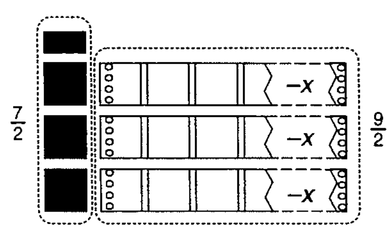
7. A master for an overhead transparency of the arrangement is attached (Master 1, bottom half). The partial tile is $\frac{1}{2}$ of a black tile—you may want to clarify this for the students—so one has $y = v(x) = \frac{7}{2} - 3x$.

Continued next page.

Actions

8. If (x, y) is on the graph of Action 7, ask the students to find x if y is:

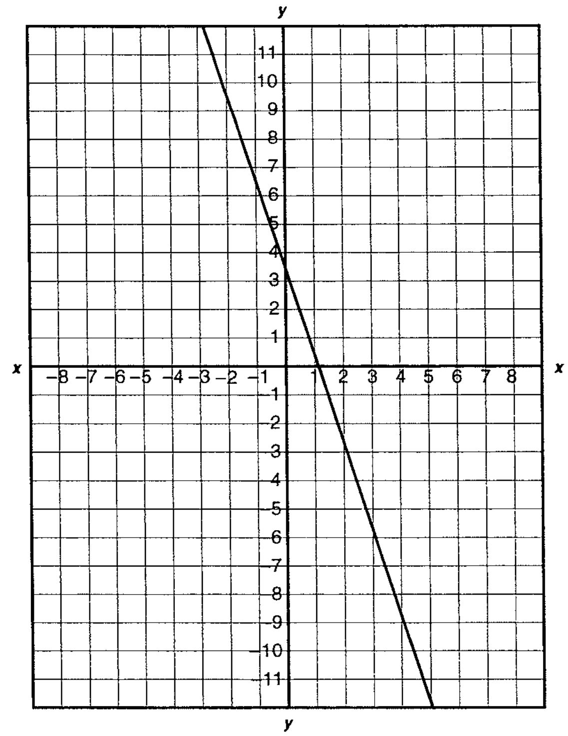
(a) 8, (b) –9, (c) –14.4, (d) 100.



$v(x) = 8 = \frac{16}{2}$

Comments

7. *Continued.* In the graph shown below the vertical axis is labeled y. It could also be labeled $v(x)$.



$y = \frac{7}{2} - 3x$

8. (a) The students might see from their graph that there is exactly one point (x, y) for which $y = 8$. For that point, one sees (if the graph is constructed carefully enough) that $x = -1.5$.

Alternatively, the students can use an Algebra Piece sketch to determine x. If the value of the xth arrangement shown here is 8, then the $-x$-frames must have a total value of $8 - {}^{7}/_{2}$ or ${}^{9}/_{2}$. Since there are 3 of them, each $-x$-frame has a value of ${}^{3}/_{2}$. Hence, the value of x is $-{}^{3}/_{2}$. At this point, students still may want to use pieces or sketches to find the value of an x-strip. Or, they may be doing it mentally and/or symbolically. We need to keep making connections between symbols and pictures.

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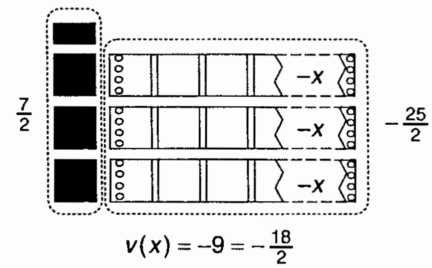
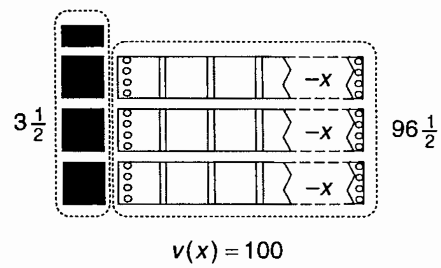




8. *Continued.*

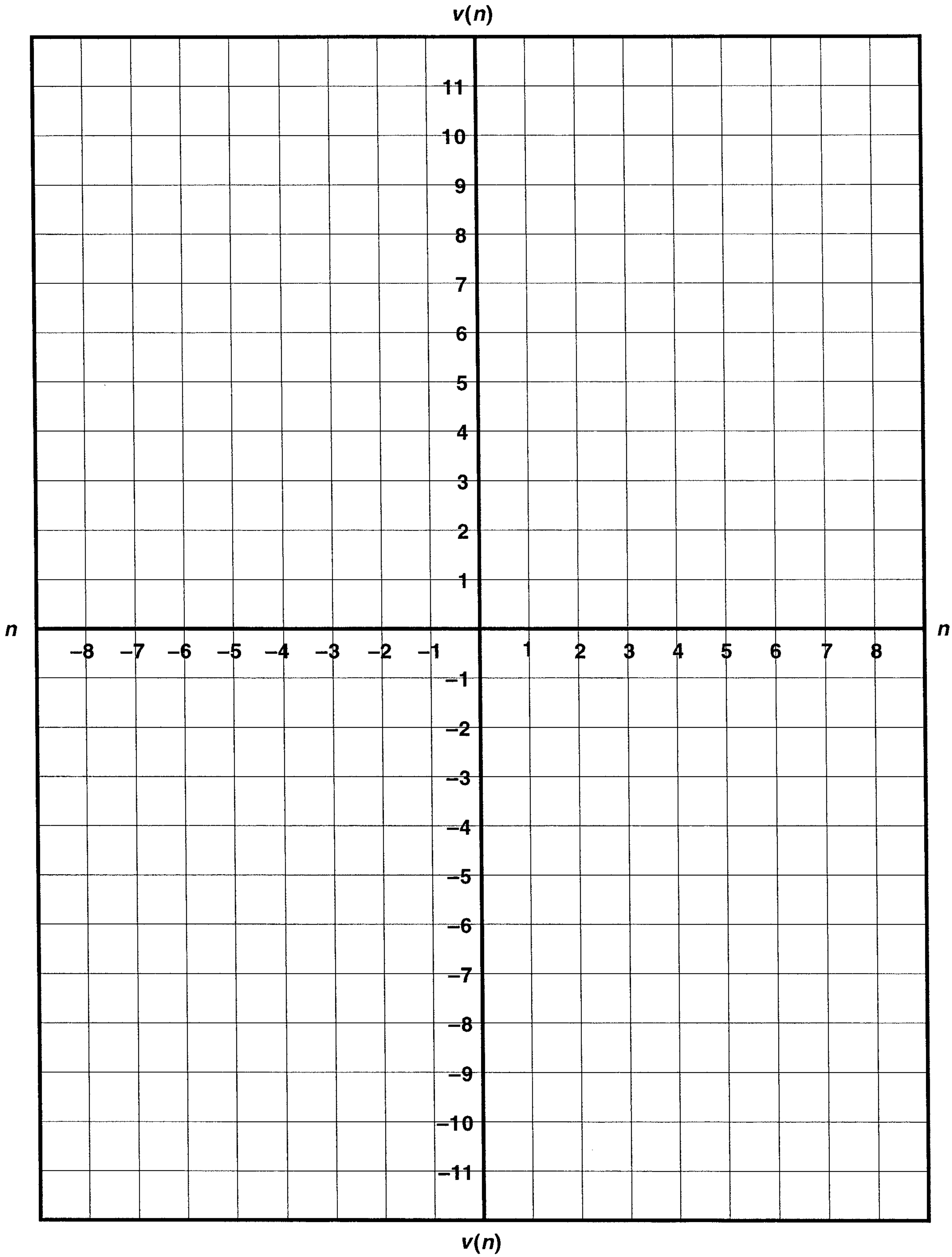
(b) If $y = -9$, it is difficult to determine the exact value of x from the graph. Referring to a picture of the xth arrangement, one sees that if its value is -9, the $-x$-frames have a total value of $-9 - 7/2$ or $-25/2$. Since there are 3 of them, each $-x$-frame has a value of $-25/6$. Hence, $x = 25/6$.

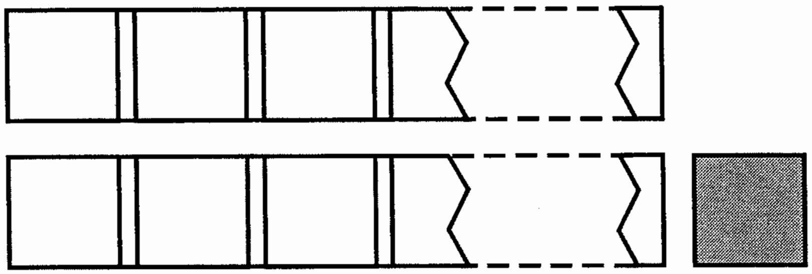
At this point, the power of using the symbols, versus tiles or graphs, to get exact values is evident.

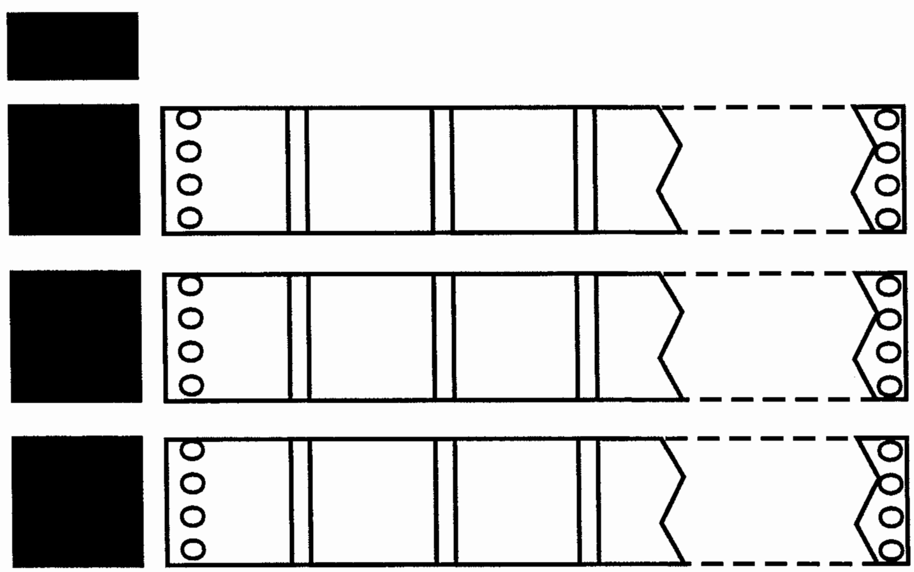
(c) Proceeding as in (b), one finds $x = 5.9$.

(d) From a picture of the xth arrangement, one sees that if its value is 100, the total value of the $-x$-frames is $96\frac{1}{2}$. Since $96 \div 3 = 32$ and $\frac{1}{2} \div 3 = \frac{1}{6}$, each $-x$-frame has value $32\frac{1}{6}$. Hence $x = -32\frac{1}{6}$.

Name ____________________ Activity Sheet XI-2







1-cm grid paper

An Introduction to Graphs, Part III

OVERVIEW

Further investigations with continua of arrangements.

Prerequisite Activity

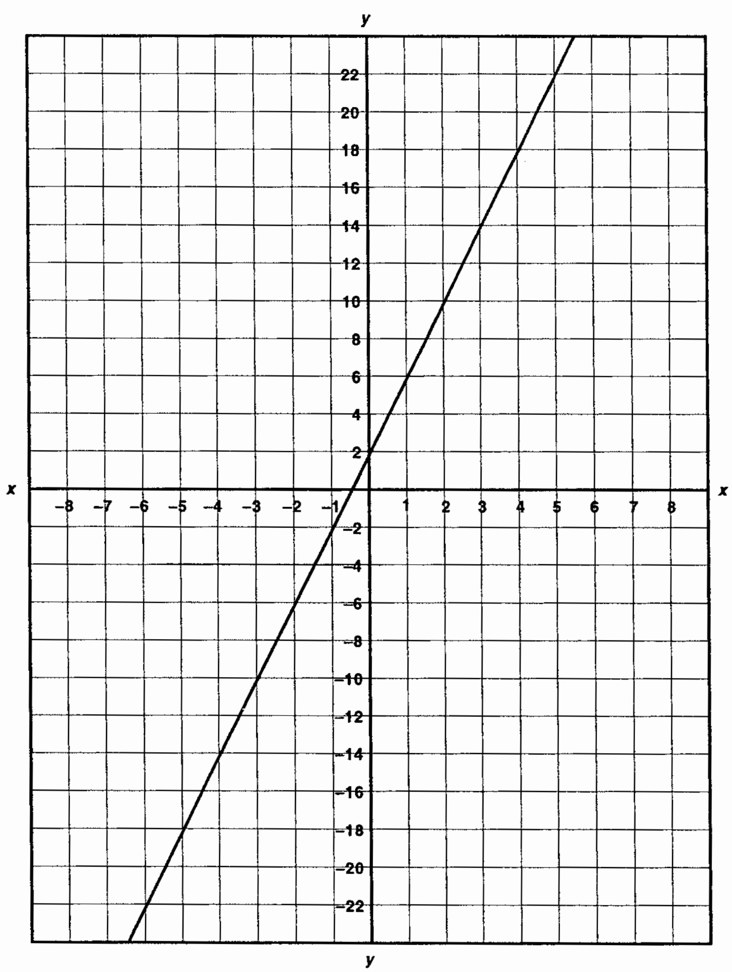
Unit XI, Activity 2, *An Introduction to Graphs, Part II.*

Materials

Graph paper, straight edges for drawing lines, Algebra Pieces, and master.

Actions

1. Show the students the following graph. Tell them it is the graph of $y = v(x)$ for a certain continuum of arrangements. Ask them to find a formula for $v(x)$ and to either sketch or construct an Algebra Piece representation of the xth arrangement. Discuss.



Comments

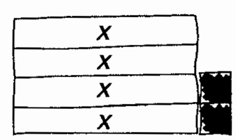
1. A master for the graph (Master 1) is attached from which an overhead transparency and/or copies for the students can be made.

The graph shows that $v(x)$ increases by 4 as x increases by 1. Thus $v(1) = 4 + v(0)$, $v(2) = 8 + v(0)$, $v(3) = 12 + v(0)$ and so forth. Since $v(0) = 2$, this suggests the formula $v(n) = 4x + 2$ which can be verified for other points on the graph. If the expression $v(x)$ is represented by y, the formula might be written $y = 4x + 2$.

The students may arrive at this result by other methods.

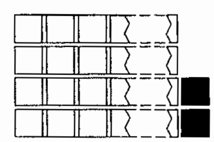
A sketch of the xth arrangement is shown on the left below. An Algebra Piece arrangement is shown on the right.

sketch:



$v(x) = 4x + 2$

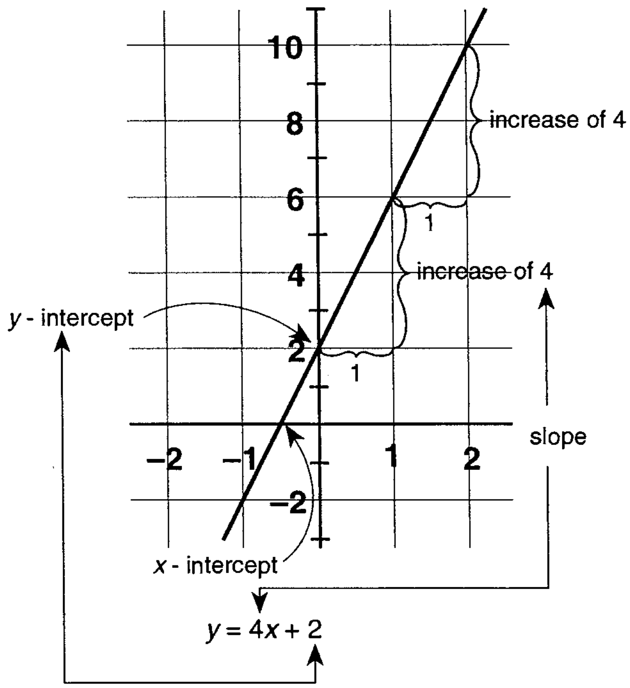
pieces:



$v(x) = 4x + 2$

Actions

2. Discuss with the students how the numerical constants, 4 and 2, in the formula for $v(x)$ relate to its graph. Introduce the terms *coefficient, slope* and *intercept*.



3. Distribute centimeter grid paper to the students. Tell them that the graph of $v(x)$ for a certain continuum of tile arrangements is a straight line which passes through the points (–2, 10) and (4, -8). Ask the students to draw the graph and find a formula for $v(x)$. Then have them construct an Algebra Piece representation or draw a sketch of the xth arrangement. Discuss.

Comments

2. You can begin the discussion by asking the students to share their ideas with one another.

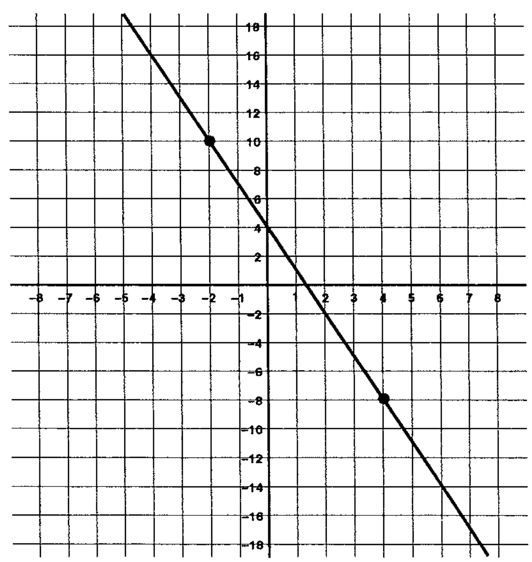
The constant 4 in the product $4x$ — called the *coefficient of* x— tells how much y values change as x values increase by 1 (for example, as x changes from 0 to 1, y changes from 2 to 6, an increase of 4). This rate of change, the change in y for each unit increase in x, is called the *slope* of the line. (Note that if y had decreased as x increased, the change, and hence the slope, would have been negative.)

The constant 2 is the value of y when x is 0 or, to put it another way, the place where the line crosses the y-axis. This value is called the *y-intercept* of the line.

A line may also cross the x-axis. If it does, the value of x at which it crosses is called the *x-intercept.* In this case the x-intercept is $-\frac{1}{2}$.

3. A master for centimeter grid paper is attached to Activity XI-2, *An Introduction to Graphs, Part II.*

The students will have to locate axes and then scale them. In the graph shown, the x-axis is scaled so that each subdivision represents 1 unit and the y-axis is scaled so that each subdivision represents 2 units. The graph will appear differently for other scales.



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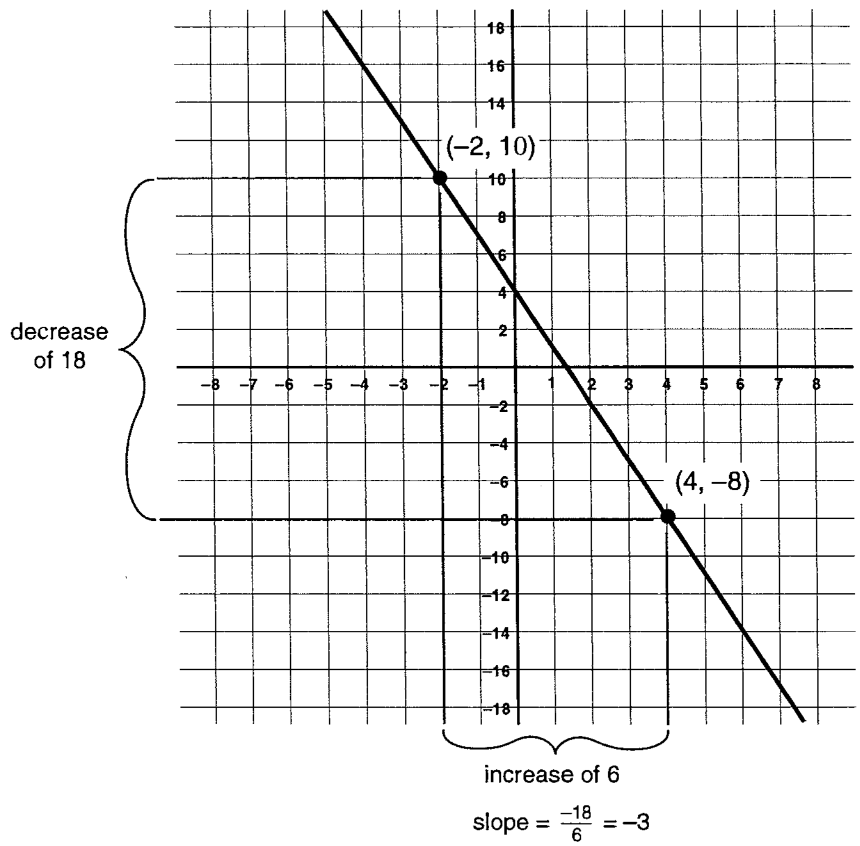
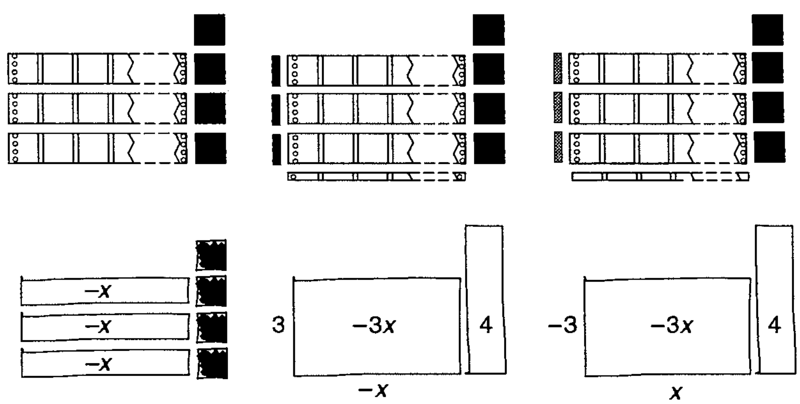
Actions





Comments

3. *(Continued)* If the points (–2, 10) and (4, –8) are located on a graph and a straightedge is used to draw a line through them, one sees that the *y*-intercept of the line is 4.

Also the slope of the line is –3 since *y* values decrease by 3 as *x* values increase by 1. One way to determine this is to note the *y* values decrease 18 units (from 10 to –8) as the *x* values increase 6 units (from –2 to 4), which is equivalent to a *y*-decrease of 3 units for every 1 unit *x*-increase.

Since the line has slope –3 and *y*-intercept 4, $y = v(x) = -3x + 4$. The students may use other methods to find a formula for $v(x)$.

Shown here are various sketches and Algebra Piece representations, some of which include edge pieces. In the sketches, a numeral alongside the edge of a rectangle denotes its value. Note that the value of an edge may differ from its length. If the value of an edge is positive, the value of the edge and its length are the same. If the value of an edge is negative, the value of the edge and its length are opposites. For example, if the value of an edge is –3, its length is 3.

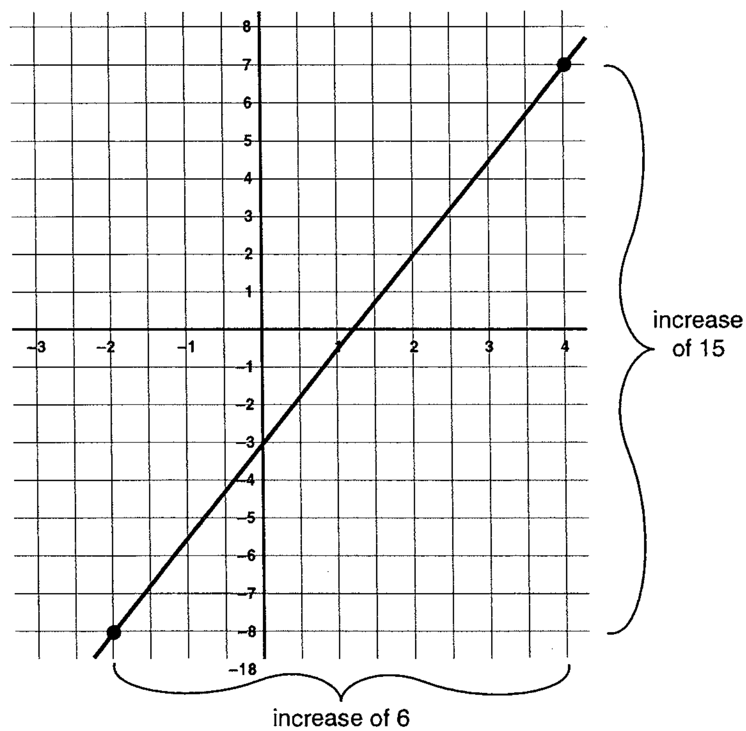
Actions

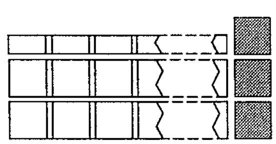
4. Repeat Action 3 for the points (–2, –8) and (4, 7).

Comments

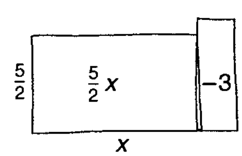
4. In the graph shown below, the x-axis is scaled so that each subdivision represents ½ unit and the y-axis is scaled so that each subdivision represents 1 unit. The graph's appearance will differ for other scales.

Using a straightedge to draw a line connecting the two given points on a graph with x and y axes, one sees that the y-intercept is –3. Also, y values increase by 15 as x values increase by 6. This is equivalent to a y-increase of $2\frac{1}{2}$ for each x-increase of 1. Thus, the line has slope $\frac{5}{2}$. Hence, $y = v(x) = \frac{5}{2}x - 3$. The students can verify this formula by showing that it provides the correct values for $v(-2)$ and $v(4)$, namely –8 and 7.





$v(x) = 2\frac{1}{2}x - 3$



$v(x) = \frac{5}{2}x - 3$

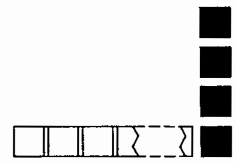
On the left is an Algebra Piece representation in which an n-frame has been cut in half. On the right is a sketch in which the values of edges and regions are shown.

Actions

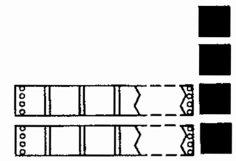
5. Ask the students to construct an Algebra Piece representation, or draw a sketch, of the xth arrangement of a continuum of arrrangements for which the graph of $v(x)$ is:
(a) a straight line whose y-intercept is 4,
(b) a straight line whose slope is 3,
(c) a straight line whose slope is –2 and y-intercept is 3.
Discuss.

Comments

5. (a) If the graph of $v(x)$ has y-intercept 4, then $v(0) = 4$. This will be the case if the "constant" part of the arrangements has value 4. Shown below are two possibilities.

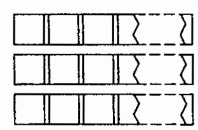


$v(x) = x + 4$

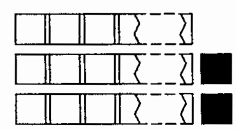


$v(x) = -2x + 4$

(b) If the graph of $v(x)$ is a straight line whose slope is 3, then the values of the arrangements must increase by 3 as x increases by 1. One possibility is that $v(0) = 0$, $v(1) = 3$, $v(2) = 6$, and so forth. This will be the case if $v(x) = 3x$. Other possibilities can be obtained by adding a constant to this expression, e.g., $v(x) = 3x + 2$. The difference in the graphs of these two expressions is that the y-intercept of the first is 0 and that of the second is 2.

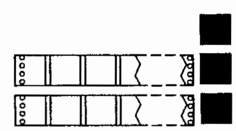


$v(x) = 3x$



$v(x) = 3x + 2$

(c) If the graph of $v(x)$ is a straight line and both the slope and y-intercept are given, there is only one possibility. In this case, $v(x) = -2x + 3$, or an equivalent expression.

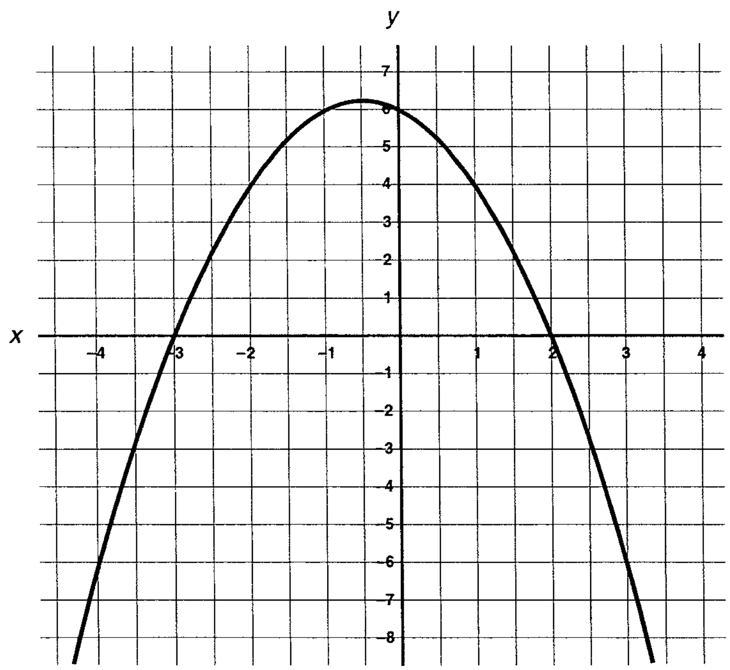


$v(x) = -2x + 3$

Actions

6. (a) Ask the students to make an Algebra Piece representation, or draw a sketch, of the xth arrangement of a continuum of arrrangements for which $v(x) = (2 - x)(3 + x)$.

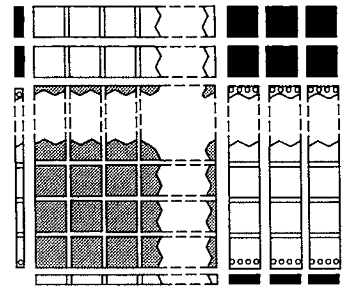
(b) Ask the students to predict what the graph of $v(x)$ looks like. Discuss their predictions and then ask them to draw the graph, starting with integer values for x. Ask the students for their observations.



$y = v(x) = (2 - x)(3 + x)$

Comments

6. (a) Shown below is an Algebra Piece representation, with edge pieces.



$v(x) = (2 - x)(3 + x)$

(b) Since $v(0) = 6$, the y-intercept is 6. The students may note that the x-intercepts are 2 and -3 since $v(x)$ is 0 for those values of x. If x is in the interval between the x-intercepts, both factors of $v(x)$ are positive and $v(x)$ is positive. Outside this interval, one factor of $v(x)$ is positive and the other is negative, so $v(x)$ is negative. An alternate form for $v(x)$ is $-x^2 - x + 6$, as can be seen from the above Algebra Piece representation. As the magnitude of x increases, in either direction, the squared term will dominate and $v(x)$ will have negative values which increase in magnitude.

In the graph of $y = v(x)$ shown here, every subdivision of the x-axis represents ½ unit and every subdivision of the y-axis represents 1 unit. The graph will look different for other scalings of the axes. The graph is a parabola symmetric about the vertical line $x = -½$ and opening downward.

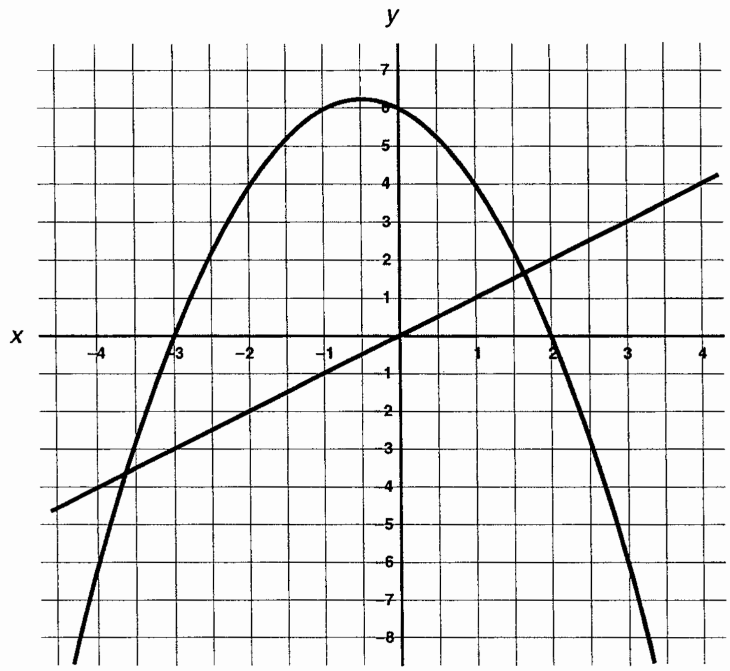
The students may find a few points on the graph and connect these points with straight line segments. You may want to point out to the students that graphs of formulas, such as the one in this Action, tend to be rounded rather than angular. This can be verified by finding more points on the graph.

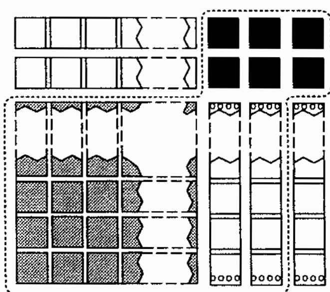
Actions

7. Ask the students to find where the graph of $v(x)$ intersects the line $y = x$.

Comments

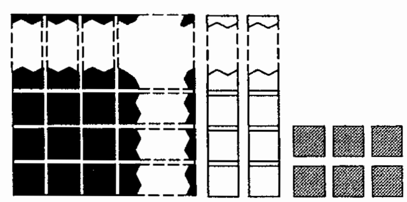
7. If $y = x$ and $y = (2 - x)(3 + x)$ are graphed on the same coordinate system, it appears that the two graphs intersect when x is about 1.7 or –3.7.

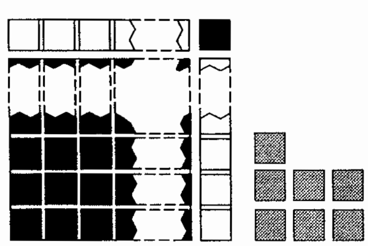




value of uncircled portion is x
value of circled portion is 0

The exact values of x where the two graphs intersect can be found by determining when $v(x) = x$. This will be the case if the value of the circled portion of the Algebra Piece representation for $v(x)$, shown here, is 0. If the value of the circled collection is 0, the value of its opposite collection, shown below, is also 0.





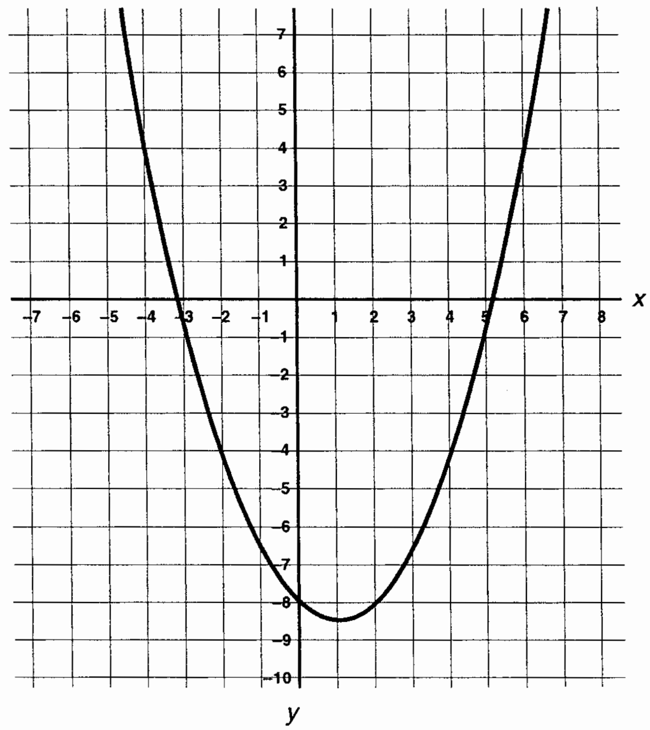
If a red and black tile is added to this collection, its value is unchanged. The resulting collection can be arranged into a square of edge $x + 1$ with 7 additional red tile. Since the total collection has value 0, the square must have value 7. Thus $x + 1$ equals $\sqrt{7}$ or $-\sqrt{7}$. Thus $x = -1 + \sqrt{7}$ or $x = -1 - \sqrt{7}$. Since $\sqrt{7} \approx 2.65$, the two graphs intersect when $x \approx 1.65$ or $x \approx -3.65$.

Actions

Comments

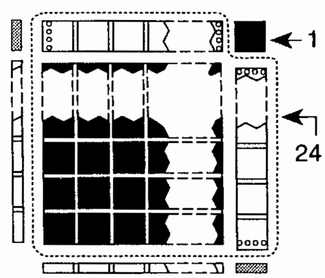
8. (a) Tell the students that, for the xth arrangement of a certain continuum of arrangements, $v(x) = {}^1/_2x^2 - x - 8$. Ask the students to graph $v(x)$.

8. (a) In the graph shown below, both the x- and y-axes are scaled so that 1 subdivision is 1 unit.



$v(x) = \frac{1}{2}x^2 - x - 8$

(b) Ask the students to find x if $v(x)$ is: (i) 4, (ii) –2.5

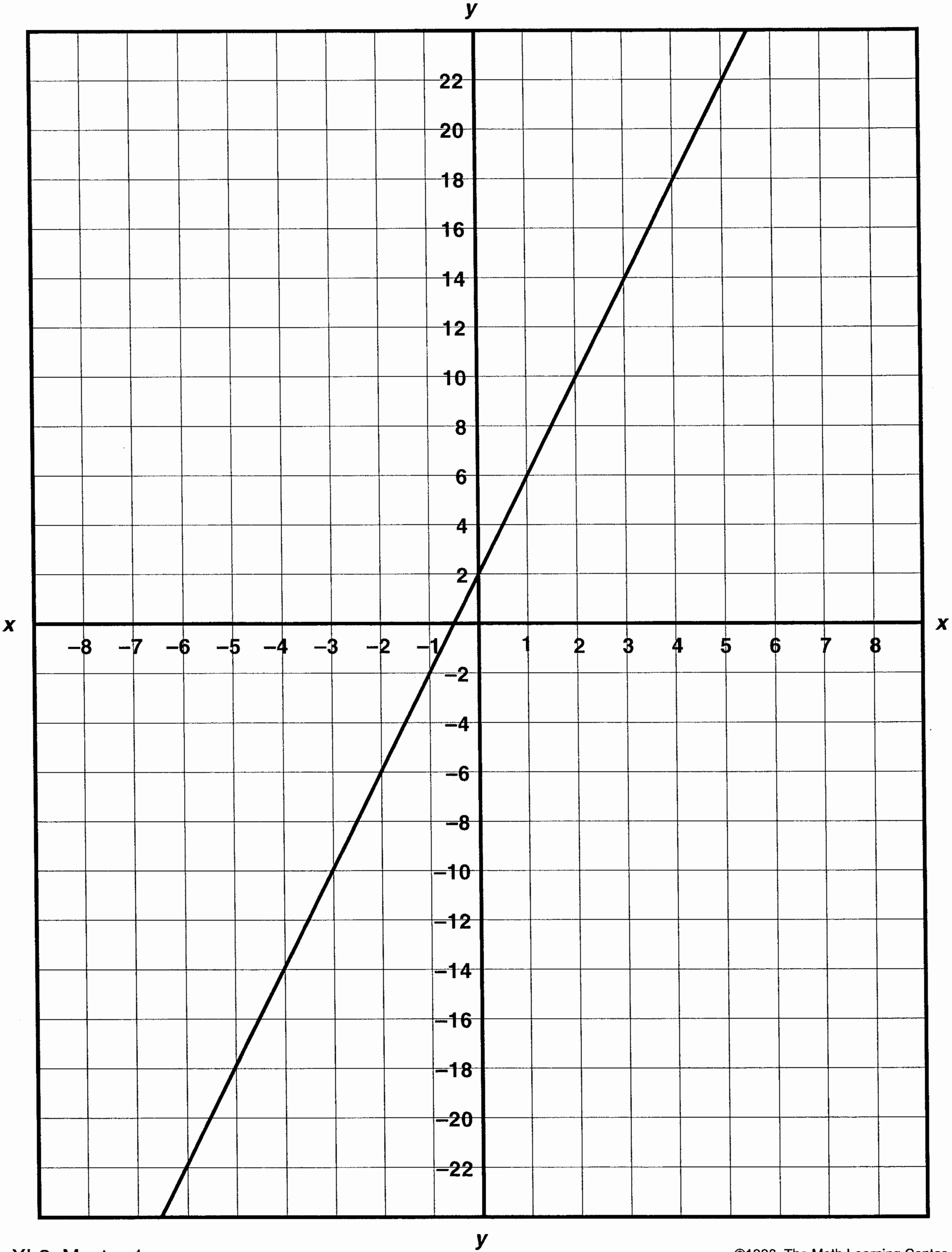


$x^2 - 2x = 24$;
$(x - 1)^2 = 25$

(b) If the students have drawn their graphs carefully, they can determine from them that $v(x)$ is 4 when x is –4 or 6.

This conclusion can also be reached by using Algebra Pieces. Because of the $\frac{1}{2}x^2$ in the expression for $v(x)$, it is easier to work with $2v(x)$. Thus, to determine when $v(x)$ is 4, one can determine when $2v(x)$ is 8, that is, when $x^2 - 2x - 16 = 8$. This will be the case if $x^2 - 2x = 24$ or, as shown, $(x - 1)^2 = 25$. Thus, $x - 1 = 5$ or $x - 1 = -5$. If $x - 1 = 5$, then $x = 6$; if $x - 1 = -5$, then $x = -4$.

When $v(x) = -2.5$, it appears from the graph that x is somewhere near 4.5 or –2.5. More precise values can be found by using the above method to find when $2v(x) = -5$. In this case, $x^2 - 2x = 16 - 5 = 11$ and the square shown in the sketch has value 12. Hence $x - 1$ has value $\sqrt{12}$ or $-\sqrt{12}$. Thus, $x = \sqrt{12} + 1$ or $-\sqrt{12} + 1$. Using a calculator, one finds $\sqrt{12} + 1 \approx 4.46$ and $-\sqrt{12} + 1 \approx -2.46$.


y
22
20
18
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14
12
10
8
6
4
2
x
−8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8
x
−2
−4
−6
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−10
−12
−14
−16
−18
−20
−22
y

Introduction to Graphing Calculators, Part I

O V E R V I E W

Graphing calculators are introduced to provide an alternative to the "by-hand" method of plotting graphs, and as a way to represent continua of arrangements in more detail. Connections between Algebra Piece, graphing, and symbolic representations of patterns are developed.

Prerequisite Activity

Unit XI, Activity 3, *An Introduction to Graphs, Part III.*

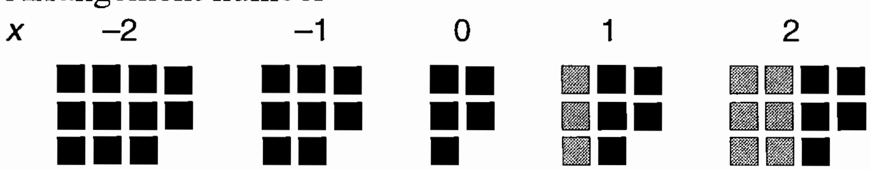
Materials

Bicolored counting pieces, Algebra Pieces, masters, a classroom set of graphing calculators, and an overhead graphing calculator that is compatible with the classroom set.

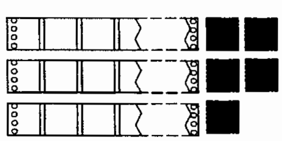
Actions

1. Pass out the bicolored counting pieces and the Algebra Pieces. Put up the formula $v(x) = -3x + 5$ on the overhead. Tell the students this formula represents a continuum of arrangements. Ask the students to use counting pieces to build arrangements for $x = -2, -1, 0, 1,$ and 2. Then ask the students to use the Algebra Pieces to build the *n*th arrangement for this expression. Ask for volunteers to share their results.

Arrangement number



*x*th arrangement



2. Ask the students to imagine in their mind's eye what the graph of $y = -3x + 5$ might look like. Ask for volunteers to share their ideas.

Comments

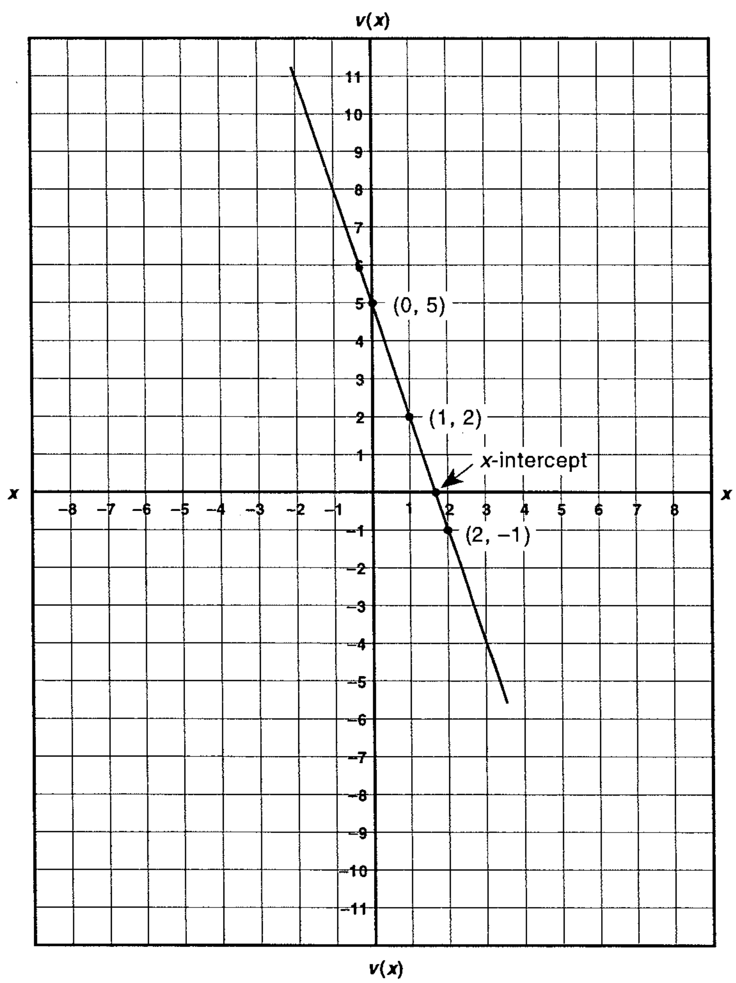
1. It is helpful to have two overhead projectors if possible, one for the overhead calculator, and one for student sharing and transparency masters. We start this activity with a counting piece and Algebra Piece representation, and then a by-hand graph, in order to build connections to the graphing calculator representation of the continuum of arrangements.

2. Students may say it's a line, that it drops—and perhaps they may trace it in the air as a falling line. They might also reference the slope of the line based on their experience in *An Introduction to Graphs, Part III.* Some may speak of "dropping three units for each unit one moves to the right."

Actions

3. Pass out a copy of Coordinate Graph Paper to each student. Ask the students to plot the graph $v(x) = -3x + 5$. Discuss the graph. Ask the students to determine the slope, and the y- and x-intercepts for this graph.

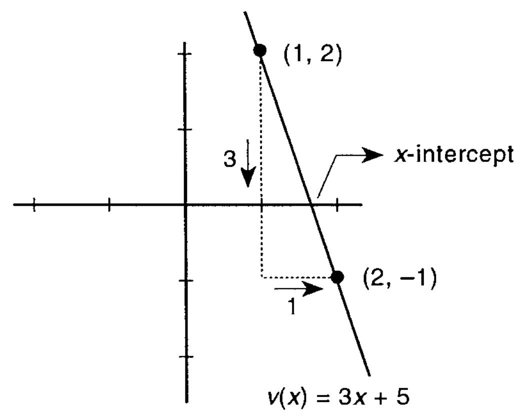
Graph of $v(x) = -3x + 5$



Comments

3. Ask for volunteers to show the graph. The problem of determining the value of the x-intercept may lead to various suggestions from the students. Some may approximate it: It is between 1 and 2. It is closer to 2. Some may suggest, based on the graph, that the line cuts the x-axis at the ⅔ mark between 1 and 2, since it has fallen 2 of the 3 vertical units from (1, 2) to (2, −1) on the graph. A transparency of Master 1 may be helpful. Some may suggest trying to find the value of x that will make $-3x + 5 = 0$. Some may even want to cut counting pieces to obtain an Algebra Piece representation.

Graph of $v(x) = -3x + 5$ "blown up"



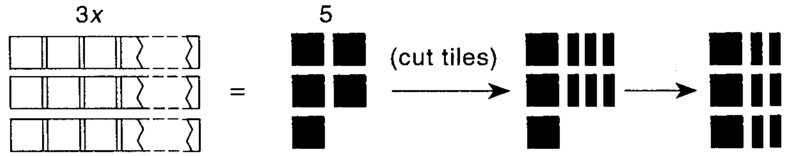
Continued next page.

Actions

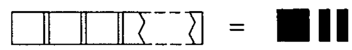
Comments

3. *Continued*

Algebra Piece representation:
The x-intercept occurs when $v(x) = -3x + 5 = 0$, or when $3x = 5$.



So $x = 1\frac{2}{3}$



4. Pass out the graphing calculators. Use the overhead calculator to help show the students how to turn the calculators on, and how to clear out or turn off any previous equations or graphs that may reside in the calculator's memory.

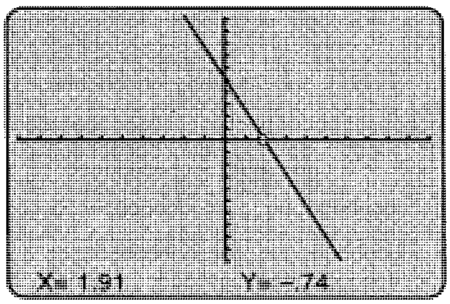
(Optional) You might ask students to graph a blank window (just the coordinate axes) on their calculator, without any equation entered. Ask them to move the cursor around the graphing window, and to try to determine the value of the tick marks on their particular set of axes.

4. An acetate master of your calculator keyboard, enlarged, is very helpful when working with students who are learning to use graphing calculators.

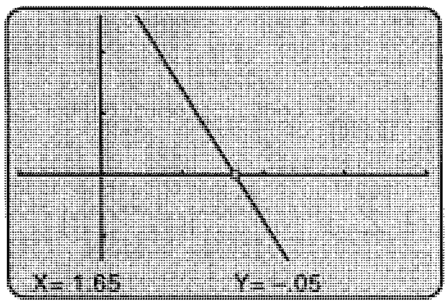
Note: When using a classroom set of graphing calculators there may be many different viewing windows (ranges) left open from the last group of students who used the devices. There also may be other graphs stored in the machine that are still turned on—functions or even statistical plots. This can lead to many different pictures showing up on your students' viewing windows. You can let some of these things just happen, and allow the students to discover them, or you can show the class how to turn off all other graphs, and/or show them how to standardize their viewing windows. This choice will depend on what you are most comfortable with. In any case it is normal for things to be a bit chaotic when students are first learning to use graphing calculators. The students try out all sorts of things on the calculators.

Actions

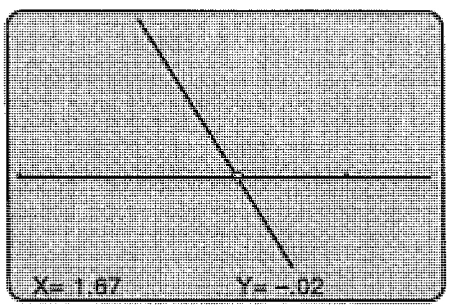
5. Ask the students to enter the expression $y = -3x + 5$ on their graphing utility, and then to GRAPH it in the "standard" viewing window. Ask them to use the TRACE function to find the x-intercept of this graph. What do they see when they trace the graph? Ask the students to zoom-in several times on this graph, and then retrace the new graph. Discuss. How does the value of the x-intercept they obtained on the calculator compare with the value they obtained in Action 3?



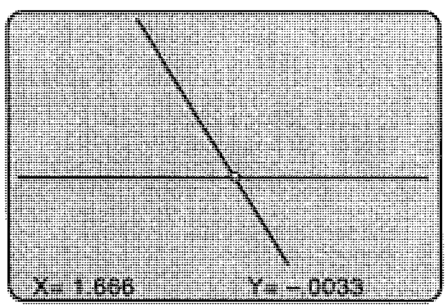
First TRACE



Second TRACE after ZOOM-IN



Third TRACE after ZOOM-IN



Fourth TRACE after ZOOM-IN

Comments

5. Ask for a student volunteer to create the graph on the overhead calculator. On many graphing calculators there is a standard default viewing window, such as [–10, 10] for y, and [–10, 10] for x.

The students can become familiar with many of the features of their graphing utility as they work through a particular example, like this one. The type and duration of instruction needed with a particular kind of graphing utility will vary depending on your machine and on your students' previous experience.

Most graphing utilities have features like GRAPH, TRACE, ZOOM, RANGE, or WINDOW, and so forth. The manual that accompanies your particular kind of calculator will provide a number of examples so you and for students can experiment with these features. To get started, students will need to know how to enter an equation, and how to get a calculator to display a graph. Many calculators have a "Y=" button to enter an equation, a special "X" button for a variable, and a GRAPH button to accomplish these tasks.

Students can experiment with ZOOM-IN and ZOOM-OUT and other ZOOM features on their calculator. Some calculators have a BOXING feature which can be used to draw a box around a particular part of the graph, and then ZOOM-IN on the box, as below. They will find that the calculator can give a very close "approximation" for the x-intercept, but not necessarily an exact value. A series of TRACES and ZOOM-INS for $y = -3x + 5$ is given in Figure 1. Each time we ZOOM-IN we obtain a closer approximation to the x-intercept value, where the graph crosses the x-axis.

Actions

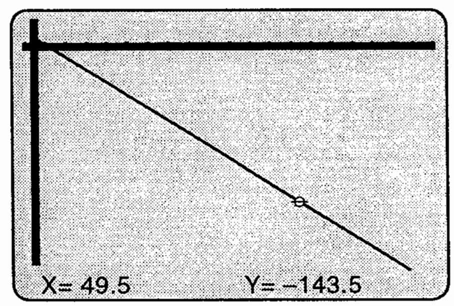
6. Ask the students to use their graphing calculators to find the value of y when x is 49 in $y = -3x + 5$. Repeat when x is -28.75. Then ask the students to find these y-values using the Algebra Pieces or symbolically. Discuss. Ask for volunteers to show their thinking. Compare the graphing calculator results with Algebra Piece/symbolic results.

Comments

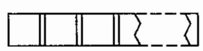
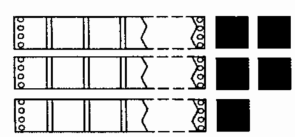
6. Students may try to graph $y = -3x + 5$ in a standard window and tracing the graph out until $x = 49$. The graph will go off the screen. Another possibility is to first resize the viewing window so that $x = 49$ appears in the window, and then trace. In order to do this, one needs a mental estimate for the y-value on the graph when $x = 49$ so the desired part of the graph will appear in the window of the calculator. For example, we would need to include at least down to $y = -150$ in order that $y = -3x + 5$ shows up in the window when $x = 49$. You and your students might want to experiment a bit with window sizes on your particular calculator to see what looks best in different situations.

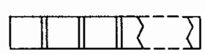
Graph of $y = -3x + 5$ for the viewing window x in [–5, 75], y in [–200, 25].

Approximation for the y-value at $x = 49$:



An Algebra Piece and symbols solution might be:

If  = 49, then  = 3(–49) + 5 = –147 + 5 = –142.

Similarly, if  = –28.75, then $-3x + 5 = -3(-28.75) + 5$
$= 86.25 + 5 = 91.25$

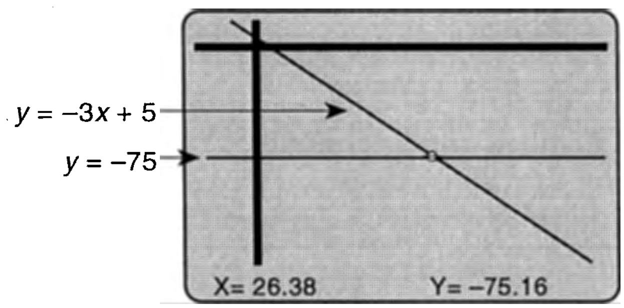
Actions

7. Ask the students to use their graphing calculators to find out for what value of x the continuum $y = -3x + 5$ has $y = -75$. Then ask the students to also find this x-value by using the Algebra Pieces or symbolically. Discuss. Ask for volunteers to show their thinking. Repeat this when $y = 10^2\!/_3$.

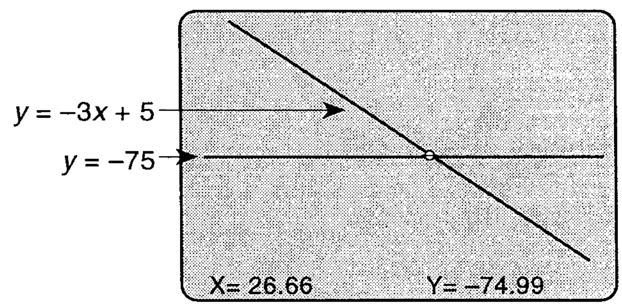
Comments

7. Students might want to resize the viewing window so that $y = -75$ appears—of course, one needs an estimate for the x-value when $y = -75$ in order to show that part of the graph in the window. Another possibility is to graph both $y = -3x + 5$ and $y = -75$ simultaneously, and check the x-value where these graphs cross.

Graphs of $y = -3x + 5$ and $y = -75$ on the window x in [−10, 50], y in [−150, 20]. We are interested in when the two graphs cross. The second graph is a ZOOM-IN and TRACE of the first one.

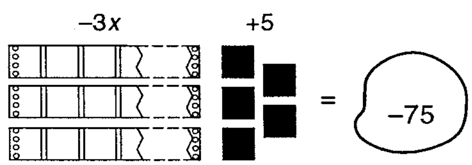


First TRACE and approximation

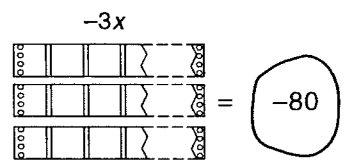


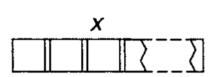
Second TRACE after ZOOM-IN

An Algebra Piece solution might be:



so,



Thus, 

$= \frac{80}{3} = 26\frac{2}{3}$

Actions

8. Point out to the students the four different representations for the continuum of arrangements of $y = -3x + 5$: (i) counting pieces and Algebra Pieces, (ii) symbols, (iii) the hand plotted graph, and (iv) the graph on the graphing calculator.

Ask the students to discuss in their groups, "What are some advantages and/or disadvantages of each of these representations for $y = -3x + 5$?" Ask the groups to share their thinking.

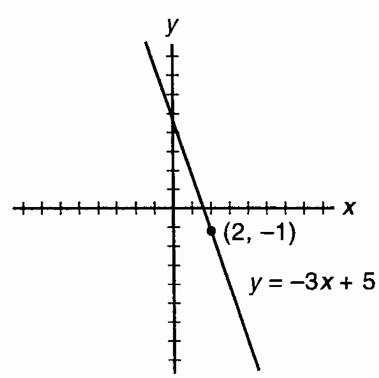
Comments

8. Possible advantages: The counting pieces and Algebra Pieces provide a visual and tactile representation of the pattern. The graphing modes, both hand drawn and graphing calculator, allow us to extend the pattern to non-integer valued arrangements, and to construct the idea of a *continuum* of arrangements. The equation provides an efficient way to write the general pattern in symbols which express the value of the counting piece arrangements, or the position on the graph. Both the equation and the graphing calculator will allow us to efficiently "evaluate" a y value given a particular xth arrangement, and vice versa.

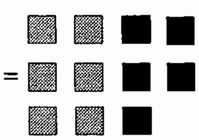
Possible disadvantages: The symbols by themselves can be devoid of meaning without the visual patterns. Graphing calculators may only give us decimal approximations for values on the graph as we TRACE the graph rather than exact values (this depends some on our choice of the viewing window). With counting pieces alone, we would have to make many cuts to represent fractional values.

Note: You might encourage the students to make connections among these various representations—a particular counting piece arrangement corresponds to a point on the graph; a point on the graph can be described by a pair (x, y) of values; the relationship between the values can be described by a general formula, in this case $y = -3x + 5$. Such connections among these various representations help build student power in algebra.

Graphically:



Symbols and pieces:

$v(2) = -3(2) + 5 =$  $= -1$

Both of the above representations indicate that the point $(2, -1)$ is on the graph; that is, when $x = 2$, $y = -1$.

Actions

9. Put up a transparency made from the top half of Master 2. Ask the students to imagine in their mind's eye what the graphs of each of these patterns looks like:
1) $y = -3x + 5$ 2) $y = -x + 5$ 3) $y = x + 5$
4) $y = 3x + 5$

Ask for volunteers to describe their visualizations of these graphs.

Then, ask the students to graph all these relationships simultaneously on their graphing calculator. Display them on the overhead graphing calculator. Discuss. Ask the students: What will the graph look like if the slope is 6? ½? –½? Which graph goes with each equation?

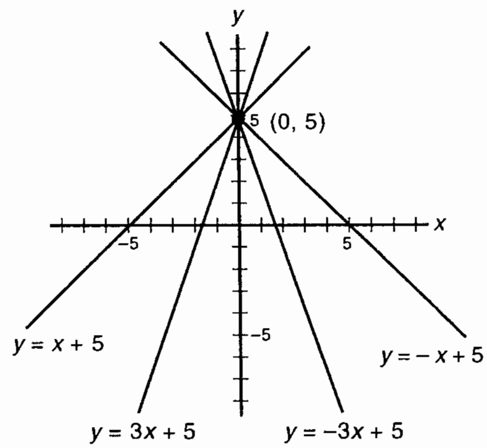
10. Put up a transparency made from the bottom half of Master 2. Ask the students to imagine in their mind's eye what the graphs of each of these patterns looks like:
1) $y = -3x + 5$ 2) $y = -3x - 5$ 3) $y = -3x + 2$
4) $y = -3x - 2$

Ask for volunteers to describe their visualizations of these graphs.

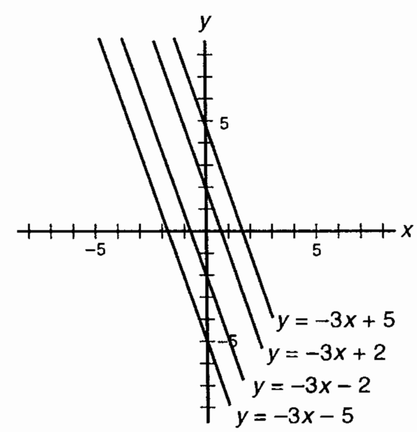
Then, ask the students to graph all these relationships simultaneously on their graphing calculator. Display them on the overhead graphing calculator. Discuss. Ask the students, "What will the graph look like if the y-intercept is 100? –25.65? 0?"

Comments

9. Students may notice the effects of positive and negative values of slope, or how the slope affects the steepness of the graph. They might also mention that 1 and 4 are mirror images of each other, and lines 2 and 3 are mirror images of each other, in the y-axis.



10. Changes in the y-intercept generate an infinite family of parallel lines, each with slope –3 in this case.

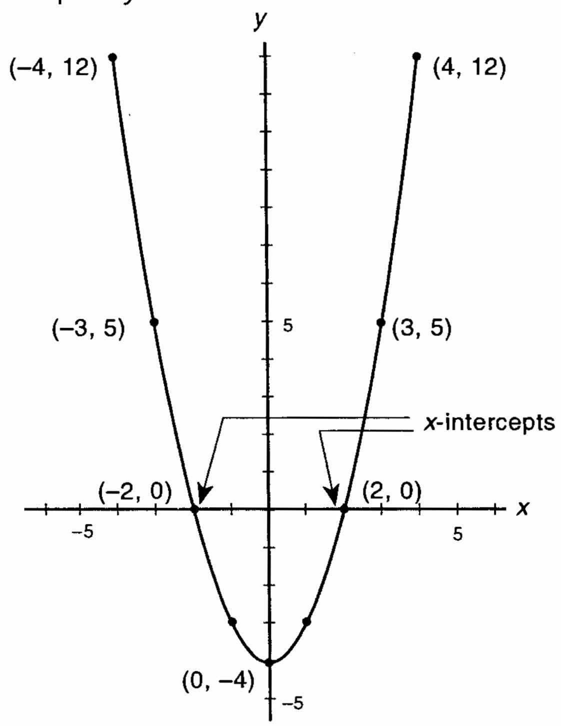


Actions

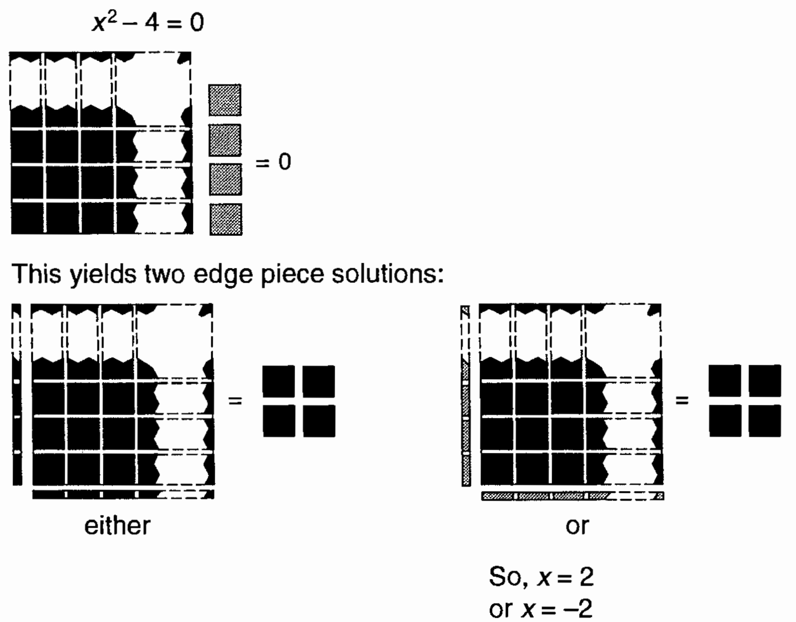
11. Ask the students to imagine in their mind's eye what the graph of the pattern $y = x^2 - 4$ looks like. Ask for volunteers to describe their visualization of this graph.

Ask the students to graph $y = x^2 - 4$ on the standard window of their graphing calculator, and to use the calculator to find where the graph crosses the x-axis. Then, ask them to use either Algebra Pieces or symbols to find the value of x for which $x^2 - 4$ takes on the y-value 0. Discuss. You may find a transparency of Master 3 helpful.

Graph of $y = x^2 - 4$



Algebra Piece solution for $0 = x^2 - 4$



Comments

11. On many graphing calculators the trace function will *not* land exactly on $x = 2$ and $x = -2$ as the x-intercepts. A decimal approximation to ± 2 will appear. ZOOMING IN will increase the accuracy of the approximation, but may not yield exactly $x = \pm 2$ when $y = 0$. The graphs on graphing calculators are made up of tiny individual points, or pixels. Each pixel has a numerical scale value, which depends on the window size, and the pixels may not come out to exact integer values.

Actions

12. Put up a transparency made from the top half of Master 4. Ask the students to imagine in their mind's eye what the graphs of each of these patterns looks like:
1) $y = x^2 - 6$ 2) $y = x^2 + 6$ 3) $y = -(x^2 - 6)$

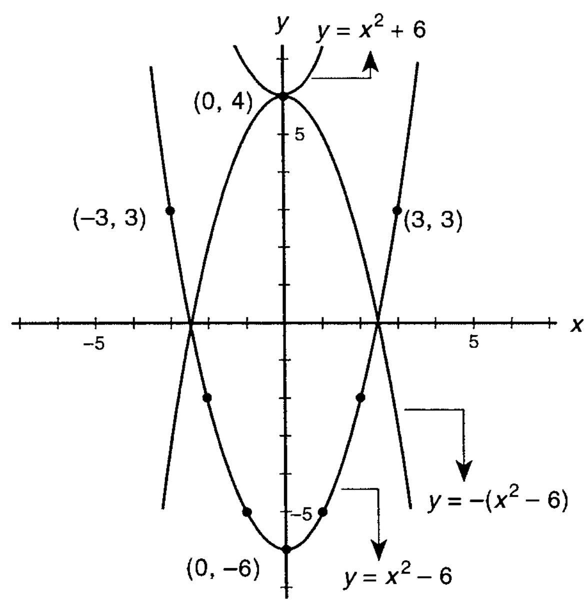
Ask for volunteers to describe their visualizations of these graphs.

Then, ask the students to graph all these relationships simultaneously on their graphing calculator. Display them on the overhead graphing calculator. Ask the students, "Which graph goes with each of the equations?" Discuss.

Comments

12. Students might notice that graphs 1 and 2 are the same, just moved up or down. They might notice that graphs 2 and 3 are reflections in the x-axis of one another. Students might predict the effects of the constant term on such graphs, or of the sign of the coefficient of the x^2 term, whether it is positive or negative, on the graph. They could quickly test any of their predictions with other similar examples on the graphing calculators.

Another exploration could be a set of equations like $y = x^2$, $y = x^2 - 2x$, and $y = x^2 + 2x$. What is the effect of the x term on these graphs? There are many possibilities for extended explorations here.



Actions

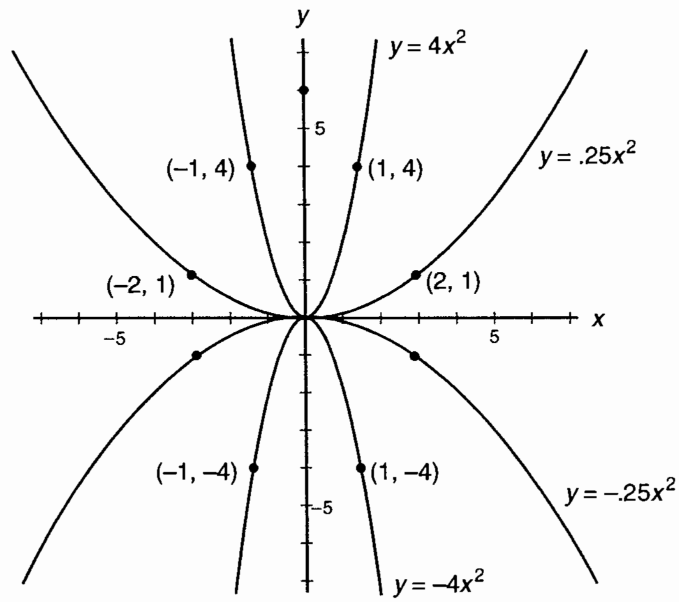
13. Put up a transparency made from the bottom half of Master 4. Ask the students to imagine in their mind's eye what the graphs of each of these patterns looks like:
1) $y = 4x^2$ 2) $y = \frac{1}{4}x^2$ 3) $y = -4x^2$
4) $y = -\frac{1}{4}x^2$

Ask for volunteers to describe their visualizations of these graphs.

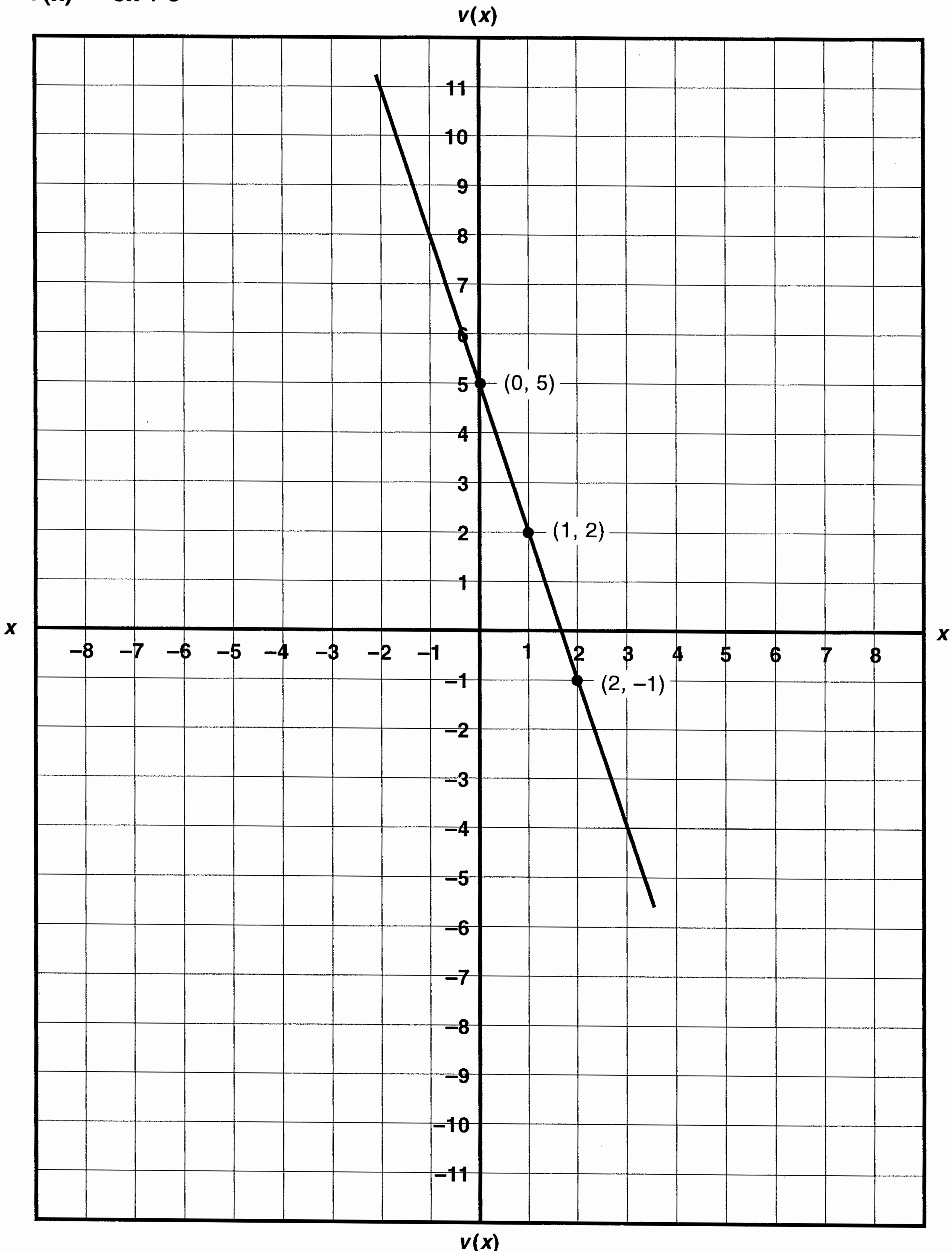
Then, ask the students to graph all these relationships simultaneously on their graphing calculator. Display them on the overhead graphing calculator. Ask the students, "Which graph goes with each equation?" Discuss. Ask the students, "What will happen to the graph if we change the coefficient of x^2 to 20? $\frac{1}{100}$? – 20?"

Comments

13. Students may wish to predict what they think will happen to the graph when these coefficient changes are made, and then check their predictions on the graphing calculator. (See graphs below.) Plotting some points on graph paper may help students to make their predictions. Fractions like ¼ must be entered as decimals on the graphing calculator, $y = .25x^2$.



$v(x) = -3x + 5$



v(x)
11
10
9
8
7
6
5
4
3
2
1
x
–8
–7
–6
–5
–4
–3
–2
–1
1
2
3
4
5
6
7
8
x
–1
–2
–3
–4
–5
–6
–7
–8
–9
–10
–11
(0, 5)
(1, 2)
(2, –1)
v(x)

1) $y = -3x + 5$

2) $y = -x + 5$

3) $y = x + 5$

4) $y = 3x + 5$

---

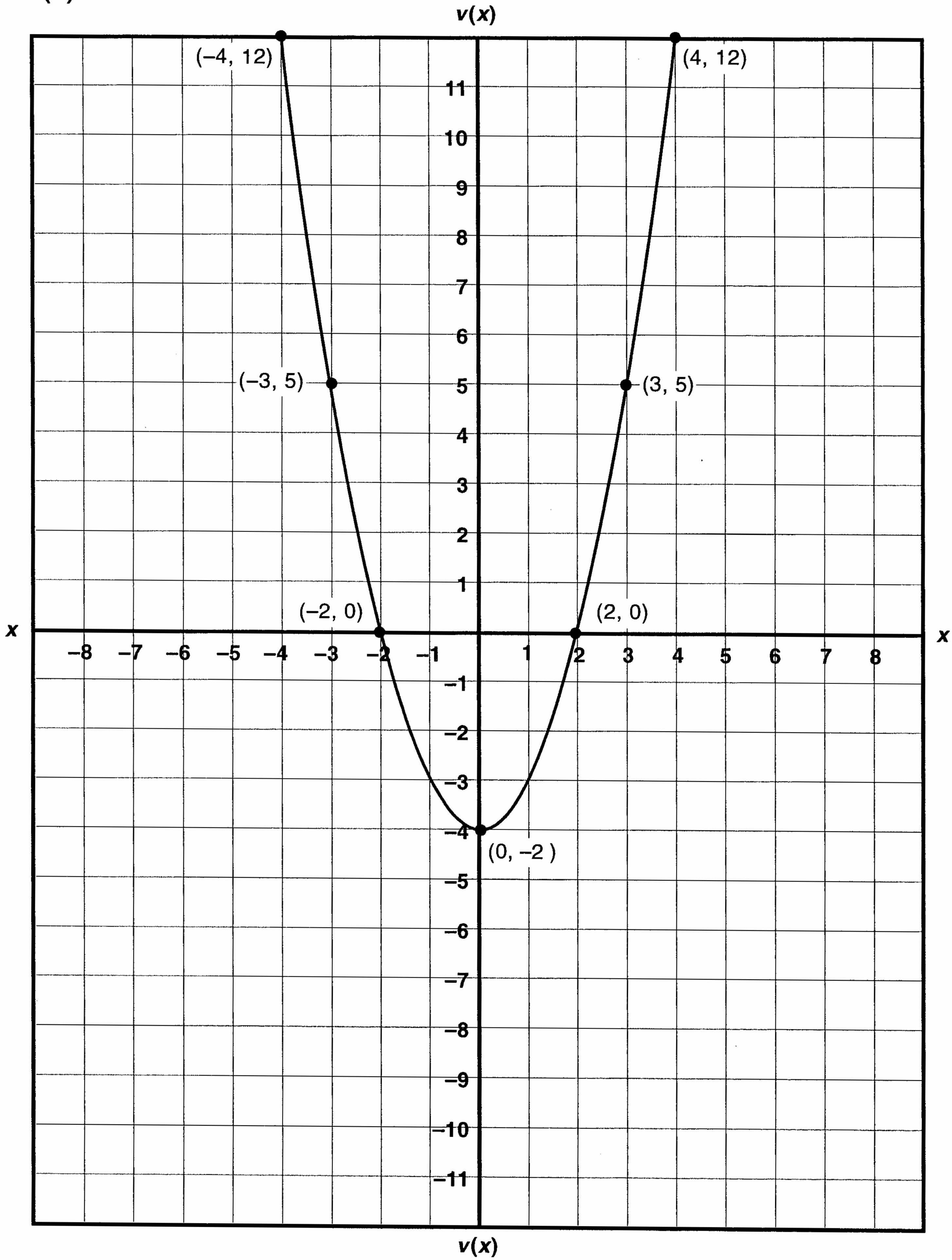
1) $y = -3x + 5$

2) $y = -3x - 5$

3) $y = -3x + 2$

4) $y = -3x - 2$

$v(x) = x^2 - 4$



v(x)
(−4, 12)
(4, 12)
11
10
9
8
7
6
(−3, 5)
5
(3, 5)
4
3
2
1
(−2, 0)
(2, 0)
x
−8
−7
−6
−5
−4
−3
−2
−1
1
2
3
4
5
6
7
8
x
−1
−2
−3
−4
(0, −2)
−5
−6
−7
−8
−9
−10
−11
v(x)

1) $y = x^2 - 6$

3) $y = -(x^2 - 6)$

2) $y = x^2 + 6$

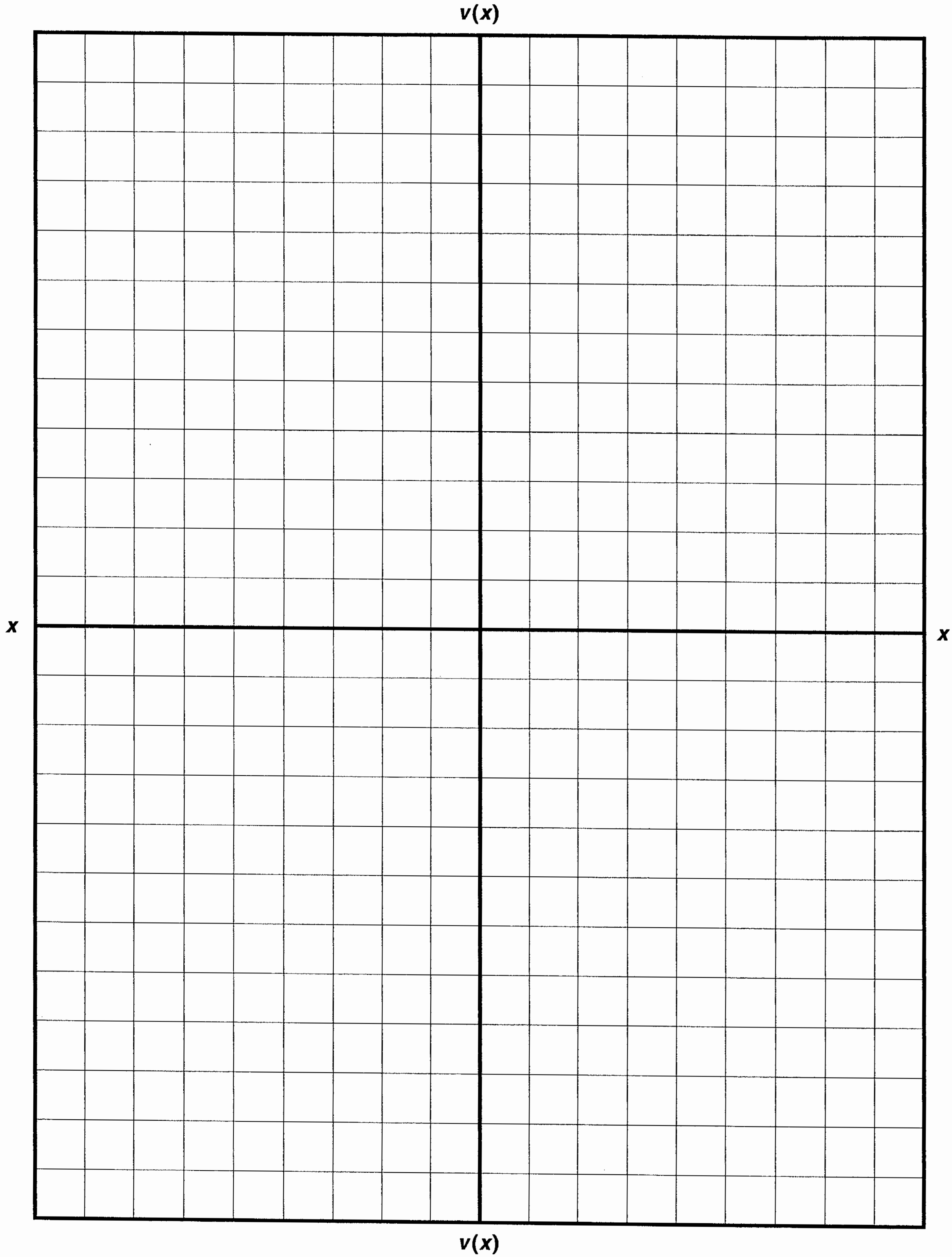
---

1) $y = 4x^2$

3) $y = -4x^2$

2) $y = \frac{1}{4}x^2$

4) $y = -\frac{1}{4}x^2$



XI-4 Coordinate Graph Paper

Unit XI • Activity 5

Introduction to Graphing Calculators, Part II

O V E R V I E W

We continue our explorations with graphing calculators, investigating systems of equations. Connections among the Algebra Piece, graphing, tabular, and symbolic representations of equations are reinforced.

Prerequisite Activity

Unit XI, Activity 4, *Introduction to Graphing Calculators, Part I.*

Materials

Bicolored counting pieces, Algebra Pieces, graph paper, masters and activity sheet, a classroom set of graphing calculators, and an overhead graphing calculator compatible with the classroom set.

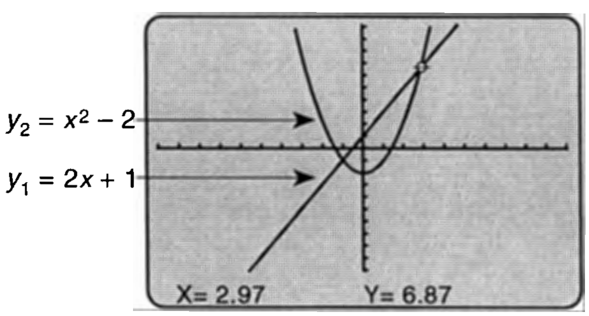
Actions

1. Write the equations $y_1 = 2x + 1$ and $y_2 = x^2 - 2$ on the overhead projector. Ask the students to graph both these equations simultaneously on their calculators. Ask the students to use the calculators to find out where the two graphs intersect. What are the coordinates of any points of intersection? Ask for volunteers to share their approaches.

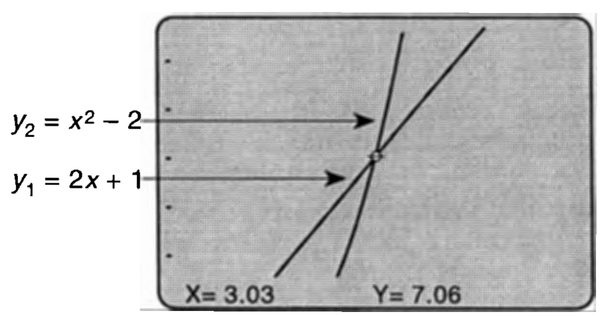
Comments

1. It is helpful to have two overhead projectors, one for the overhead calculator, and one for student sharing and transparency masters.

Students may need to first rescale the viewing windows so they can see the graphs. Students might try various approaches to finding the intersection points with the calculator. Some students might TRACE and "eyeball" the intersections. This approach may yield something like (–1.06, –.868) and (2.97, 6.87) for the intersection points. Other students may ZOOM-IN several times on the intersection points, and obtain values such as (3.005, 7.031) for one of the intersection points. These approximate solutions will vary depending on the type of calculator and the scaling of the window the students are using. They might decide the coordinates shown on the graphing calculator are very close to (3, 7), and notice that that pair "works" exactly in both the equations $y = 2x + 1$ and $y = x^2 - 2$.



Solution: Using TRACE



After ZOOM-IN and TRACE-ING again.

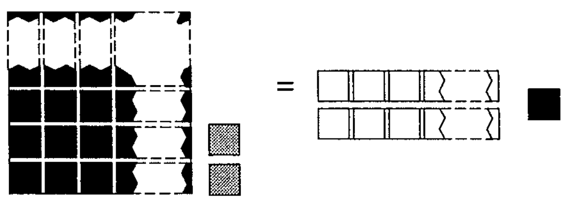
Actions

2. Ask the students to use Algebra Pieces to build the general xth arrangement for both $y = 2x + 1$ and $y = x^2 - 2$. Then ask them to use the Algebra Pieces to determine which arrangement(s), if any, will have the same value. The students might record each move of the Algebra Pieces in symbolic form.

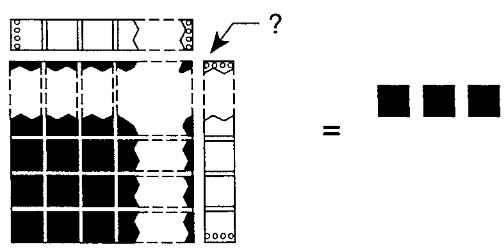
Comments

2. The purpose of this Action is to continue to build connections between the graphical, symbolic and visual representations of equations.

Some students might want to build several particular arrangements of these two patterns before forming the general arrangement with the Algebra Pieces. They might want to build the –2nd, –1st, 0th, 1st and 2nd arrangements, for example. The Algebra Piece solution provides convincing evidence that the points (3, 7) and (–1, 1) are the exact places where these two graphs intersect. Depending on the scaling window, the graphing calculator may not have provided these exact coordinates, but rather gave only closer and closer decimal approximations to this exact solution as students ZOOMED-IN. An possible Algebra Piece solution of the problem is shown here.

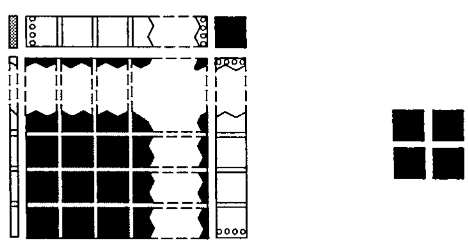


$x^2 - 2$ $\qquad$ $2x + 1$



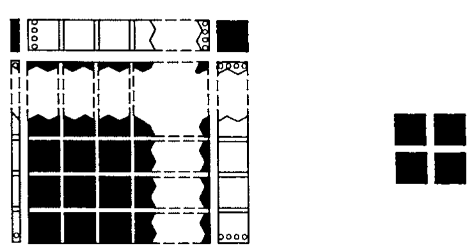
$x^2 - 2x = 3$

Making trades and trying to complete a square.



$x^2 - 2x + 1$ or $(x - 1)^2$ $= 4$

Completing a square, one possible set of edges is $(x - 1)^2$. So $x - 1 = 2$, $x = 3$. When $x = 3$, $y = (3)^2 - 2 = 7$. Thus (3, 7) is one exact solution pair.



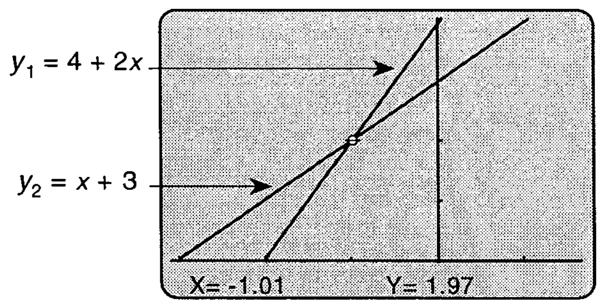
$(x^2 - 2x + 1)$ or $(-x + 1)^2$ $= 4$

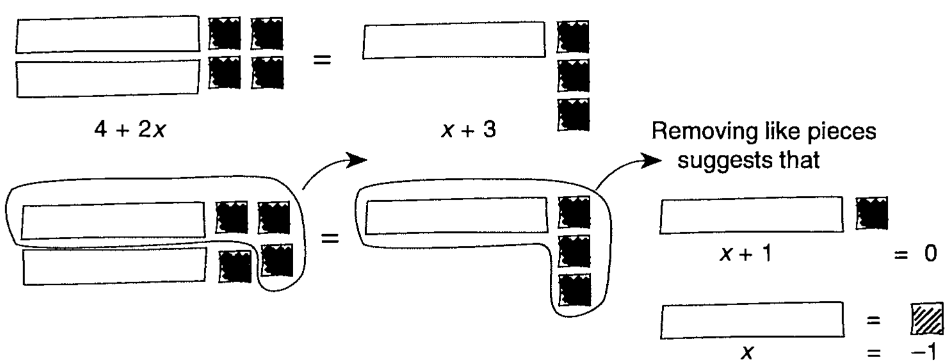
Another possible set of edges is $(-x + 1)^2$. So, if $-x + 1 = 2$, $-x = 1$, so $x = -1$. When $x = -1$, $y = (-1)^2 - 2 = -1$. Thus (–1, –1) is another exact solution pair where the graphs cross.

Actions

3. Pass out copies of Master 1 to each group of students. For each pair of equations, ask the students to use any approach they wish to find those instances where the two equations have the same value.

4. Ask the students to share their approaches to the systems of equations in Action 3.





Comments

3. This question can also be posed; "Are there any arrangements (xth arrangements) for which each pair of equations has the same y-value? Or, "What are the points (if any) of intersection of the graphs of each pair of equations?" This will take the students some time. Students might use a graphing calculator approach, or they might use the Algebra Pieces, or perhaps some students will record movements symbolically. Students may wish to use multiple approaches on some of the pairs to verify the results that they obtain with one approach. You might tell the students that these are examples of *systems of equations*, more specifically, *systems of two equations in two variables.*

4. Throughout this activity, graphs can be displayed by the students on the overhead graphing calculator when they share their approaches.

Here are some sample solution approaches.

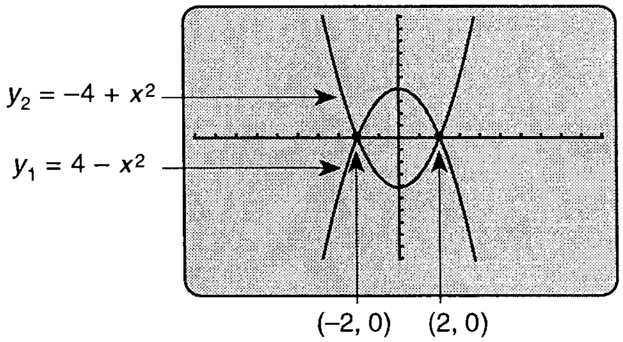
A. Using a graphing calculator, and ZOOMING-IN, the TRACE function yields this solution estimate (–1.01, 1.97). The calculator suggests that the lines cross at, or very near, $x = -1$. In fact, when $x = -1$, $y_1 = 4 + 2(-1) = 2$ and $y_2 = -1 + 3 = 2$, so the lines meet at exactly the point (–1, 2).

A sketch for system A might look like this:

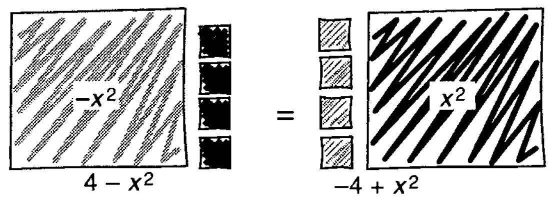
There are other possible Algebra Piece solutions.

Continued next page.

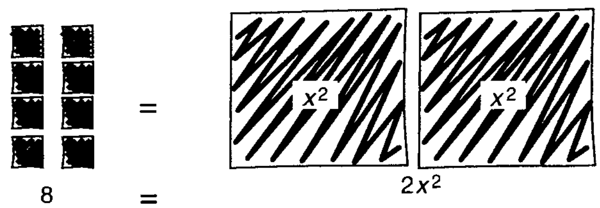
Actions



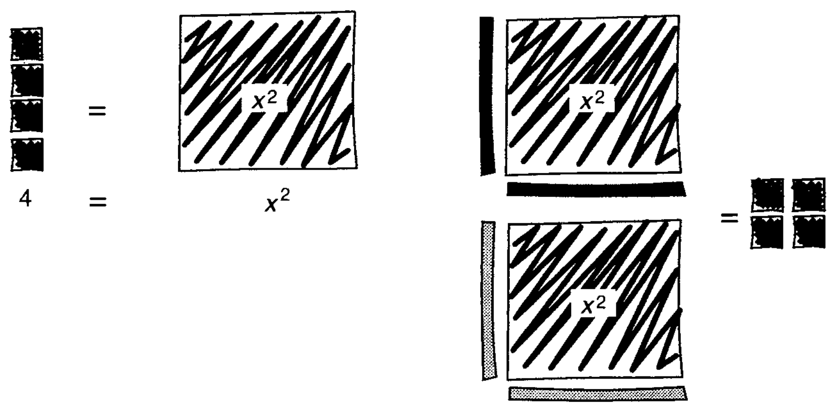
Using a sketch



Trading we obtain



So,



The edges are
$x = 2$ or $x = -2$.

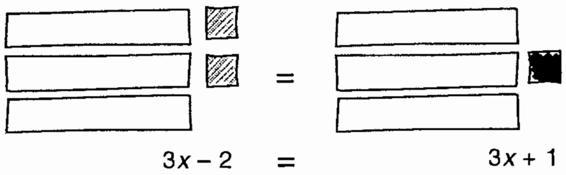
Comments

4. *Continued.*

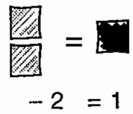
B. Graphing on a standard graphing calculator window suggests that these two graphs cross one another on the x-axis at (−2, 0) and (2, 0). *Note*: On some calculators, the trace function will show exactly (±2, 0) for the intersections. On other calculators a decimal approximation such as (2.01, −.03), which is very close to (2, 0), will appear on the TRACE.

Continued next page.

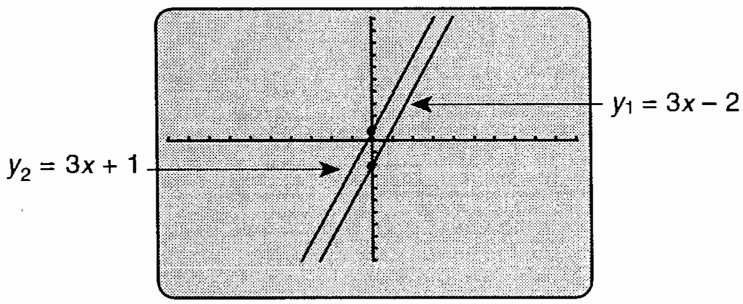
Actions

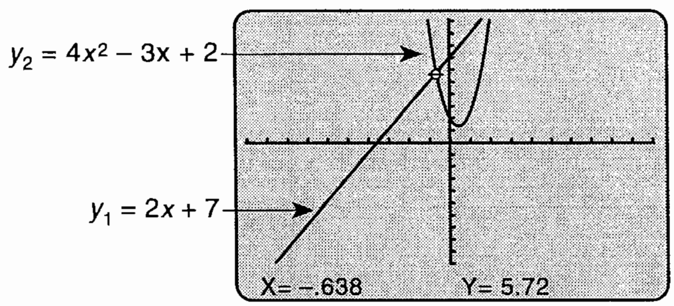


This would hold only if

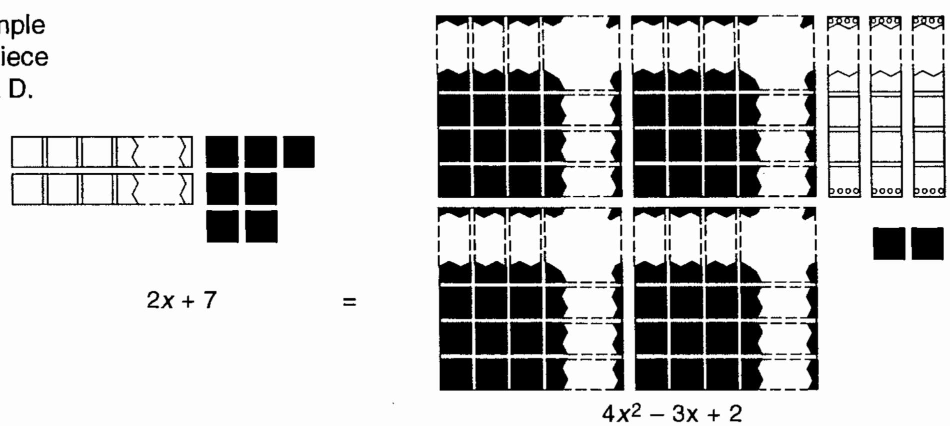


since we can remove 3x from each pile.





Here is an example of an Algebra Piece solution for part D.



Comments

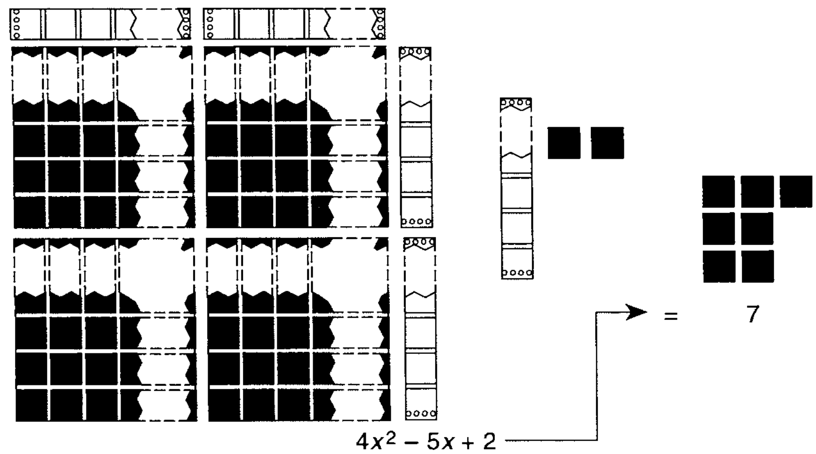
4. *Continued.*

C. There is no value of x for which these two arrangements will be equal. We can verify this on a graphing calculator as these two lines have the same slope and will never intersect. You might want to have students verify this on the calculator by ZOOMING-OUT to a larger scale. Sometimes graphs do intersect, but we might not catch the intersection in the particular window we are using.

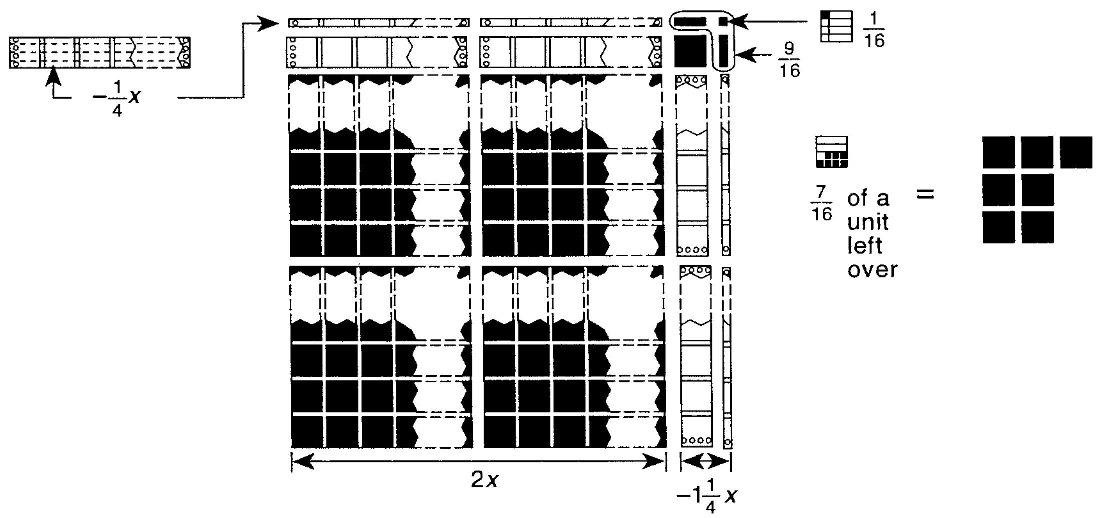
D. Using a graphing calculator we obtain an estimate for one intersection point at (–.638, 5.72). The other intersection point is off the screen in this window. Students may need to be reminded to change the x and y min/max ranges in the WINDOWS menu to find the other intersection point. The exact algebraic solution for part D is a bit messy. It will depend on your students as to how far you wish to explore it. Using Algebra Pieces to complete the square for this quadratic equation, we find that these graphs do not cross each other at exact whole numbers or even at exact fractional (rational) number points. The solutions are at irrational numbers in this case.

Continued next page.

Trading x-frames and beginning to arrange into a square, we obtain:



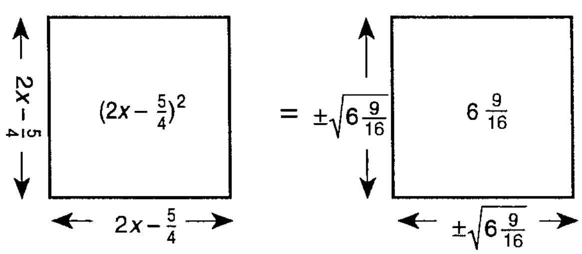
We find we must cut the remaining $-x$-frame and the tile, to form a square. The extra $-x$-frame is cut into $-\frac{1}{4}$ strips, and the tile must be cut into $\frac{1}{4}$ strips, and then into $\frac{1}{16}$ pieces to make a square. The square will then be $2x - \frac{5}{4}$ on each side.



$$\left(2x - \frac{5}{4}\right)^2 + \frac{7}{16} \quad = \quad 7$$

So, $\left(2x - \frac{5}{4}\right)^2 = 6\frac{9}{16}$.

Trading $\frac{7}{16}$ of a unit we get the picture:



The square on the right will have edges either $\pm\sqrt{6\frac{9}{16}} \approx \pm 2.56$. Students may wish to work through order of operation on a calculator for this. Thus, $2x - 1.25 = 2.56$ so $2x = 3.81$, or $2x - 1.25 = -2.56$ so $2x = -1.31$.

4. *Continued.*

The graphs cross at approximately $x = 1.95$ and $x = -.655$. When $x = -.655$, $y_1 = 2(-.655) + 7 = 5.69 = y_2$. When $x = 1.95$, $y_1 = 2(1.95) + 7 = 10.9 = y_2$. Thus, the two points of intersection for these graphs occur at $(-.655, 5.69)$ and $(1.95, 10.9)$, approximately. Remind the students that these are rational approximations to irrational solutions. This example may help to show how useful the graphing calculator is for finding approximate solution points quickly. Note that the TRACE function obtained $x = -.638$, which is quite close to the Algebra Piece/symbolic solution $x = -.655$.

Actions

5. Pass out copies of Master 2. Ask the groups to create some problems from this situation, then have groups share the problems they have posed.

6. Ask the students to focus in their groups on the question "Who would be favored to win a 100 meter race, the man or the child?" When the students are ready, ask them to share their approaches to this problem.

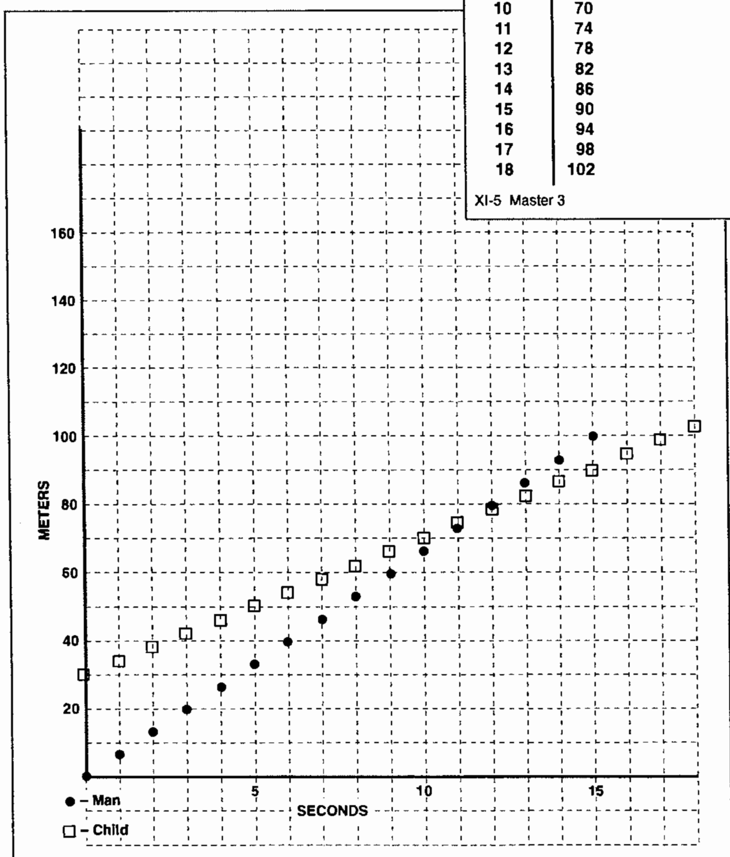
seconds	child meters
0	30
5	30 + 20 = 50
10	70
15	90

seconds	man meters
0	0
3	20
6	40
9	60

seconds	child meters
0	30
1	34
2	38
3	42
⋮	⋮
10	70
11	74
12	78
13	82
14	86
15	90
16	94
17	98
18	102

seconds	man meters
0	0
1	$6\frac{2}{3}$
2	$13\frac{1}{3}$
3	20
⋮	⋮
9	60
10	$66\frac{2}{3}$
11	$73\frac{1}{3}$
12	80
13	$86\frac{1}{3}$
14	$93\frac{2}{3}$
15	100

XI-5 Master 3



XI-5 Master 5

Comments

5. Students may wonder if this if a fair head start for the child, for example. Some possible questions might include: "Who would win a 50 meter race?" "Who would win a 100 meter race?" "When would the man catch the child?" If questions like these don't arise in the groups, you might suggest some of them.

6. This particular problem is just a suggestion. There may be a question that the students raised themselves that would be more interesting to them than this particular one. The students can be encouraged to use anything they want to solve this problem. Some of them might like to make tables of values. Some might graph the distances traveled by both runners at various time intervals. If the students don't think of this, you might suggest it to them. Transparencies made from Masters 3 and 4 may be helpful *after* the students have shared their own approaches to the problem. Some students may find it helpful to determine how far each runner travels in one second, and use that information in a table or a graph:

Child—20 meters in 5 seconds is 4 meters in 1 second.

Man—20 meters in 3 seconds is $\frac{20}{3}$ meters in 1 second, or $6\frac{2}{3}$ meters in 1 second.

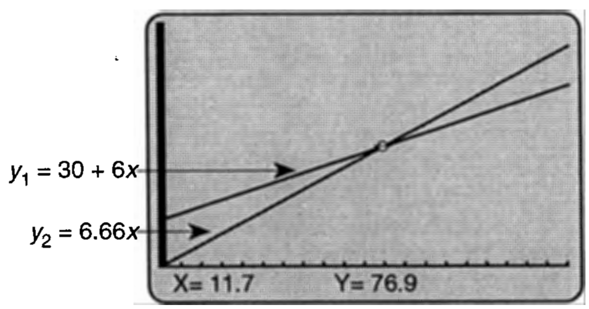
The tables and/or graph show that the man catches up to the child somewhere between 11 and 12 seconds. Also, at 12 seconds, the man is at the 80 meter mark, and the child is at the 78 meter mark, so the man is favored at 100 meters.

Actions

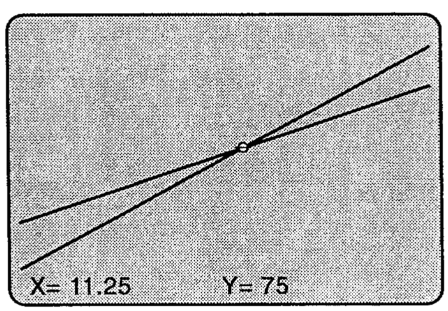
7. What race length would be "fair" for these runners?

8. Ask the students to write a general expression (equation) for the number of meters (m) the child has moved from the starting line in (s) seconds, and also for the number of meters (m) that the man has moved from the starting line in (s) seconds. Ask for volunteers to share their expressions. When the students have settled on some expressions for the distances traveled by the man and the child, ask them to graph these expressions on their graphing calculators, and to use the calculator to find a fair race length for these runners.

TRACE on
graphing calculator window
x min 0 x max 20
y min 0 y max 150



After several ZOOM-INS and TRACE.



Comments

7. In order to find the race length that makes the 30 meter head start fair, students will need to know where the two graphs cross, or where the tables will have the same values. They may do some eyeballing on the graphs, or calculate some approximations using 4 meters per second for the child, and $6\frac{1}{3}$ meters per second for the man.

8. This process may take some time. Ask students to test out their expressions for various numbers of seconds to see if the meters traveled makes sense. For example, their expression for the child should yield 50 meters at 5 seconds (the initial 30 plus 20 more), 70 meters at 10 seconds, and so forth. Any tables or graphs they made in Action 5 may be helpful in determining how far the runners go in 1 second.

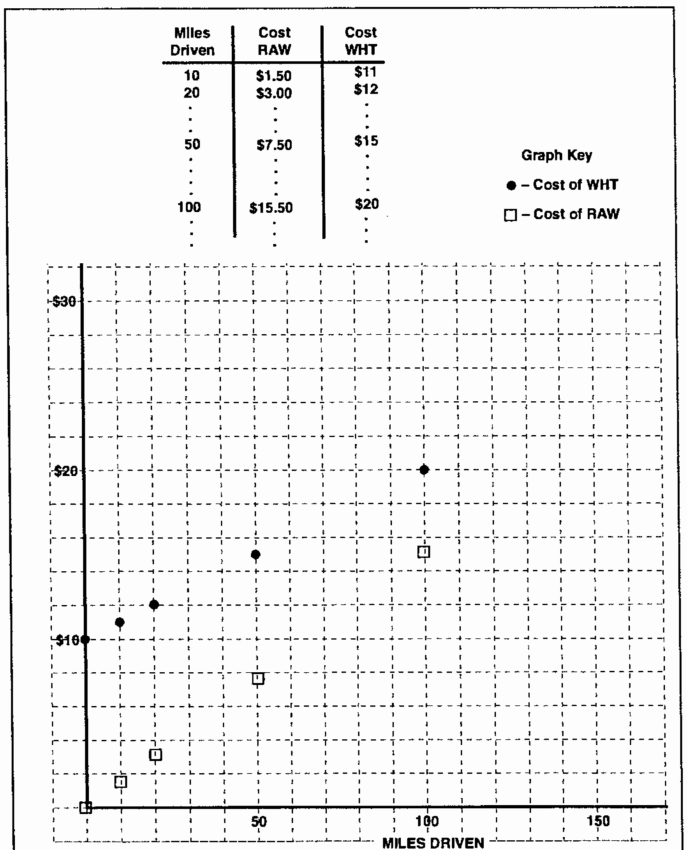
One possible pair of expressions is child meters = $s \times 4 + 30$, man meters = $s \times 6\frac{2}{3}$.

The students can set $y_1 = 4s + 30$ for the child and $y_2 = (6\frac{2}{3})s$ or $y_2 = 6.666s$ for the man, graph the equations, and use trace to find out where they cross. You might wish to have them create an Algebra Piece sketch of a solution as well. Notice that the students will have to experiment with the window values so that they can get the intersection point to appear in the window of the graphing calculator. If they use a standard window, the intersection is likely to be off the screen. If their calculators have a TABLE key, extended tables of values for each of the two expressions are automatically generated for the equations entered, and students can look through the tables for the place(s) where the running distances are nearly equal. (See diagrams to the left.)

Thus, a "fair" race appears to be about 75 meters. A follow-up question for students might be, "How much of a head start does the child need for 100 meters to be fair?"

Actions

9. Pass out a copy of Activity Sheet XI-5 to each student. Ask them to use any approach they wish on these problems, and to write up a problem-solving summary for each problem. Ask groups of students to present their approaches to the whole class.



XI-5 Master 6

Comments

9. These problems could be used in class, and/or as part of an extended homework activity. A problem-solving summary could include written information on how the students thought about the problem, various strategies they tried, any places they felt stuck or where an "aha" occurred, their solution attempts, and a reflection on what they learned in the problem. Students can be encouraged to write general expressions for each of the situations, and to use the graphing calculator to graph these general expressions. They may wish to use Algebra Pieces and/or symbolic approaches to get an exact solution for the questions they raise. You may want to pick and choose from among the situations on Activity Sheet XI-5, as each one will take some time.

Situation 1). Examples of questions for the Car Rental situation: How much does it cost to drive each vehicle 10 miles? 20 miles? etc. Which one has the better deal for 100 miles? Is there a number of miles for which the cost will be the same for both companies?

In the car rental situation, students might ask which is the better deal. Since this depends on the number of miles driven, you might ask which is the better deal if we drive 50 miles? If we drive 500 miles? Students might make a table of values, or a graph of the values. Use a transparency of Master 5 to show part of a table of values and part of the graph of those values for the cost of each rental car depending on the miles driven. This may help students see that, for a long mile span, the cost of RAW is lower than WHT. Students can extend the table or the graph to find where the two companies would cost the same, at 200 miles both cost \$30. Below 200, WHT is better, while above 200, RAW is a better deal.

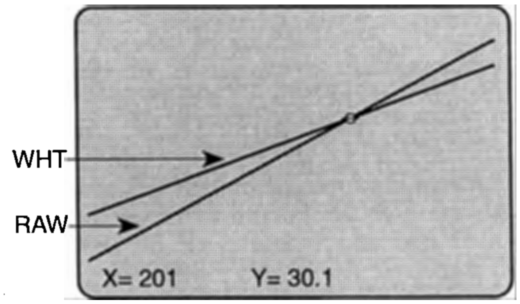
Writing general expressions for the cost of each car depending on the number of miles driven is a worthy goal in the car rental problem.

RAW\$ = \$10 + .10 × *m*

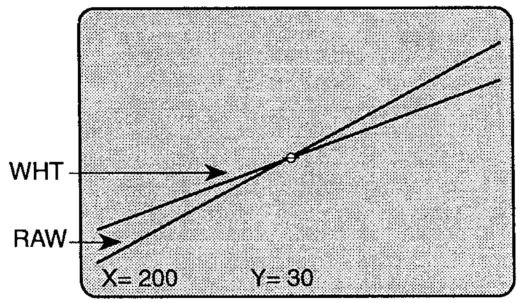
WHT\$ = .15 × *m*, where m is the number of miles driven.

Continued next page.

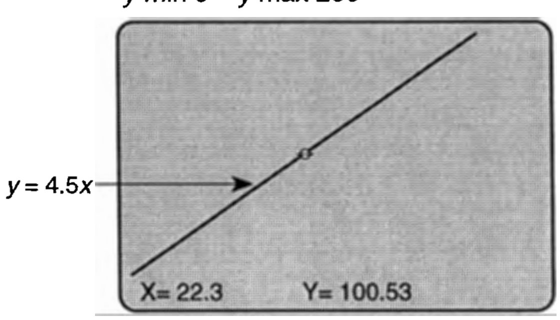
TRACE on
graphing calculator window
x min 0 *x* max 300
y min 0 *y* max 50



TRACE on, graphing calculator window after several ZOOM-INS.



TRACE on
graphing calculator window
x min 0 *x* max 50
y min 0 *y* max 200



This suggests that Saucey needs to sell at least 23 pizzas to make money on one day.

9. *Continued.* Once general expressions are written for the miles driven, students can use a graphing calculator. Students will have to explore changing the size of the viewing WINDOW order to see where the graphs of these two car rental situations actually cross. If their graphing calculators have a TABLE function, students can also use this feature to find the miles driven when the two companies will cost the same.

You might discuss with students the advantages and/or disadvantages of each approach to this problem: tables, making a graph by hand, or using a graphing calculator.

Situation 2). Examples of pizza questions: If they sell 100 pizzas, how much money will they make or lose that day? How many pizzas do they need to sell in a day in order to make money?

Some students might work with profit on pizzas, and figure out how many $4.50's they need to make up $100. This could be done with a table, or some students might divide, or find some other way of keeping track of how many $4.50's fit in $100. If students approach the problem this way, you might ask them to write a general expression for the amount of money made if they sell P pizzas at $4.50 a piece.

Profit = $4.50 × P. Then, ask students to graph this on a graphing calculator, and to use the calculator to verify the number of pizzas needed to cover the $100 cost.

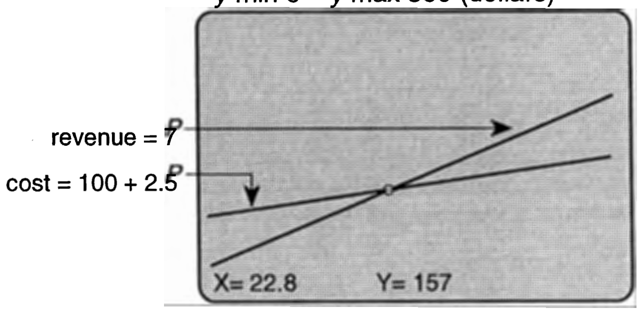
A graphing calculator window for $y = 4.50x$ might look like the illustration to the left.

Continued next page.

Actions

Expression for daily cost of pizzas:
Cost = \$100 + 2.50P.
Expression for daily revenue of pizzas:
R = \$7P.

TRACE on
graphing calculator window
x min 0 x max 50 (pizzas)
y min 0 y max 500 (dollars)



This suggests Saucey would have to sell at least 23 pizzas to make money on any day. Possible follow-up: How much would they make if they sold 100 pizzas?

seconds x	height in feet $y_1 = 80x - 16x^2$	elevated tee height in feet $y_2 = 80x - 16x^2 + 20$
0	0	20
0.5	36	56
1.0	64	84
1.5	84	104
2.0	96	116
2.5	100	120
3.0	96	116
3.5	84	104
4.0	64	84
4.5	36	56
5.0	0	20
5.5	–44	–24

Comments

9. *Continued.* Other students may wish to make separate tables of values and graphs for the cost of making pizzas and for the money they collect from selling the pizzas (the total *revenue).*

Encourage the students to write a general expression for the cost of *n* pizzas and for the revenue from *n* pizzas.

Once they have general expressions written, they can graph both the cost expression and the revenue expression on graphing calculators. You might ask how the graphing calculator can show us the number of pizzas that Saucy must sell to make money in one day.

Situation 3). Examples of golf ball questions: How high is the golf ball in 1 second, 2 seconds, etc.? At what time does the golf ball reach its highest point? How long does it take before the golf ball hits the ground? How high up is the loft for the elevated tees? Where did those formulas come from?

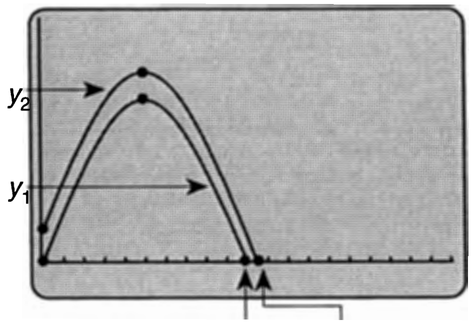
Making a table of values from the expressions for height of the golf ball can be tedious for middle school students. If their graphing calculators have a TABLE function, students can look at the table after they enter the equation on the calculator. Such a table might look like the one on the left and is on Master 6.

The table suggests the ball reaches a high point after 2.5 seconds, and then starts back down again. You might ask the students when the ball hits the ground, does the table help? Also, you might ask them what they think about those negative heights. Some equations only make physical sense for a certain range of values.

Continued next page.

Actions

TRACE on
graphing calculator window
x min 0 x max 10
y min 0 y max 150



$x = 2.55$ $x = 5$ $x = 5.24$
$y_1 = 99.95$ $y_1 = 0$ $y_2 = -.06$
$y_2 = 119.95$

$y_1 = 80x - 16x^2$
$y_2 = 80x - 16x^2 + 20$
The elevated ball stays 20 feet higher, and hits the ground about a quarter second later.

10. Discuss with the students the relative advantages and disadvantages of various approaches to solving the problems that were used throughout this activity. For example, tables, Algebra Pieces, graphs by hand, and graphing calculator approaches all were used to help us solve these problems. What is lost or gained in each of these approaches?

Comments

9. *Continued.* Plotting both equations on a graphing calculator, we can find the high point of the golf ball and the time when the ball hits the ground for each graph using the TRACE key. The students will need to experiment with the window size in order to get the entire graph on the window. Looking at the table of values first, before graphing, can help students get a feel for the needed window range. You might ask if the students notice anything about the two graphs. How do they compare? (Same shape, one is always 20 feet higher than the other, takes longer for the elevated ball to hit the ground, etc.) The numbers in the equations, the coefficients of the variables, have physical significance. The "80" means the golf ball rises 80 feet/sec when it is first hit. On the other hand, the "–16" is the effect of the deceleration due to the earth's gravity on the golf ball, it slows the height gain of the ball over time and eventually the ball starts to descend. The "20" in the elevated tee equation is the height of the ball at time 0, so the elevated tee is 20 feet up.

10. The visual representation with the Algebra Pieces helps us to "see" the solution. On the other hand, an Algebra Piece sketch can become somewhat messy, like part D in Action 4. Making tables or plotting graphs by hand can help us begin to get a rough idea about solutions, but we may need a more complete graph to get a better approximation to the answer, as in the problem with the two runners. The graphing calculator can show us the complete graph, and we can find close approximations using a TRACE.

On the other hand, sometimes it is difficult to know what values we should set for the WINDOW ranges in order to see the graph in the graphing calculator window (as in the golf ball problem). Using tables can help us find benchmark values to put in for our graphing windows, on the other hand, sometimes making tables by hand is tedious. A TABLE key on a graphing calculator can make table building more efficient. It is valuable for students to realize there are various approaches to solving problems that involve general expressions. These approaches include tables, Algebra Pieces, graphs by hand, graphing calculators, and purely symbolic approaches.

A) $y_1 = 4 + 2x$ $\quad$ $y_2 = x + 3$

B) $y_1 = 4 - x^2$ $\quad$ $y_2 = -4 + x^2$

C) $y_1 = 3x - 2$ $\quad$ $y_2 = 3x + 1$

D) $y_1 = 2x + 7$ $\quad$ $y_2 = 4x^2 - 3x + 2$

A man and his child are racing one another on a track.

The man can run 20 meters in 3 seconds.

His child can run 20 meters in 5 seconds.

They decide to give the child a 30 meter head start.

Make up some questions based on this situation.

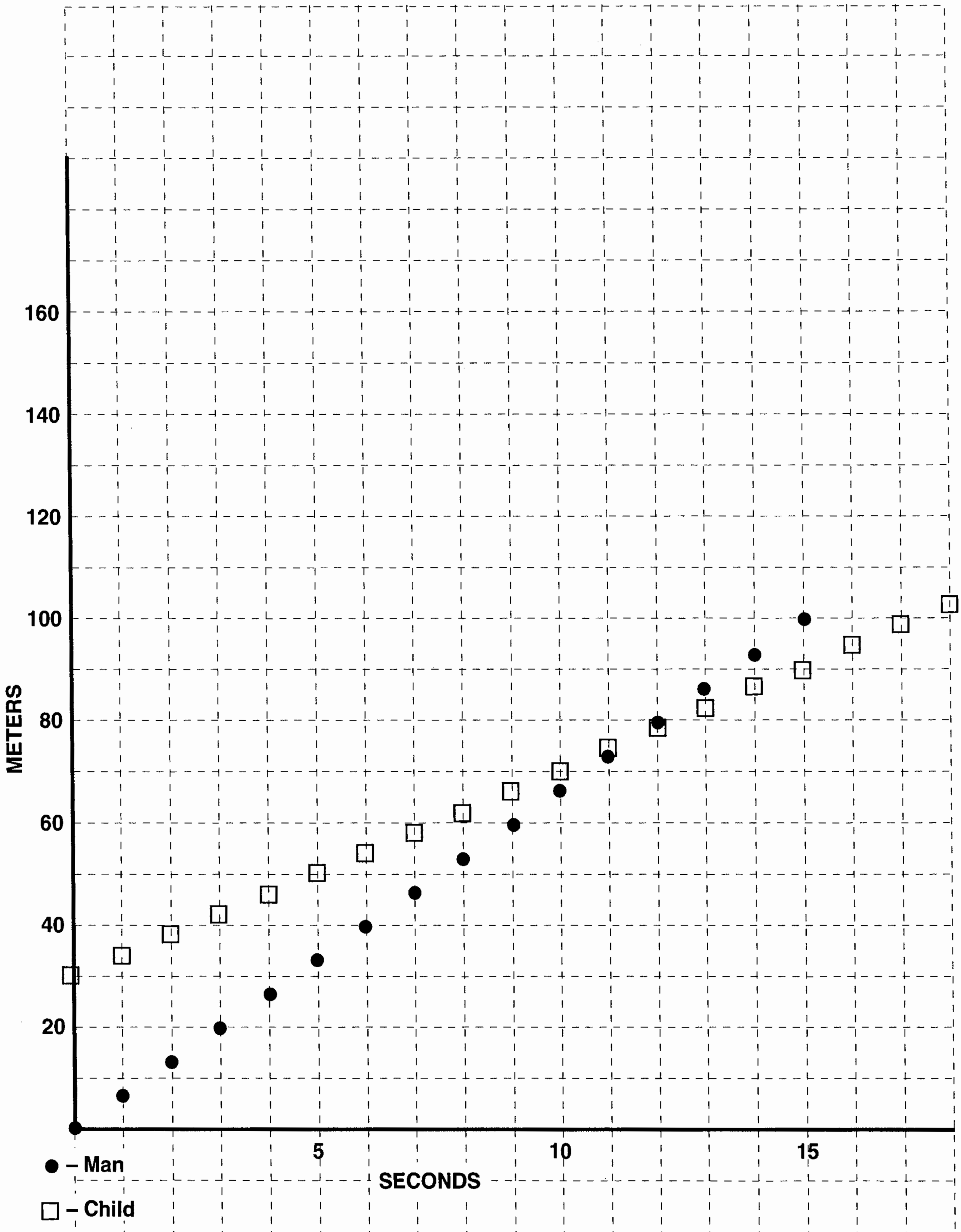
Share and record your questions in your group.

seconds	child meters
0	30
5	30 + 20 = 50
10	70
15	90

seconds	man meters
0	0
3	20
6	40
9	60

seconds	child meters
0	30
1	34
2	38
3	42
⋮	⋮
10	70
11	74
12	78
13	82
14	86
15	90
16	94
17	98
18	102

seconds	man meters
0	0
1	$6\frac{2}{3}$
2	$13\frac{1}{3}$
3	20
⋮	⋮
9	60
10	$66\frac{2}{3}$
11	$73\frac{1}{3}$
12	80
13	$86\frac{1}{3}$
14	$93\frac{2}{3}$
15	100

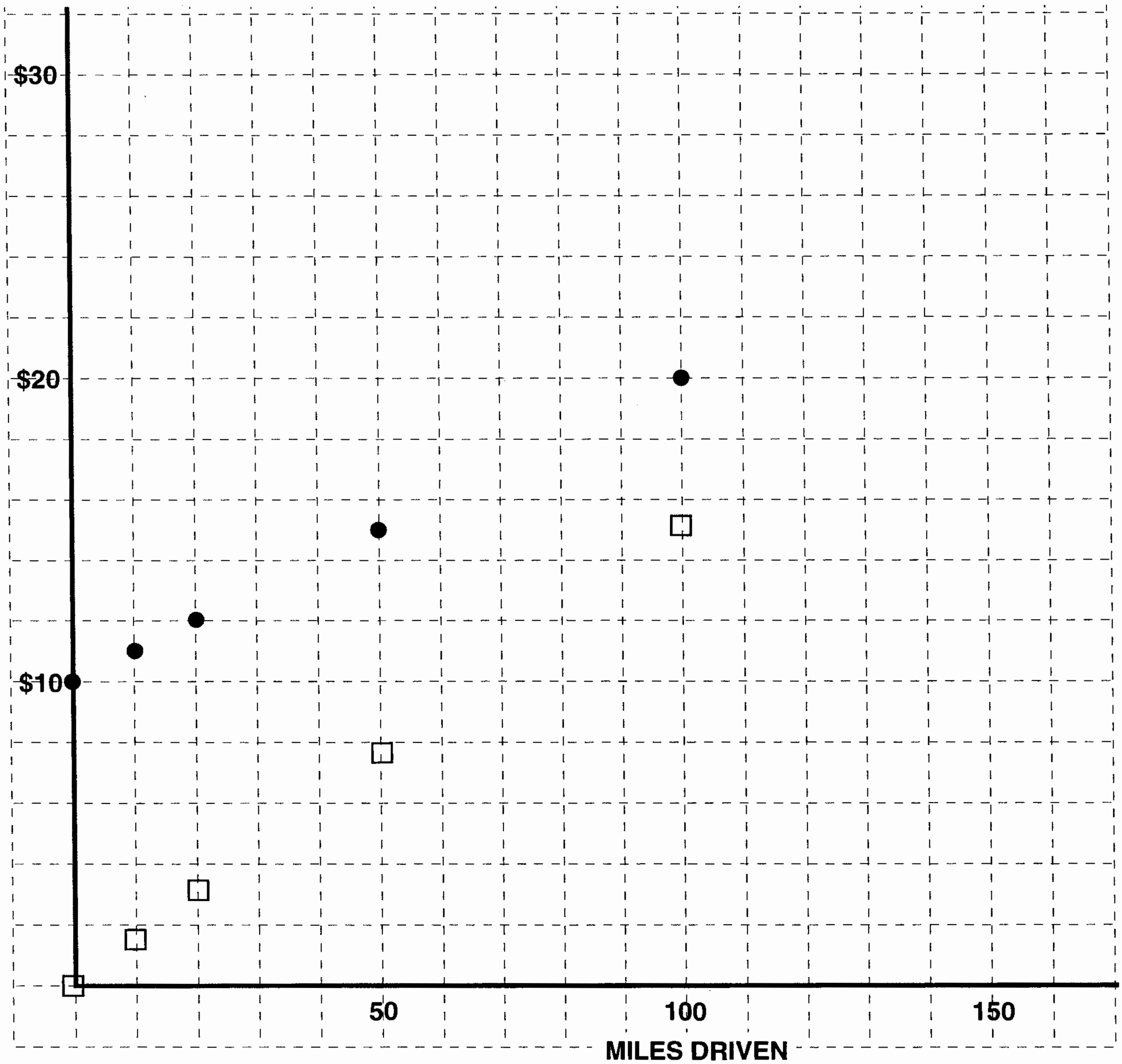

160
140
120
100
80
60
40
20
METERS
5
10
15
SECONDS
● – Man
□ – Child

Miles Driven	Cost RAW	Cost WHT
10	$1.50	$11
20	$3.00	$12
.	.	.
.	.	.
.	.	.
50	$7.50	$15
.	.	.
.	.	.
.	.	.
.	.	.
100	$15.50	$20
.	.	.
.	.	.
.	.	.

Graph Key

● – Cost of WHT

□ – Cost of RAW



seconds x	height in feet $y_1 = 80x - 16x^2$	elevated tee height in feet $y_2 = 80x - 16x^2 + 20$
0	0	20
0.5	36	56
1.0	64	84
1.5	84	104
2.0	96	116
2.5	100	120
3.0	96	116
3.5	84	104
4.0	64	84
4.5	36	56
5.0	0	20
5.5	–44	–24

Name ____________________________

Pose some questions about each of the following situations and then answer them using any approach that you wish. Write up a problem solving summary for each situation.

1) The Rent-a-Wreck (RAW) and the We Hardly Try (WHT) car rental companies are competing for business and change their prices as follows:

WTH has an initial rental charge of $10 and then charges $.10/mile.

RAW eliminates the initial rental charge altogether but charges $.15/mile.

2) The Saucy Pizza company is currently charging $7 for their pizzas. The ingredients and labor for each pizza they make cost $2.50. Each day that the company operates costs them $100 in overhead (lights, water, heat, rent, etc.).

3) The U-Drive-It golf range claims that the height (H) in feet above the ground of a golf ball at (T) seconds after their pro hits it can be determined from the expression

$$H = 80T - 16T^2$$

They also claim that when the ball is hit from their elevated loft tees that the height can be determined by

$$H = 80T - 16T^2 + 20$$

1-cm grid paper

Unit XI / Math and the Mind's Eye

Transparencies

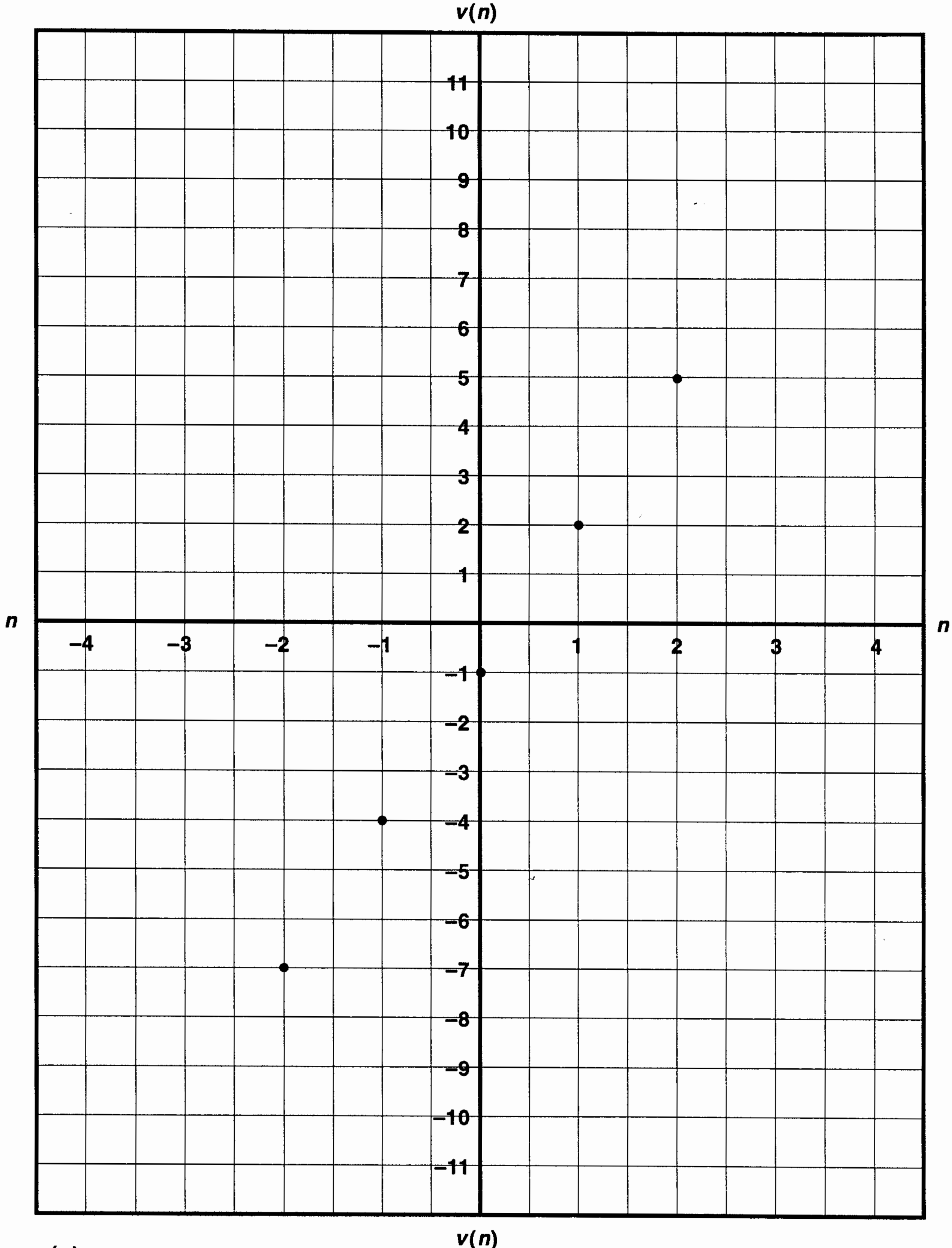
for Graphing Algebraic Relationships

The Math Learning Center
PO Box 3226
Salem, Oregon 97302

Catalog #MET11

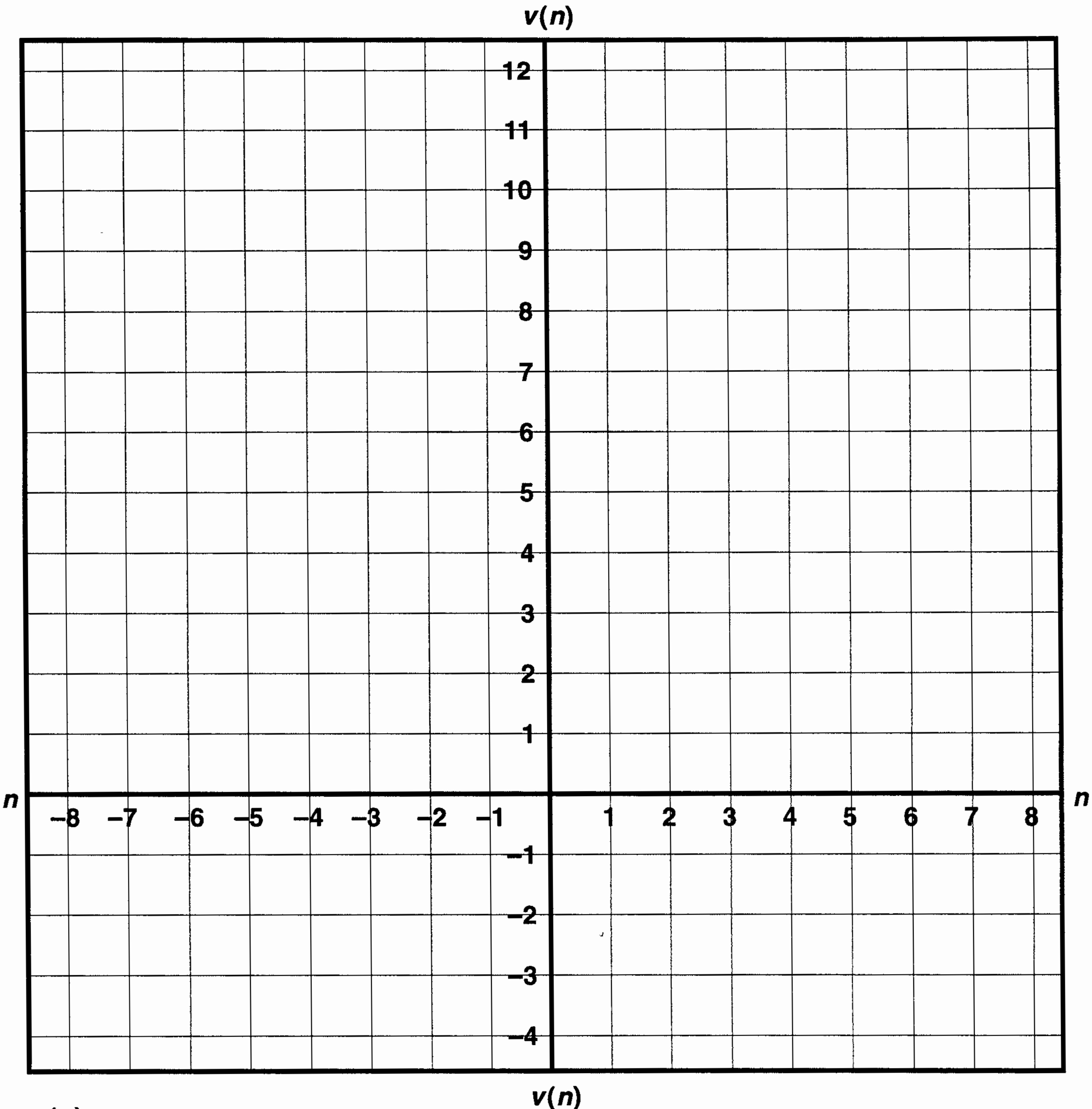
Name ____________________

Activity Sheet XI-1-A



$v(n) =$

Name ______________________________

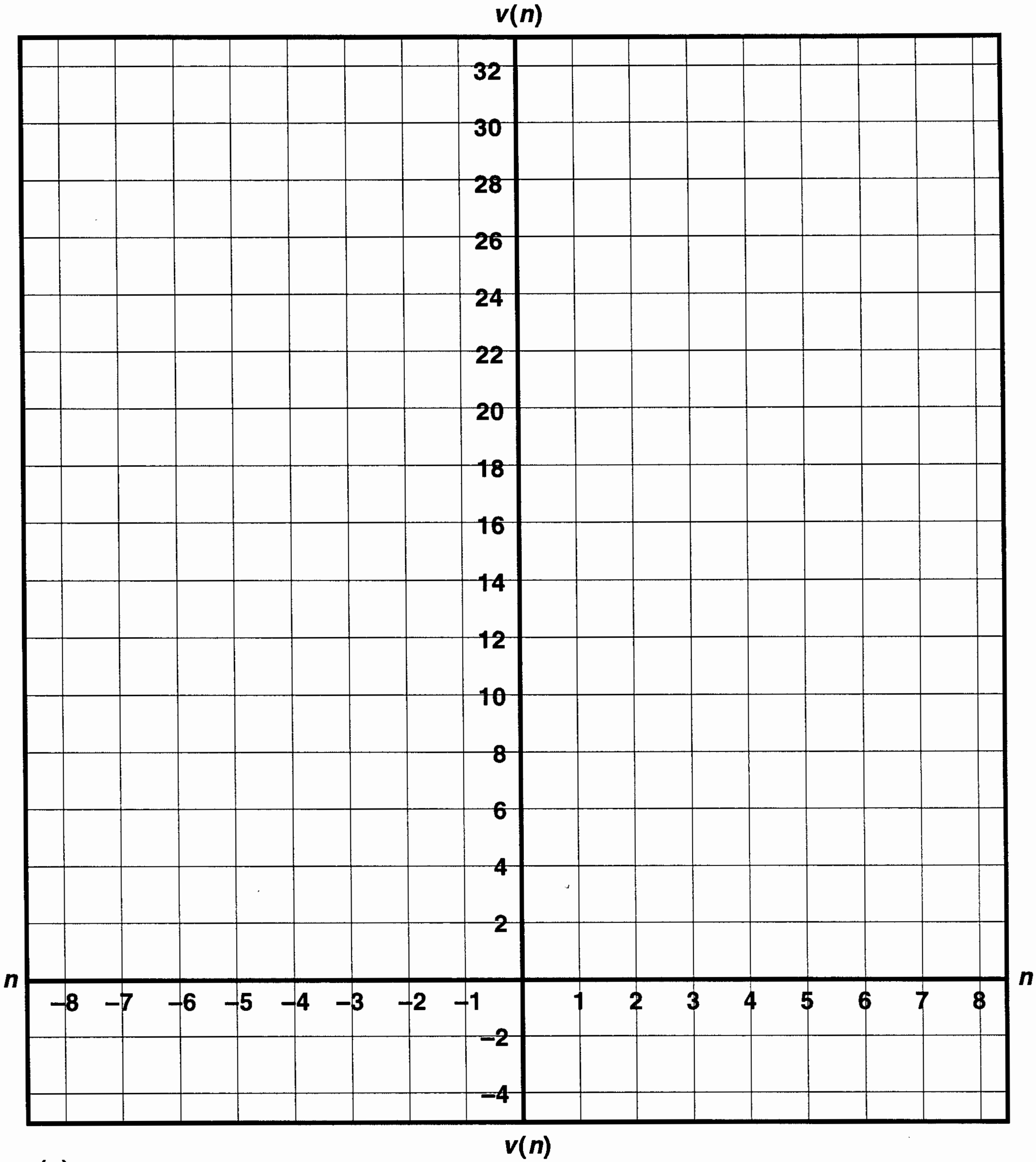


***v*(*n*) =**

Observations about the graph:

Activity Sheet XI-1-C

Name ______________________________

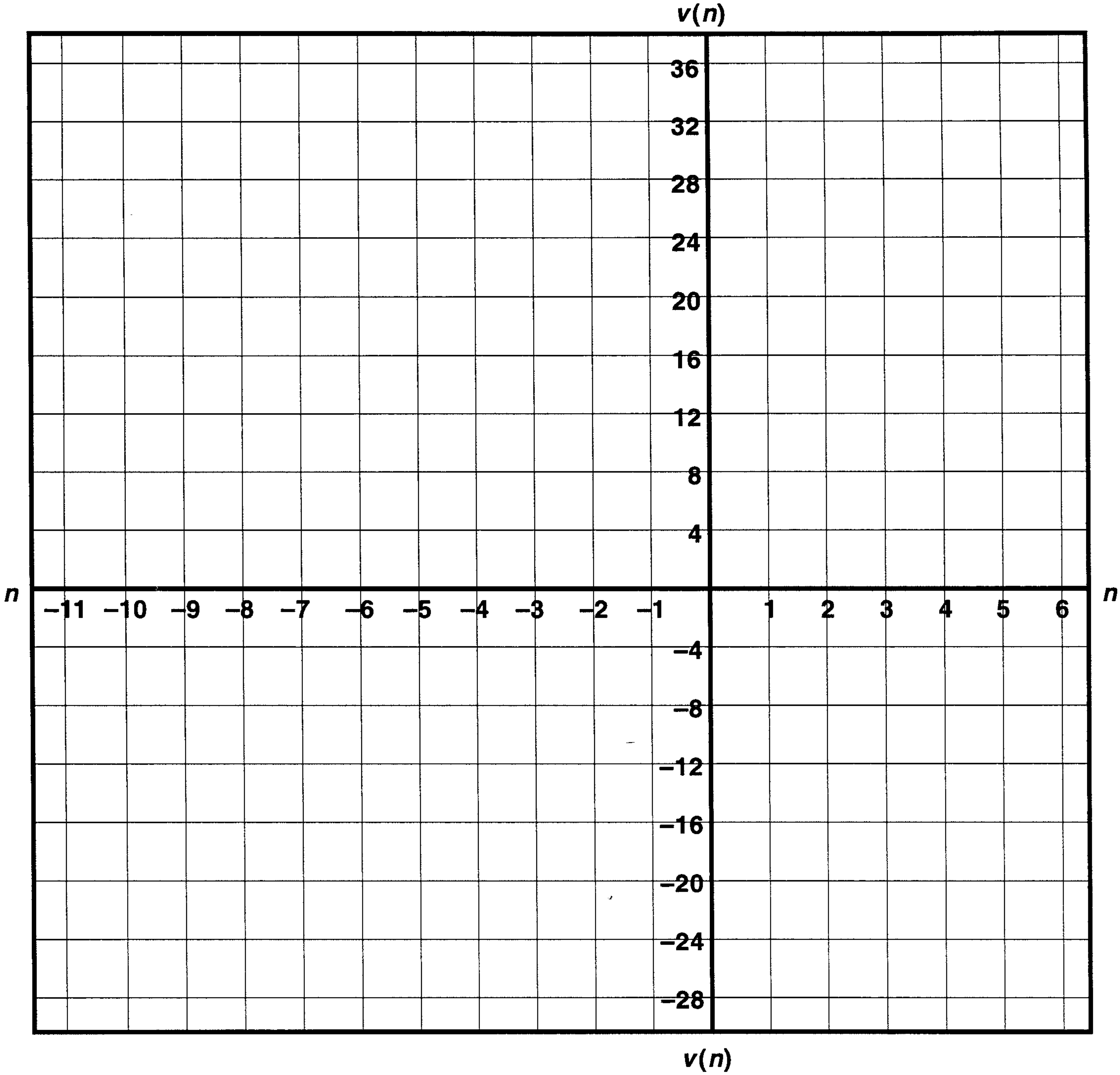


$v(n)$ =

Observations about the graph:

Activity Sheet XI-1-D

Name ______________________________



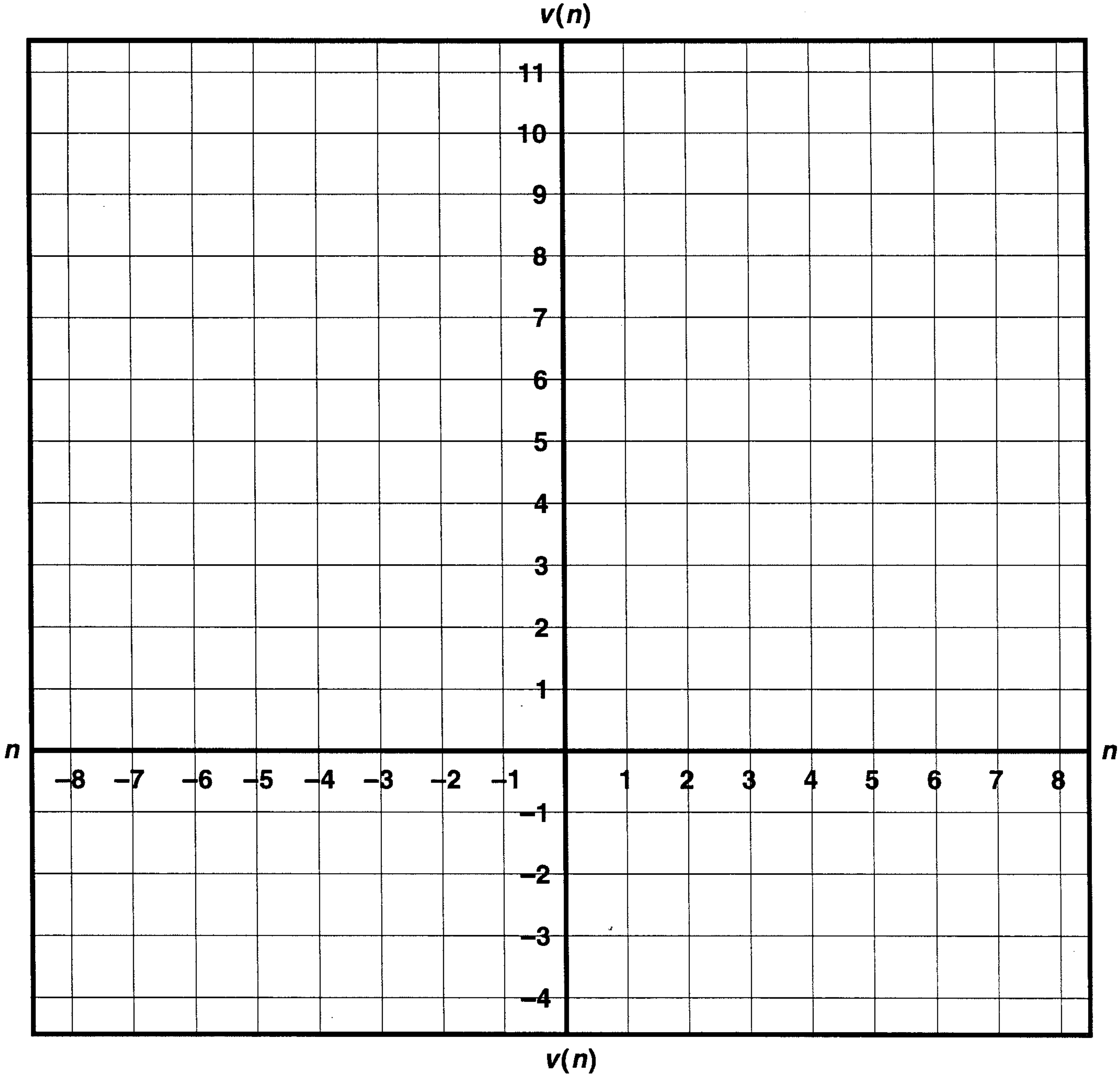
X $v_1(n)$ =

O $v_2(n)$ =

Observations:

Activity Sheet XI-1-E

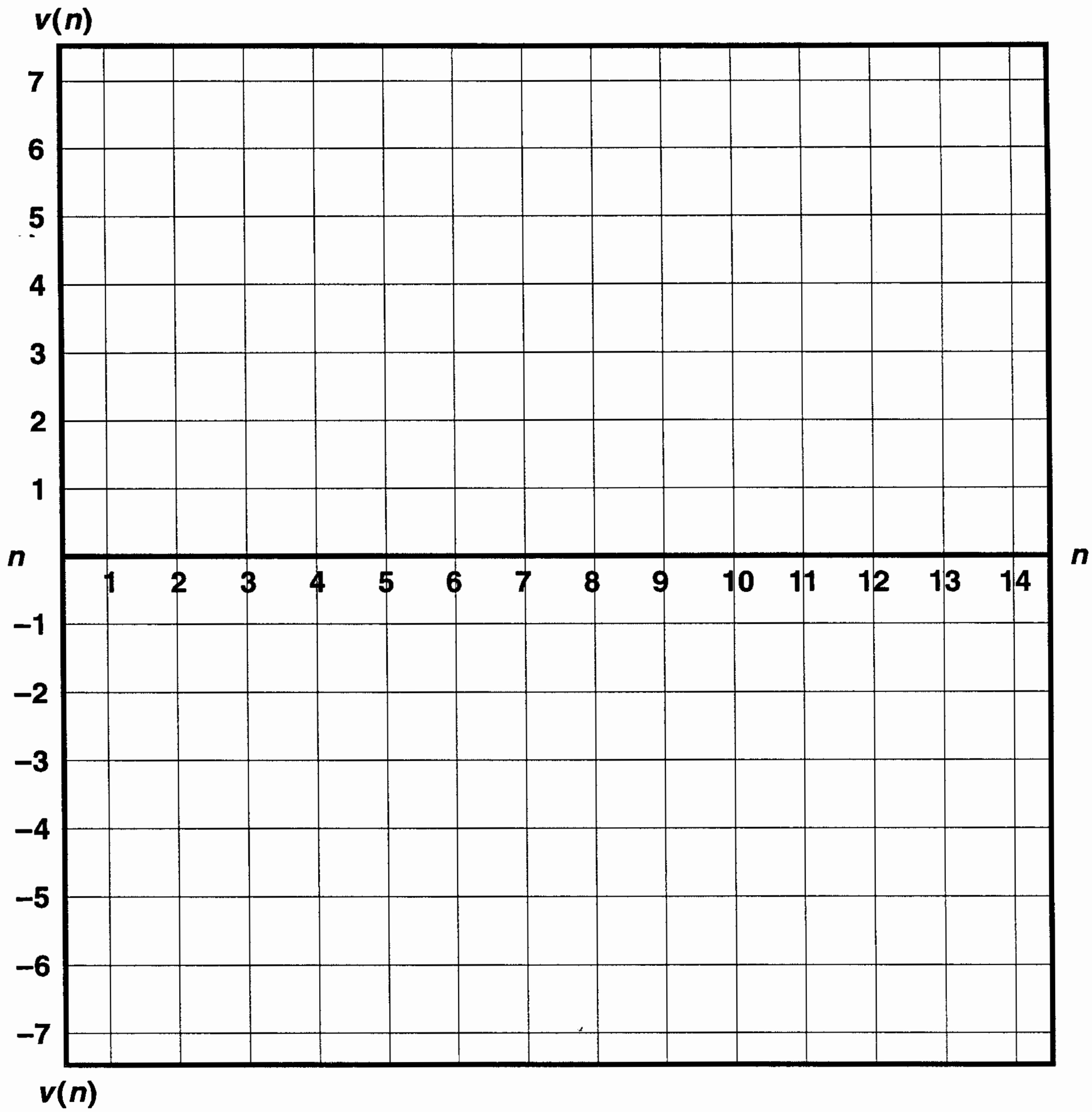
Name ______________________________



X $v_1(n) =$ O $v_2(n) =$

Observations:

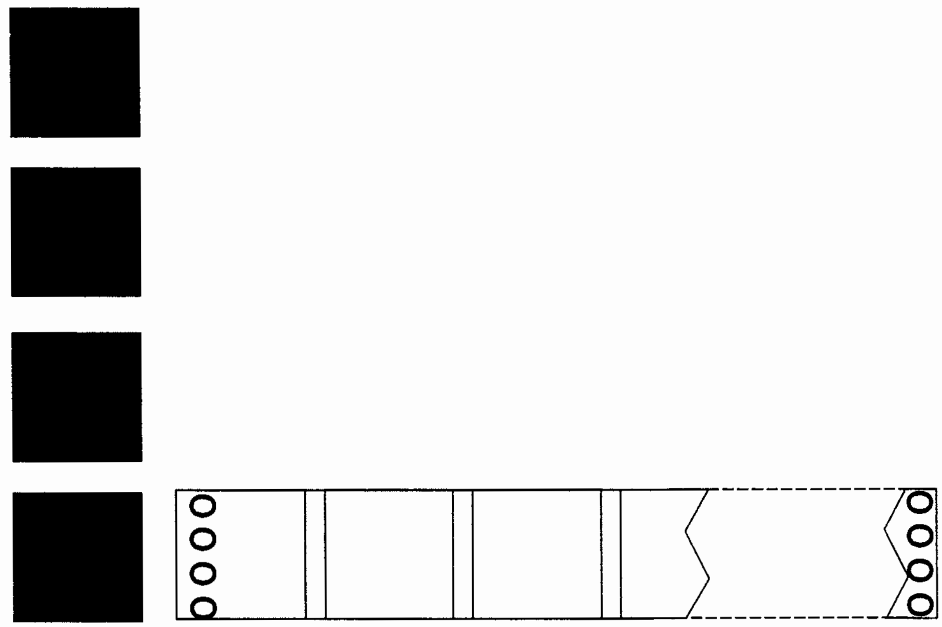
Name ____________________________



X $v_1(n) =$

O $v_2(n) =$

Observations:



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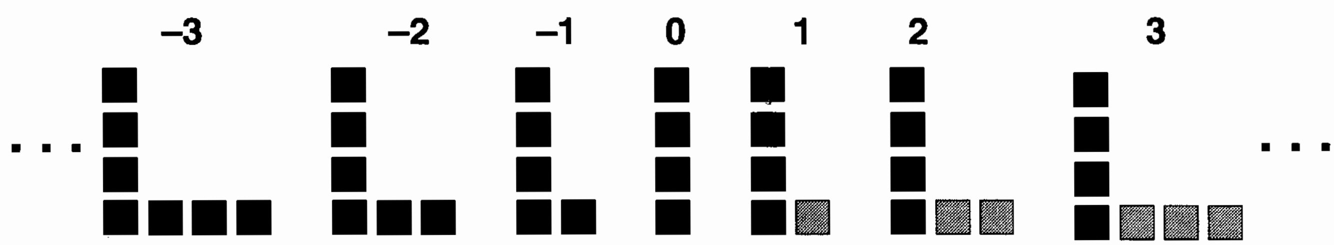
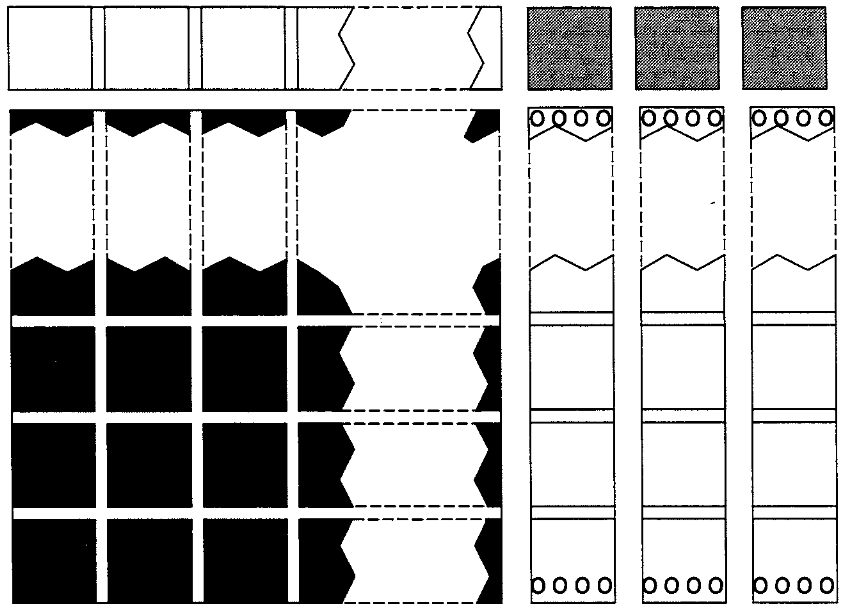
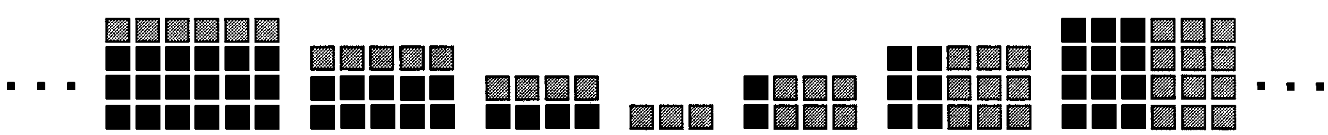


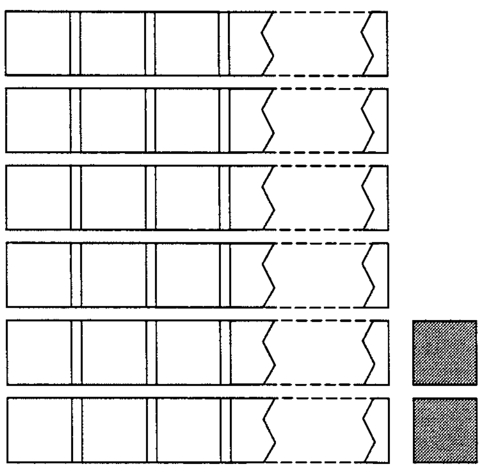


Arrangement number:

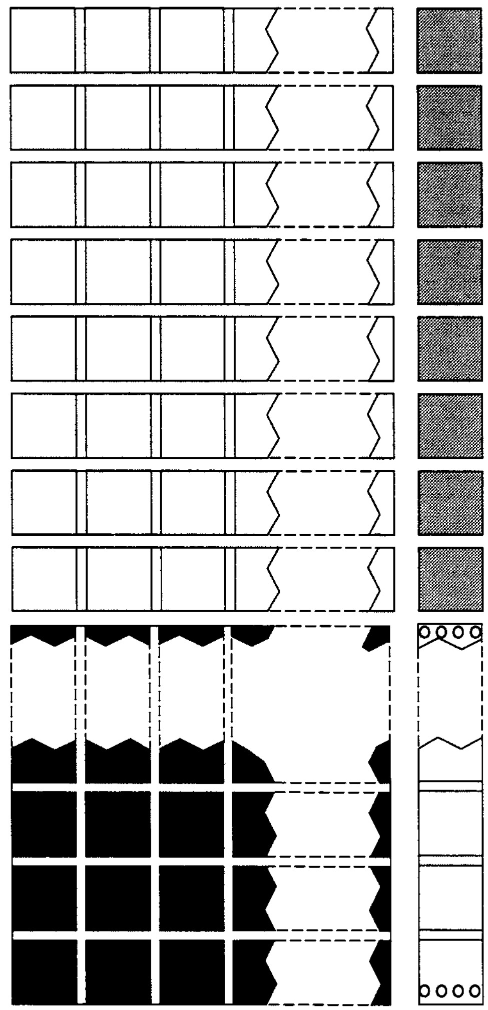
–3 –2 –1 0 1 2 3

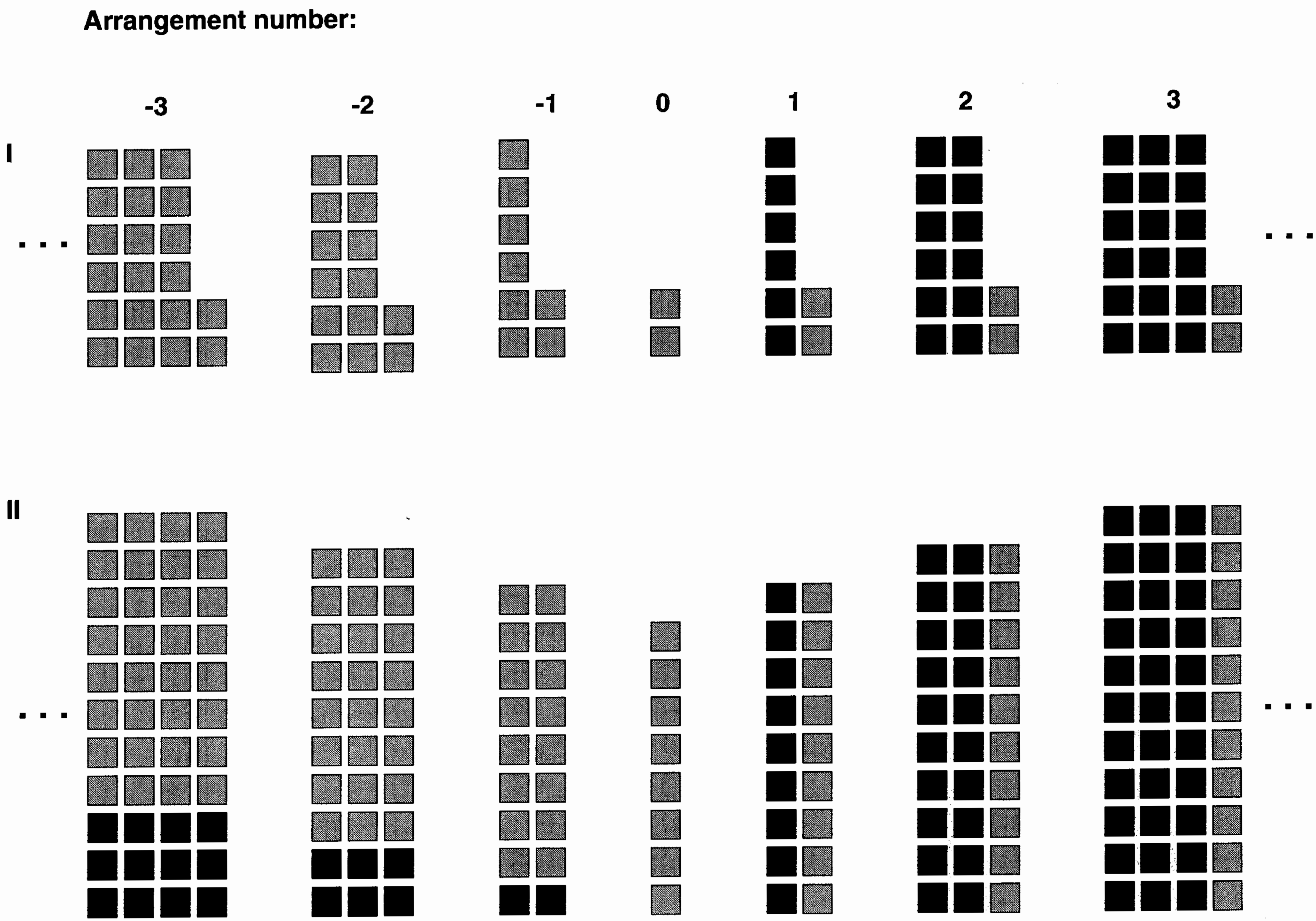


I

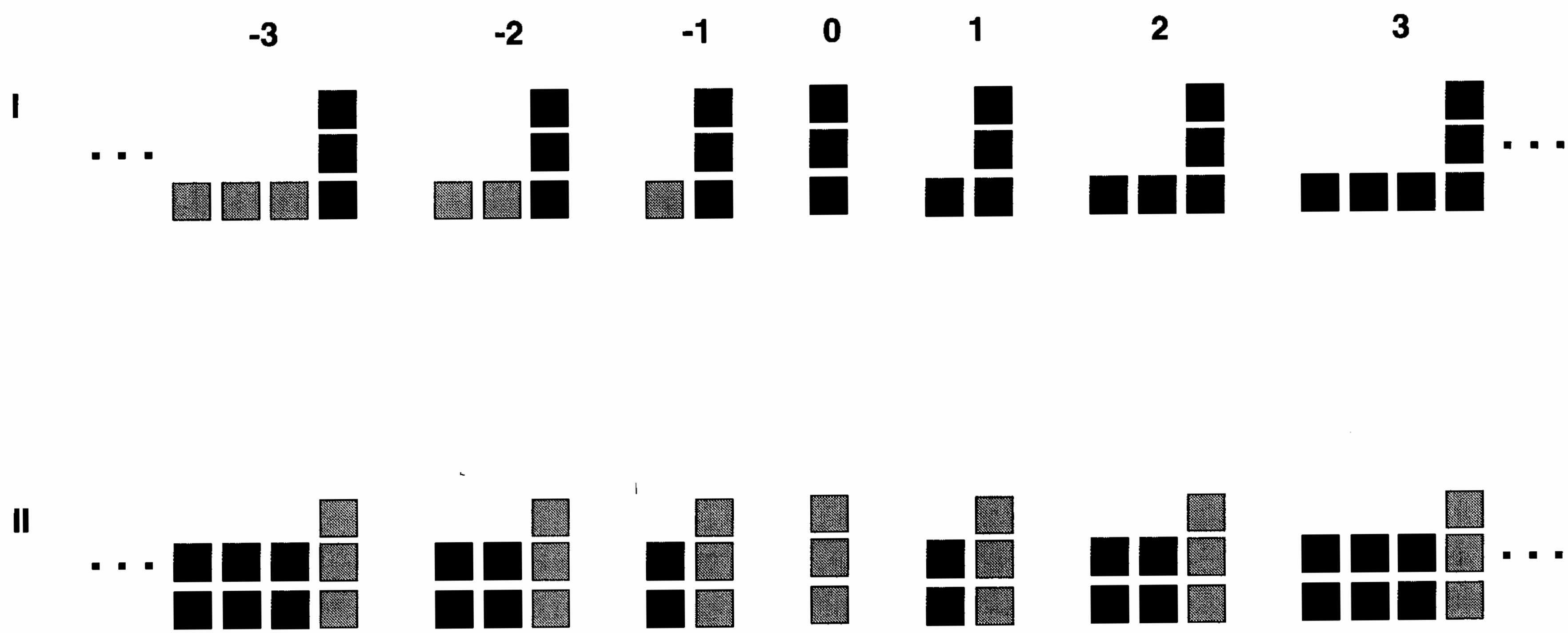


II




Arrangement number:
-3
-2
-1
0
1
2
3
I
II

Arrangement number:



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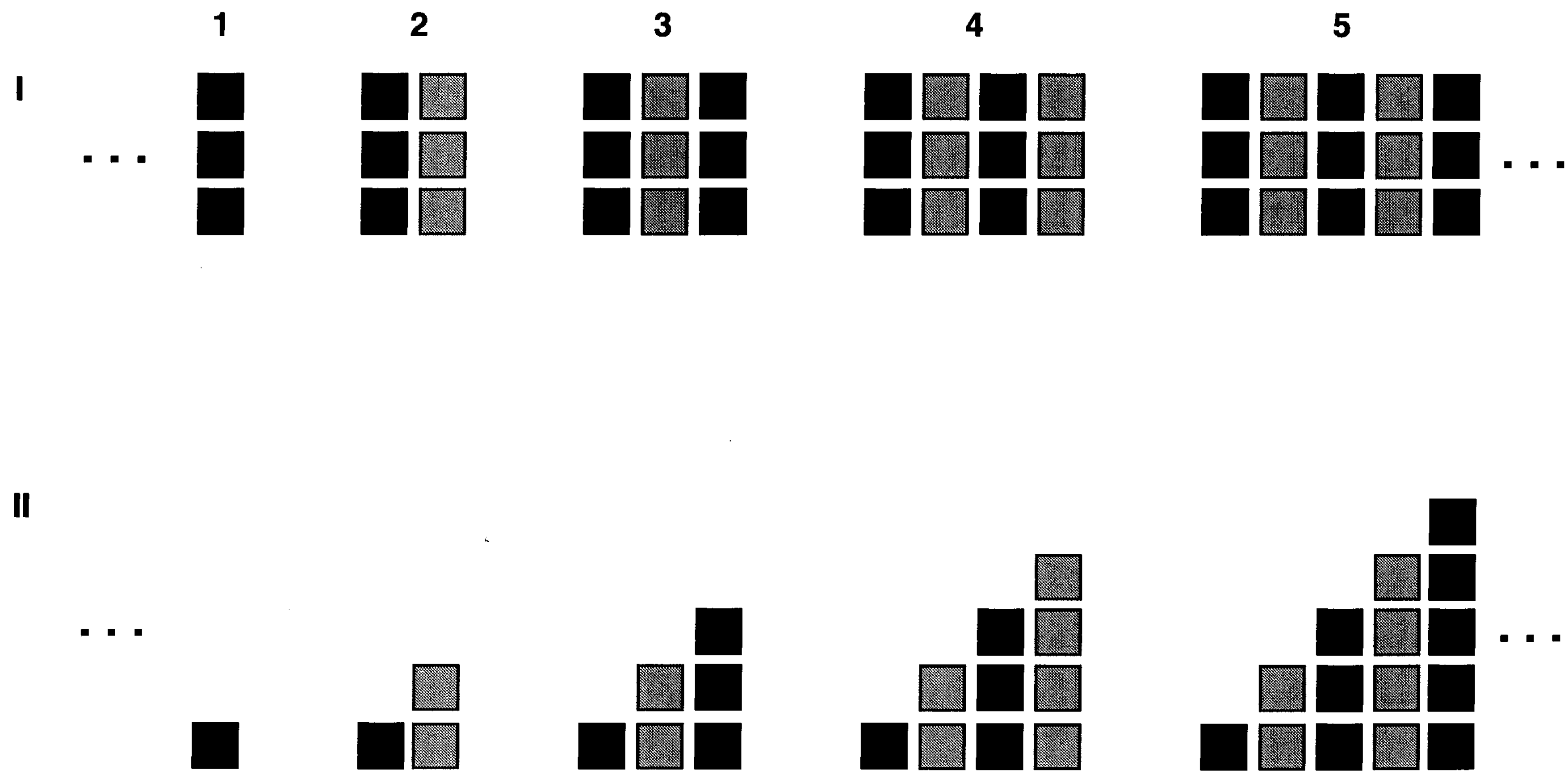
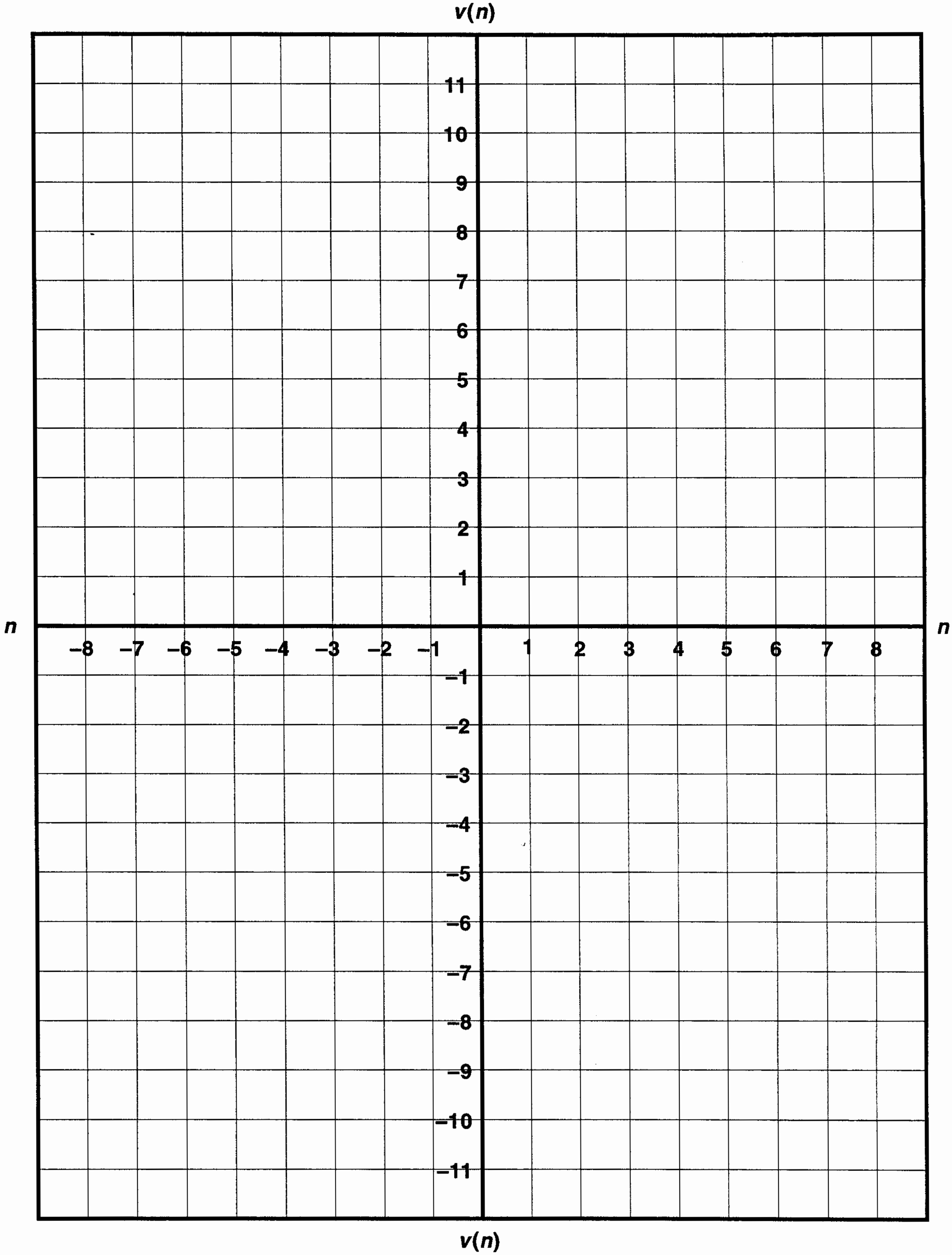
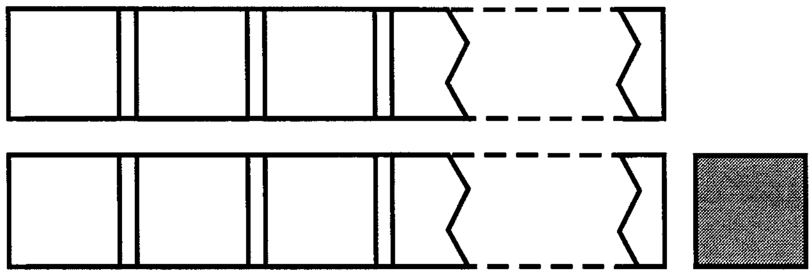
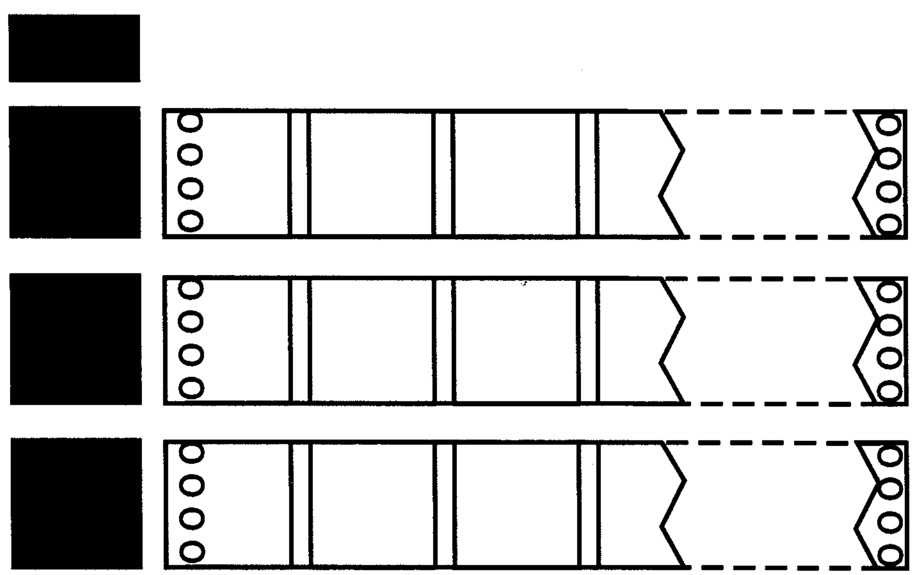


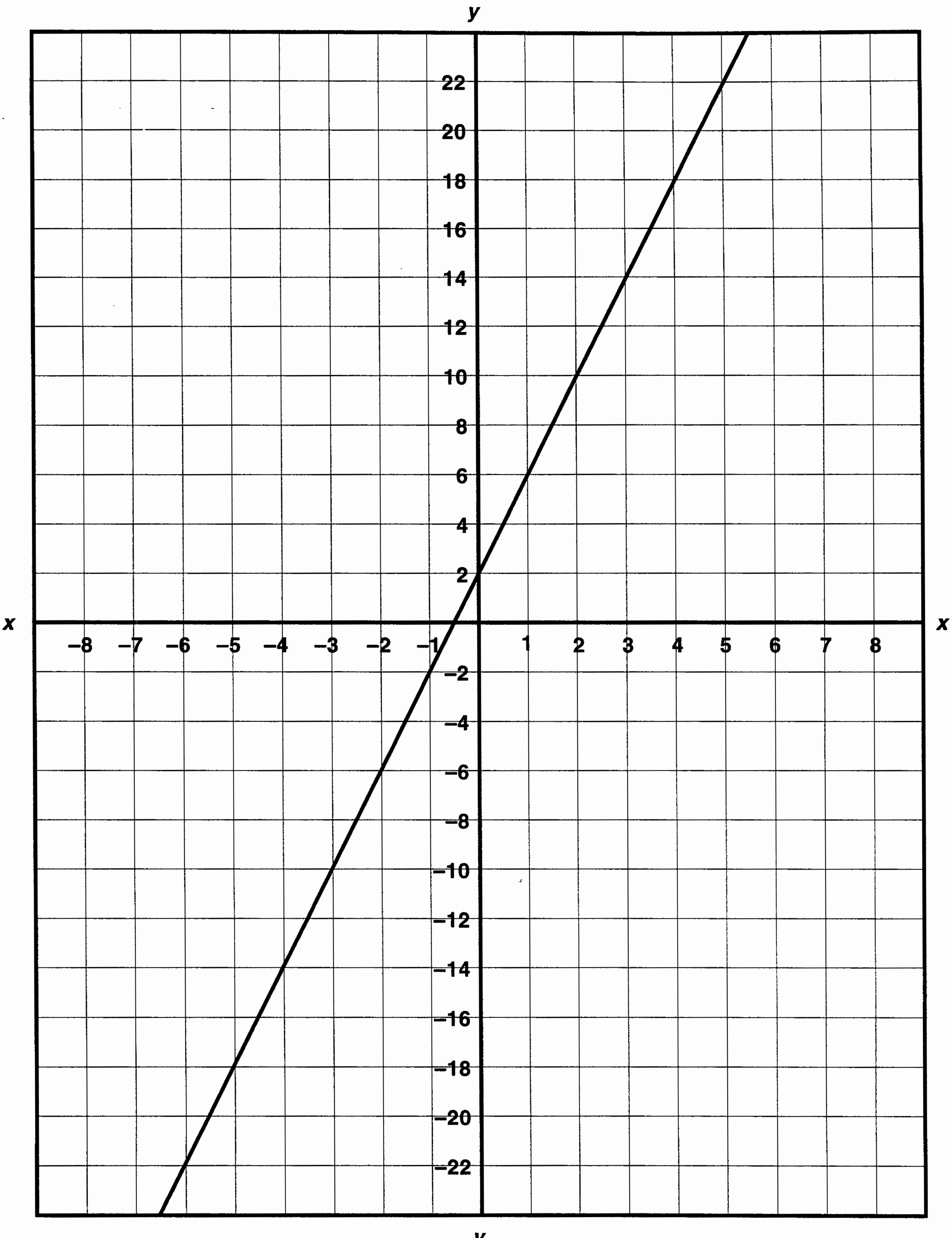
Name ______________________________ *Activity Sheet XI-2*



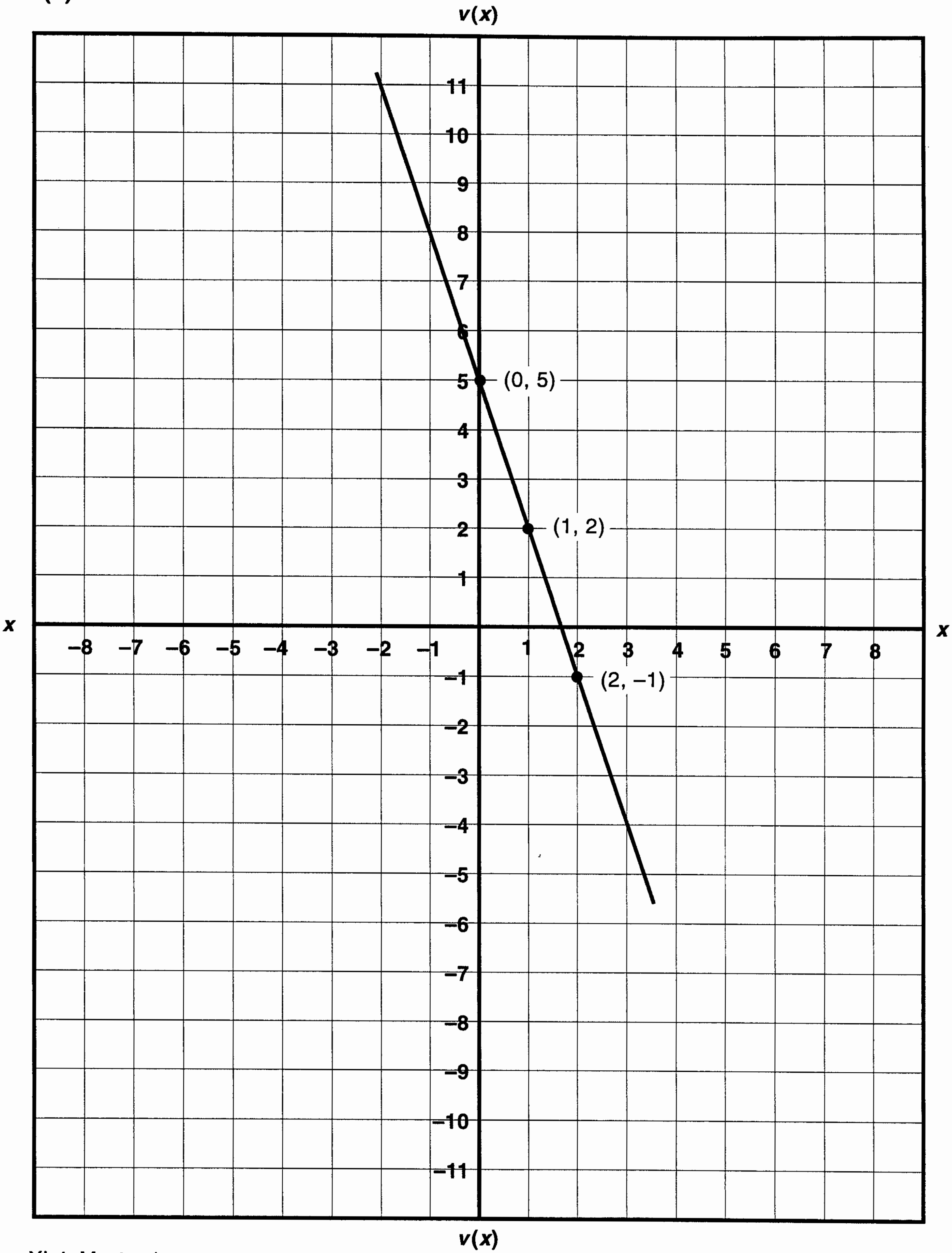




1-cm grid paper


y
22
20
18
16
14
12
10
8
6
4
2
x
−8 −7 −6 −5 −4 −3 −2 −1
1 2 3 4 5 6 7 8
x
−2
−4
−6
−8
−10
−12
−14
−16
−18
−20
−22
y

$v(x) = -3x + 5$



v(x)
x
x
−8 −7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 8
11 10 9 8 7 6 5 4 3 2 1
−1 −2 −3 −4 −5 −6 −7 −8 −9 −10 −11
(0, 5)
(1, 2)
(2, −1)
v(x)

1) $y = -3x + 5$

3) $y = x + 5$

2) $y = -x + 5$

4) $y = 3x + 5$

---

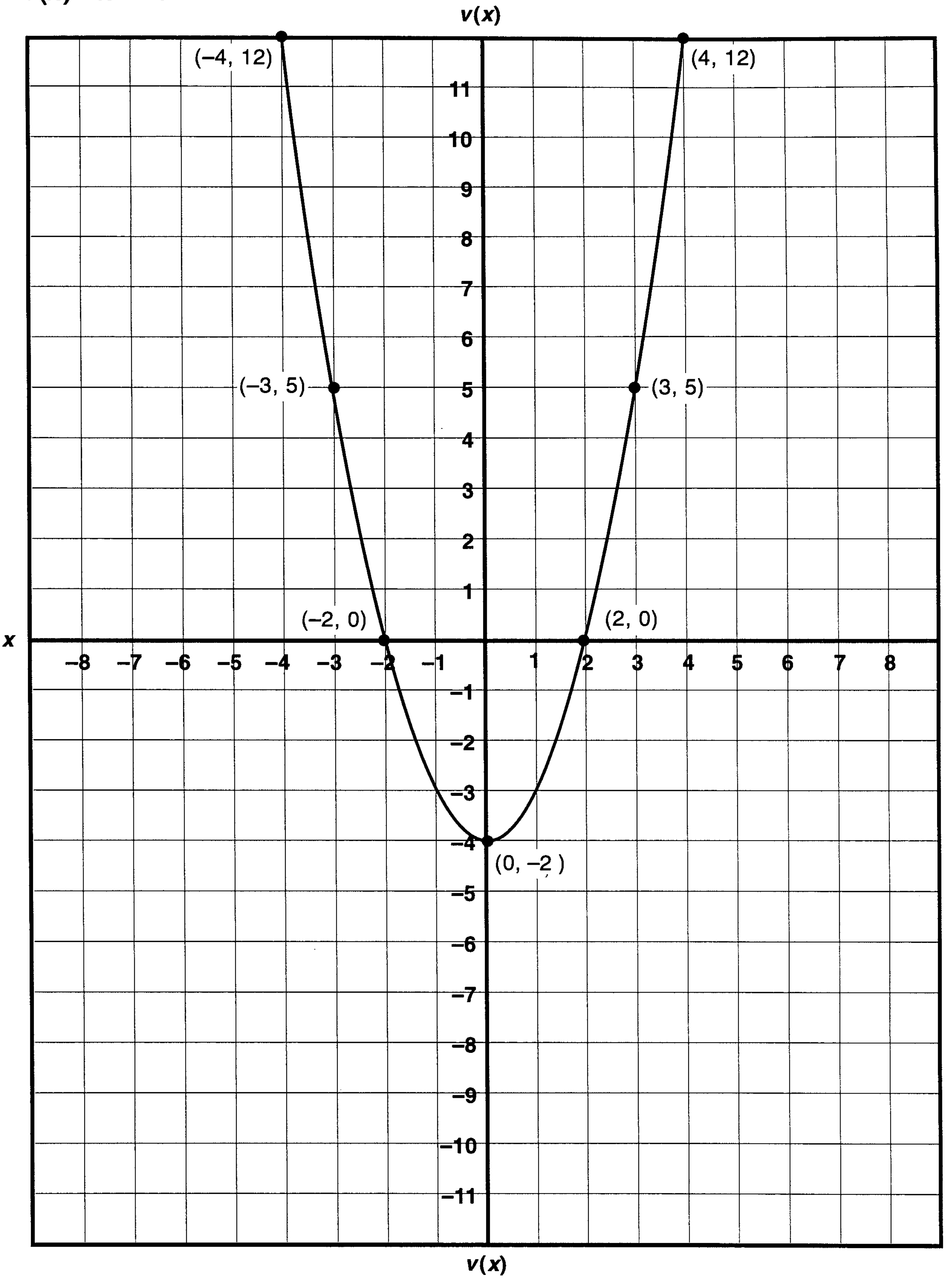
1) $y = -3x + 5$

3) $y = -3x + 2$

2) $y = -3x - 5$

4) $y = -3x - 2$

$v(x) = x^2 - 4$


v(x)
(–4, 12)
(4, 12)
11
10
9
8
7
6
5
4
3
2
1
(–3, 5)
(3, 5)
(–2, 0)
(2, 0)
x
x
–8
–7
–6
–5
–4
–3
–2
–1
1
2
3
4
5
6
7
8
–1
–2
–3
–4
(0, –2)
–5
–6
–7
–8
–9
–10
–11
v(x)

1) $y = x^2 - 6$

3) $y = -(x^2 - 6)$

2) $y = x^2 + 6$

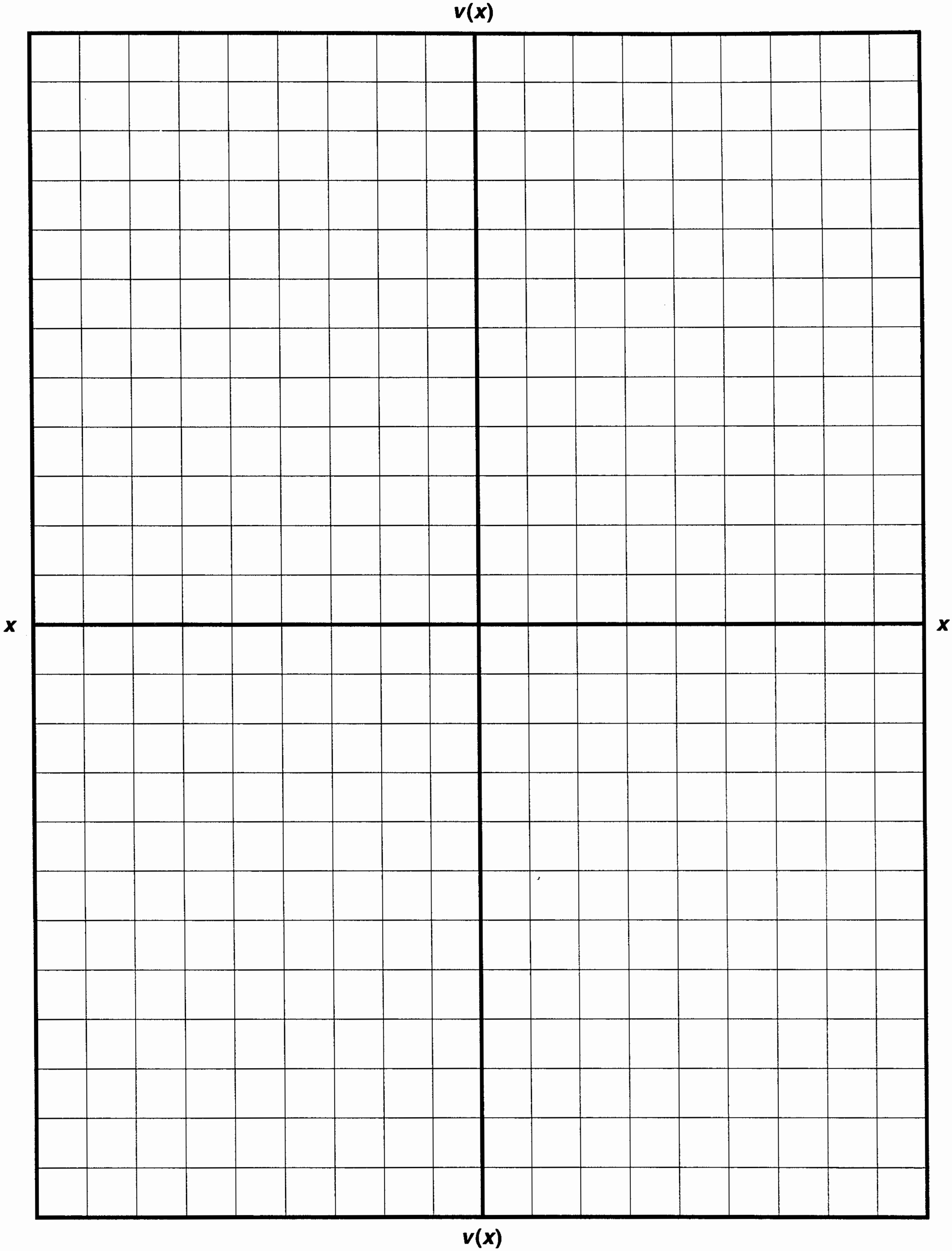
---

1) $y = 4x^2$

3) $y = -4x^2$

2) $y = \frac{1}{4}x^2$

4) $y = -\frac{1}{4}x^2$


v(x)
x
x
v(x)

A) $y_1 = 4 + 2x$ $y_2 = x + 3$

B) $y_1 = 4 - x^2$ $y_2 = -4 + x^2$

C) $y_1 = 3x - 2$ $y_2 = 3x + 1$

D) $y_1 = 2x + 7$ $y_2 = 4x^2 - 3x + 2$

A man and his child are racing one another on a track.

The man can run 20 meters in 3 seconds.

His child can run 20 meters in 5 seconds.

They decide to give the child a 30 meter head start.

Make up some questions based on this situation.

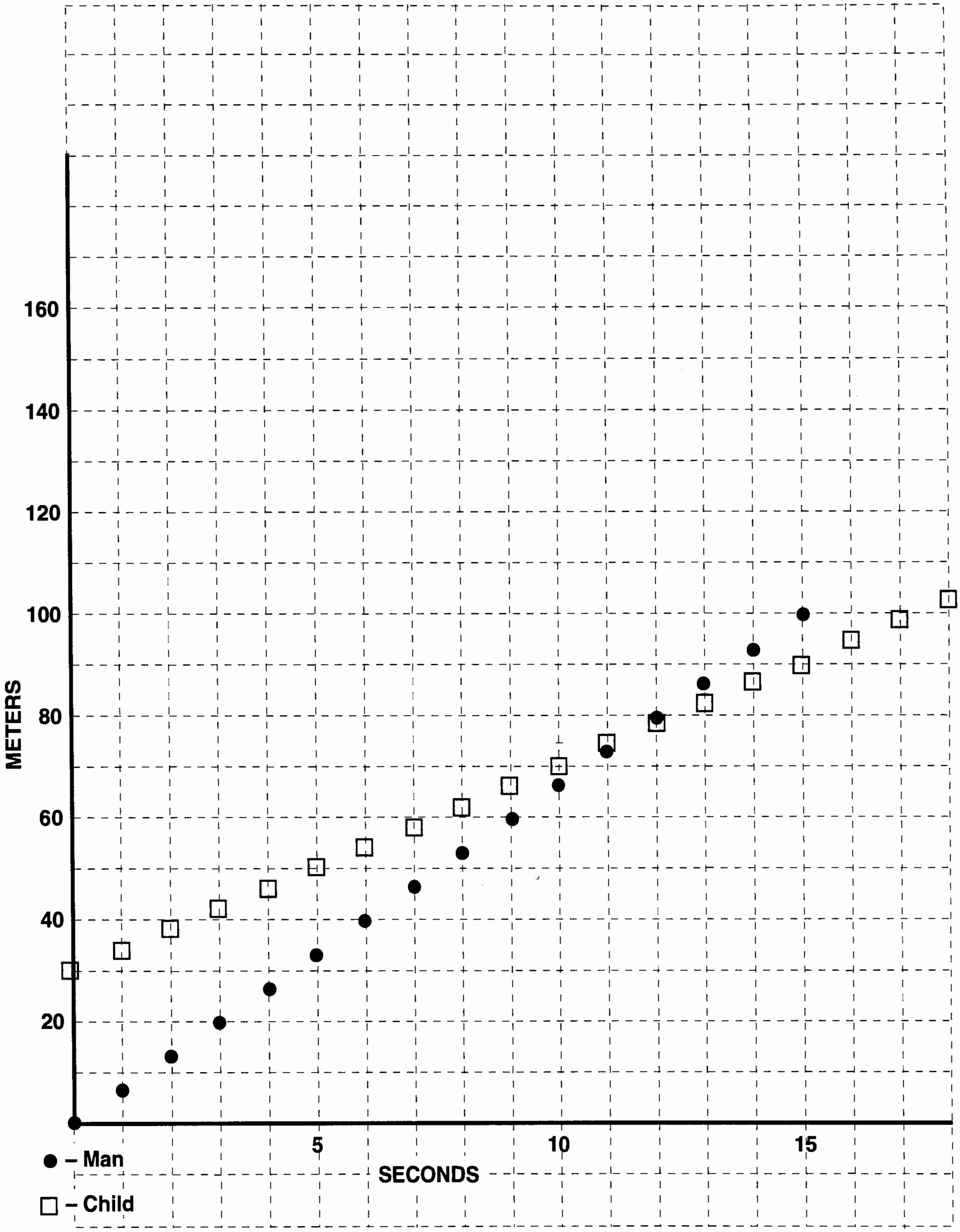
Share and record your questions in your group.

seconds	child meters
0	30
5	30 + 20 = 50
10	70
15	90

seconds	man meters
0	0
3	20
6	40
9	60

seconds	child meters
0	30
1	34
2	38
3	42
⋮	⋮
10	70
11	74
12	78
13	82
14	86
15	90
16	94
17	98
18	102

seconds	man meters
0	0
1	$6\frac{2}{3}$
2	$13\frac{1}{3}$
3	20
⋮	⋮
9	60
10	$66\frac{2}{3}$
11	$73\frac{1}{3}$
12	80
13	$86\frac{1}{3}$
14	$93\frac{2}{3}$
15	100

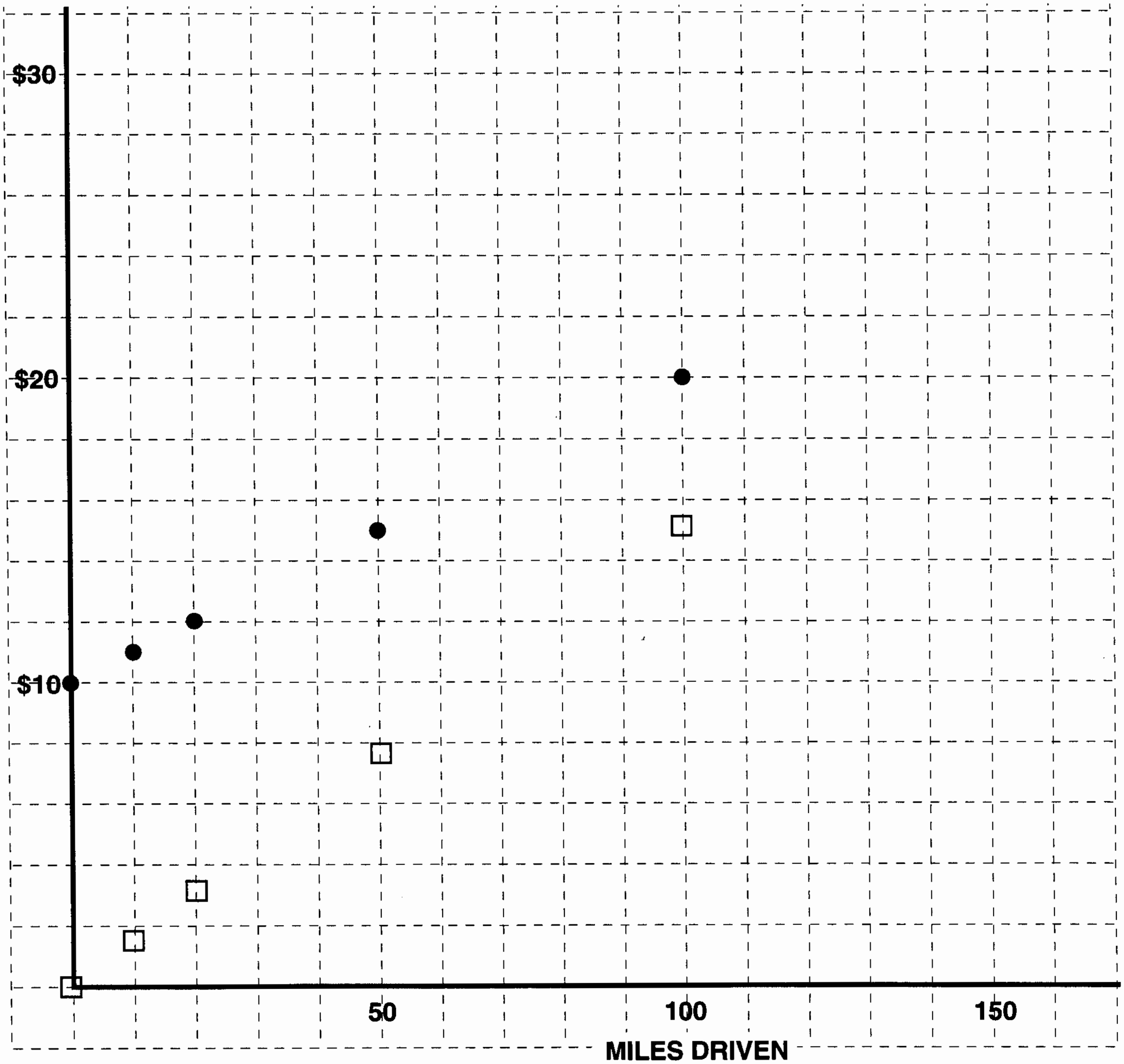

160
140
120
100
80
60
40
20
METERS
5
10
15
SECONDS
● – Man
□ – Child

Miles Driven	Cost RAW	Cost WHT
10	$1.50	$11
20	$3.00	$12
.	.	.
.	.	.
.	.	.
50	$7.50	$15
.	.	.
.	.	.
.	.	.
100	$15.50	$20
.	.	.
.	.	.
.	.	.

Graph Key

● – Cost of WHT

□ – Cost of RAW



seconds x	height in feet $y_1 = 80x - 16x^2$	elevated tee height in feet $y_2 = 80x - 16x^2 + 20$
0	0	20
0.5	36	56
1.0	64	84
1.5	84	104
2.0	96	116
2.5	100	120
3.0	96	116
3.5	84	104
4.0	64	84
4.5	36	56
5.0	0	20
5.5	–44	–24

Name ____________________________

Pose some questions about each of the following situations and then answer them using any approach that you wish. Write up a problem solving summary for each situation.

1) The Rent-a-Wreck (RAW) and the We Hardly Try (WHT) car rental companies are competing for business and change their prices as follows:

WTH has an initial rental charge of $10 and then charges $.10/mile.

RAW eliminates the initial rental charge altogether but charges $.15/mile.

2) The Saucy Pizza company is currently charging $7 for their pizzas. The ingredients and labor for each pizza they make cost $2.50. Each day that the company operates costs them $100 in overhead (lights, water, heat, rent, etc.).

3) The U-Drive-It golf range claims that the height (H) in feet above the ground of a golf ball at (T) seconds after their pro hits it can be determined from the expression

$$H = 80T - 16T^2$$

They also claim that when the ball is hit from their elevated loft tees that the height can be determined by

$$H = 80T - 16T^2 + 20$$

1-cm grid paper